Review 5A

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1. For the system $\begin{array}{rl} x+2y & = 5 \\ x \quad -4z & = -11. \\ 2x+7y+z & = 19 \end{array}$

   (a) Write the system as an augmented matrix.

   (b) Use the Gauss-Jordan elimination method on the augmented matrix to show that the solution is $(1, 2, 3)$.

2. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & -2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$, evaluate the following.

   (a) $\mathbf{AB}$

   (b) $\mathbf{BA}$

   (c) $\mathbf{C} + \mathbf{A}$

   (d) $\mathbf{C} - \mathbf{B}$

   (e) $\mathbf{CB}$

   (f) $\mathbf{BC}$

3. Use augmented matrices to show that the inverse of $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is $\begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$.

4. Write $\begin{cases} z = 8 \\ -x+y-z = -100 \\ 2x-y+z = 50 \end{cases}$ as a matrix equation and use the result of the previous problem to solve.

5. Find the determinant $\begin{vmatrix} 1 & 6 & 1 & 0 \\ 3 & 0 & 4 & x \\ 5 & 5 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix}$.

6. Use Cramer's rule to solve the system $\begin{cases} x-3y-z = 1 \\ 2x+y+2z = 0 \\ x-y-3z = 4 \end{cases}$ for x.

7. If $a_n = -3a_{n-1}$ for $n > 1$ and $a_1 = 2$, then *(a)* Find the first four terms and *(b)* Find the general term.

8. If a_n is a arithmetic sequence with $a_5 = 11$ and $a_8 = 20$, what are the first four terms of the sequence?

9. Find the sum of the integers from 1 to one million.

10. Evaluate $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots + \frac{1}{4^{12}}$.

11. Evaluate $\dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \cdots$.

Answers to problems above: 1. in class 2. (a) $\mathbf{AB} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ -2 & 4 \end{bmatrix}$, (b) $\mathbf{BA}$ DNE, (c) $\mathbf{C} + \mathbf{A}$ DNE, (d) $\mathbf{C} - \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix}$, (e) $\mathbf{CB} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$, (f) $\mathbf{BC} = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ 3. $\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \overset{(1)\leftrightarrow(3)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \overset{(2)\leftrightarrow(3)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \end{array}\right] \overset{-1(1)+(3)}{\Longrightarrow}$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & -1 \end{array}\right] \overset{-2(2)+(3)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & -1 \end{array}\right] \overset{-1(3)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{array}\right] \overset{-1(3)+(2)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{array}\right]$

4. $\begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -100 \\ 50 \end{bmatrix}$; So $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ -100 \\ 50 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 8 \\ -100 \\ 50 \end{bmatrix} = \begin{bmatrix} -50 \\ -142 \\ 8 \end{bmatrix}$ 5. $5x + 115$

6. $x = \frac{21}{22}$ 7. *(a)* $a_1 = 2$, $a_2 = -6$, $a_3 = 18$, $a_4 = -54$, *(b)* $a_n = 2(-3)^{n-1}$ 8. 2, 5, 8, 11 9. $500{,}000{,}500{,}000$ 10. $\frac{1}{3}\left(1 - \frac{1}{4^{12}}\right)$ 11. $\frac{1}{2}$

Review 5B

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1. For the system $\begin{aligned} x+3y+\ z&=10 \\ x+\ y+\ z&=\ 6. \\ x+8y+2z&=23 \end{aligned}$

   (a) Write the system as an augmented matrix.

   (b) Use the Gauss-Jordan elimination method on the augmented matrix to show that the solution is $(1, 2, 3)$.

2. If $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & -2 & 1 \\ -3 & 1 & 1 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, evaluate the following.

   (a) $\mathbf{AB}$

   (b) $\mathbf{BA}$

   (c) $\mathbf{C} + \mathbf{A}$

   (d) $\mathbf{C} - \mathbf{B}$

   (e) $\mathbf{CA}$

   (f) $\mathbf{2A} - \mathbf{CA}$

3. Use augmented matrices to show that the inverse of $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ is $\begin{bmatrix} 1 & -1 & -3 \\ 0 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}$.

4. Write $\begin{cases} 2x+3y+z=-10 \\ x \quad\ \ +z=\ \ \ 4 \\ \quad\ \ y \quad\ =\ \ 40 \end{cases}$ as a matrix equation and use the result of the previous problem to solve.

5. Find the determinant $\begin{vmatrix} 1 & 0 & 4 & -1 \\ 2 & 1 & x & 0 \\ 0 & 2 & 0 & 1 \\ 3 & 0 & 4 & 2 \end{vmatrix}$.

6. Use Cramer's rule to solve the system $\begin{cases} x+3y+\ z=0 \\ x+\ y+2z=3 \\ 2x-\ y+3z=4 \end{cases}$ for z.

7. If $a_n = a_{n-1} + 6$ for $n > 1$ and $a_1 = -5$, then *(a)* Find the first four terms and *(b)* Find the general term.

8. If a_n is a geometric sequence with $a_3 = 1$ and $a_6 = 8$, what are the first four terms of the sequence?

9. Evaluate $\displaystyle\sum_{k=1}^{1000} 2k - 3$

10. Evaluate $3 + 6 + 12 + \cdots + 3 \cdot 2^{400}$.

11. Evaluate $\dfrac{4}{5} + \dfrac{4}{25} + \dfrac{4}{125} + \cdots$.

Answers to problems above: 1. in class 2. (a) $\mathbf{AB} = \begin{bmatrix} -3 & -1 & 2 \\ -3 & 1 & 1 \end{bmatrix}$, (b) $\mathbf{BA}$ DNE, (c) $\mathbf{C} + \mathbf{A} = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$, (d) $\mathbf{C} - \mathbf{B}$ DNE, (e) $\mathbf{CA} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, (f) $\mathbf{2C} - \mathbf{AC} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 3. $\left[\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \overset{(2)\leftrightarrow(1)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \overset{(2)\leftrightarrow(3)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 2 & 3 & 1 & 1 & 0 & 0 \end{array}\right] \overset{-2(1)+(3)}{\Longrightarrow}$
$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 3 & -1 & 1 & -2 & 0 \end{array}\right] \overset{-3(2)+(3)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & -2 & -3 \end{array}\right] \overset{-1(3)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 2 & 3 \end{array}\right] \overset{-1(3)+(1)}{\Longrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & -3 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 2 & 3 \end{array}\right]$
4. $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -10 \\ 4 \\ 40 \end{bmatrix}$; So $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -10 \\ 4 \\ 40 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ 0 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -10 \\ 4 \\ 40 \end{bmatrix} = \begin{bmatrix} -134 \\ 40 \\ 138 \end{bmatrix}$ 5. $-10x + 56$ 6. $z = \frac{13}{5}$
7. *(a)* $a_1 = -5$, $a_2 = 1$, $a_3 = 7$, $a_4 = 13$, *(b)* $a_n = 6n - 11$ 8. $\frac{1}{4}$, $\frac{1}{2}$, 1, 2 9. 998000 10. $3(2^{401} - 1)$ 11. 1