CHAPTER-7

1. As the mass is not disturbed from its original position (state of rest) there will not be any movement.

2. $m = 2$, $b = 4$, $K = 7$. Hence characteristic equation is,

$$2s^2 + 4s + 7 = 0.$$

$$\therefore s_1 = \frac{-4+\sqrt{16-56}}{4} \quad \& \quad s_2 = \frac{-4-\sqrt{16-56}}{4}$$

$$\therefore s_1 = -1 + i\sqrt{10} \quad \& \quad s_2 = -1 - i\sqrt{10}$$

As the roots are complex, the system is underdamped.

For equation of motion, let,

$$x(t) = C_1 e^{(-1+i\sqrt{10})t} + C_2 e^{(-1-i\sqrt{10})t}$$

$$= C_1 e^{-t} e^{\sqrt{10}\,it} + C_2 e^{-t} e^{-\sqrt{10}\,it}$$

substituting $e^{it} = \cos t + i \sin t$

Hence $e^{\sqrt{10}\,it} = \cos\sqrt{10}\,t + i \sin\sqrt{10}\,t.$

& $e^{-\sqrt{10}\,it} = \cos\sqrt{10}\,t - i\sin\sqrt{10}\,t$

$$\therefore x(t) = C_1 e^{-t}(\cos\sqrt{10}\,t + i\sin\sqrt{10}\,t) + C_2 e^{-t}(\cos\sqrt{10}\,t - i\sin\sqrt{10}\,t)$$

$$= (C_1 + C_2)\, e^{-t}\cos\sqrt{10}\,t + (C_1 - C_2)\, e^{-t}\, i\sin\sqrt{10}\,t$$

$$= C_3\, e^{-t}\cos\sqrt{10}\,t + C_4\, e^{-t}\, i\sin\sqrt{10}\,t$$

Let $C_3 = r\cos\theta$ & $C_4 = r\sin\theta$

~~$\therefore x(t) =$~~ where, $r = \sqrt{C_3^2 + C_4^2}$ & $\theta = \tan^{-1} C_4/C_3$

$$\therefore x(t) = r e^{-t}[\cos\theta\cos\sqrt{10}\,t + i\sin\theta\sin\sqrt{10}\,t]$$

$$= r e^{-t}[\cos(\theta - \sqrt{10}\,t)]$$

For $x(0) = 2.5$ we get,

$$2.5 = r e^0[\cos(\theta - 0)]$$

$$\therefore 2.5 = r\cos\theta$$

Similarly $\dot{x}(0) = 0$. To get expression for $\dot{x}(t)$, differentiate $x(t)$ wrt time.

$$\therefore \dot{x}(t) = -re^{-t}\cos(\theta - \sqrt{10}\,t) + re^{-t}\sqrt{10}\sin(\theta - \sqrt{10}\,t)$$

$$\therefore \dot{x}(0) = 0 = -r\cos\theta + r\sin\theta$$

$$\therefore r\cos\theta = r\sin\theta \Rightarrow \theta = \pi/4 \text{ radians or } 45^\circ$$

& $r = 2.5.$

$\therefore\ x(t) = 2.5\, e^{-t} \cos(\pi/4 - \sqrt{10}\, t)$

~~For~~ $t = 0, 0.1, 0.2, \ldots$ we get following ~~motion~~ positions of the system,

t	$x(t)$
0	2.5
0.1	2.193
0.2	1.799
0.3	1.361
0.4	0.915
0.5	0.490
0.6	0.110
0.7	−0.207
0.8	−0.454
0.9	−0.627

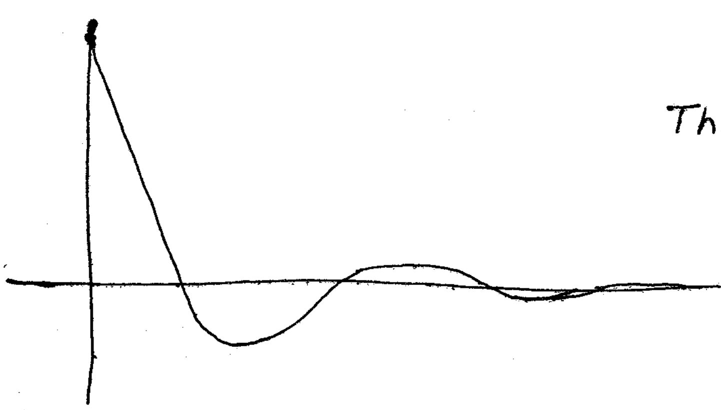
t	$x(t)$
1.0	−0.728
1.1	−0.763
1.2	−0.743
1.3	−0.679
1.4	−0.582
1.5	−0.465
1.6	−0.340
1.7	−0.215
1.8	−0.099
1.9	0.002

t	$x(t)$
2.0	0.085
2.1	0.148
2.2	0.189
2.3	0.211
2.4	0.215
2.5	0.205
2.6	0.183
2.7	0.153
2.8	0.119
2.9	0.083

t	$x(t)$
3.0	0.049
3.1	0.017
3.2	−0.010
3.3	−0.031
3.4	−0.046
3.5	−0.056
3.6	−0.060
3.7	−0.060
3.8	−0.056
3.9	−0.049

t	$x(t)$
4.0	−0.040
4.1	−0.030
4.2	−0.020
4.3	−0.011
4.4	−0.002
4.5	0.005
4.6	0.010
4.7	0.014
4.8	0.016
4.9	0.017

t	$x(t)$
5.0	0.017
5.1	0.015
5.2	0.013
5.3	0.010
5.4	0.007
5.5	0.005
5.6	0.002
5.7	−0.0002
5.8	−0.002
5.9	−0.003



This is behavior of the system

3. $m = 2,\ b = 9,\ K = 8$ hence characteristic equation of the system is

$$2s^2 + 9s + 8 = 0$$

$$\therefore \quad s_1 = \frac{-9 + \sqrt{81 - 64}}{4} \quad \& \quad s_2 = \frac{-9 - \sqrt{81-64}}{4}$$

$$\therefore \quad s_1 = \frac{-9 + 4.123}{4} \quad \& \quad s_2 = \frac{-9 - 4.123}{4}$$

$$\therefore \quad s_1 = -1.21925 \quad \& \quad s_2 = -3.28075$$

As the roots are real and unequal, the system is overdamped

$$x(t) = C_1 e^{-1.21925t} + C_2 e^{-3.28075t}$$

$$\& \ \dot{x}(t) = -1.21925\, C_1 e^{-1.21925t} - 3.28075\, C_2 e^{-3.28075t}$$

At $t = 0$, $x(0) = -1.5$ & $\dot{x}(0) = 0$.

$$\therefore \quad -1.5 = C_1 + C_2$$

$$\& \quad 0 = -1.21925\, C_1 - 3.28075\, C_2$$

$$\therefore \quad C_1 = -\frac{3.28075}{1.21925} C_2 = 2.6908\, C_2$$

$$\therefore \quad -1.5 = 3.6908\, C_2 \Rightarrow C_2 = -0.4064$$

$$\& \quad C_1 = -1.0935.$$

$$\therefore \quad x(t) = -1.0935\, e^{-1.21925t} - 0.4064\, e^{-3.28075t}$$

At $t = 0, 0.1, 0.2, \ldots$ following positions will be attained

t	x(t)	t	x(t)	t	x(t)
0	−1.5	1.0	−0.338	2.0	−0.096
0.1	−1.261	1.1	−0.297	2.1	−0.085
0.2	−1.068	1.2	−0.261	2.2	−0.075
0.3	−0.910	1.3	−0.230	2.3	−0.066
0.4	−0.781	1.4	−0.202	2.4	−0.059
0.5	−0.673	1.5	−0.179	2.5	−0.052
0.6	−0.583	1.6	−0.158	2.6	−0.046
0.7	−0.507	1.7	−0.139	2.7	−0.041
0.8	−0.442	1.8	−0.123	2.8	−0.036
0.9	−0.386	1.9	−0.109	2.9	−0.032