

Exam 1

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Use the paper you feel most comfortable with to answer all questions. No answer should appear on this exam sheet. You are not allowed to leave or use the restroom until the end of the exam period.

1. (20 points) Answer the following either **True** or **False**.
 - (a) For any function $f(x)$, if $\lim_{x \rightarrow a} f(x) = c$, then $f(a) = c$.
 - (b) For any function $f(x)$, if $\lim_{x \rightarrow a} f(x)$ exists, then $f(a)$ exists.
 - (c) If $f(x)$ is continuous at $x = a$, then $f'(a)$ exists.
2. (15 points) Use the Intermediate Value Theorem to show that

$$x^2 \tan\left(\frac{\pi x}{4}\right) = \frac{1}{2}$$

for some x in $[0, 1]$ (hint, $\sin(\pi/4) = \cos(\pi/4)$).

3. (25 points) Find the derivatives of the following functions. Be sure to state which of the basic derivative laws that you use.

(a)

$$\frac{1}{1+x^2}$$

(b)

$$\frac{e^x + e^{-x}}{2}$$

(c)

$$(x^2 + 1)(1 + e^x)$$

(d)

$$\frac{x^2 + 1}{x + 1}$$

(e)

$$\frac{\sqrt{x}}{x + 1}$$

4. (25 points) Evaluate the following limits

(a)

$$\lim_{x \rightarrow a} (x^2 + 2)$$

(b)

$$\lim_{x \rightarrow 0} \frac{\tan(x)}{x}$$

(c)

$$\lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{x + 1}$$

5. (15 points) Use the **formal definition** of the derivative to find the derivative of the function x^3 (hint: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$).
6. (5 points) **Bonus, do not attempt until last!** Use the definition of the derivative to find the derivative of $\sin(x)$.