Tabular Integration – A Special Case of Integration By Parts

Rather than applying several applications of integration by parts, a shortcut method exists for evaluating integrals of the form:

$$\int polynomial \times \begin{Bmatrix} \text{sinusoid} \\ \text{exponential} \end{Bmatrix} dx$$

In this method the polynomial is referenced as the function f and the sinusoid or exponential as g. The method proceeds by constructing a table of two columns. In the first column we place f and the derivatives of f, until we reach 0. In the second column we place g and the derivatives of g, until we reach the 0 row of the previous column. These derivatives and integrals are then paired with one another through multiplication in an offset manner. Finally, alternately adding and subtracting these pairs we have the integral sought.

For example, consider: $\int x^2 \sin(2x)dx$.

Again this integral could be done by parts, but would require two applications. Instead we assign $f = x^2$ and $g = \sin(2x)$ and construct the following table:

	f and its derivatives	g and its integrals
+	x^2	$\sin(2x)$
-	$2x$	$\frac{-1}{2}\cos(2x)$
+	2	$\frac{-1}{4}\sin(2x)$
	0	$\frac{1}{8}\cos(2x)$

Thus from this table we can easily determine:

$$\int x^2 \sin(2x)dx = x^2 \cdot \frac{-1}{2}\cos(2x) - 2x \cdot \frac{-1}{4}\sin(2x) + 2 \cdot \frac{1}{8}\cos(2x) + C$$

$$= \frac{-x^2}{2}\cos(2x) + \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C. \qquad \zeta$$