

A Table For Trigonometric Substitution

Recall the Pythagorean trigonometric identities:	So if an integrand involves: $I(x)$	We may make the inverse substitution:	Along with the differential	In this case	Also note	So that the integrand now involves: $I(x)$
$\cos^2 \mathbf{q} = 1 - \sin^2 \mathbf{q}$	$\sqrt{a^2 - x^2}$	$x = a \sin \mathbf{q}$	$dx = a \cos \mathbf{q} \cdot d\mathbf{q}$	$\mathbf{q} = \sin^{-1} \frac{x}{a}$	$\mathbf{q} \in \frac{-\mathbf{p}}{2}, \frac{\mathbf{p}}{2}$ $\mathbf{q}\hat{\mathbf{I}}$ QI or QIV	$\sqrt{a^2 - (a \sin \mathbf{q})^2}$ $= \sqrt{a^2 - a^2 \sin^2 \mathbf{q}}$ $= \sqrt{a^2 (1 - \sin^2 \mathbf{q})}$ $= \sqrt{a^2 \cos^2 \mathbf{q}}$ $= a \cos \mathbf{q}$
$\sec^2 \mathbf{q} = 1 + \tan^2 \mathbf{q}$	$\sqrt{a^2 + x^2}$	$x = a \tan \mathbf{q}$	$dx = a \sec^2 \mathbf{q} \cdot d\mathbf{q}$	$\mathbf{q} = \tan^{-1} \frac{x}{a}$	$\mathbf{q} \in \frac{-\mathbf{p}}{2}, \frac{\mathbf{p}}{2}$ $\mathbf{q}\hat{\mathbf{I}}$ QI or QIV	$\sqrt{a^2 + (a \tan \mathbf{q})^2}$ $= \sqrt{a^2 + a^2 \tan^2 \mathbf{q}}$ $= \sqrt{a^2 (1 + \tan^2 \mathbf{q})}$ $= \sqrt{a^2 \sec^2 \mathbf{q}}$ $= a \sec \mathbf{q}$
$\tan^2 \mathbf{q} = \sec^2 \mathbf{q} - 1$	$\sqrt{x^2 - a^2}$	$x = a \sec \mathbf{q}$	$dx = a \sec \mathbf{q} \cdot \tan \mathbf{q} \cdot d\mathbf{q}$	$\mathbf{q} = \sec^{-1} \frac{x}{a}$	$\mathbf{q} \in 0, \frac{\mathbf{p}}{2}$ $\cup \mathbf{p}, \frac{3\mathbf{p}}{2}$ $\mathbf{q}\hat{\mathbf{I}}$ QI or QIII	$\sqrt{(a \sec \mathbf{q})^2 - a^2}$ $= \sqrt{a^2 \sec^2 \mathbf{q} - a^2}$ $= \sqrt{a^2 (\sec^2 \mathbf{q} - 1)}$ $= \sqrt{a^2 \tan^2 \mathbf{q}}$ $= a \tan \mathbf{q}$

Here we assume $a > 0$ is a positive, real constant. Furthermore, we should first consider the effectiveness of a straight forward substitution, such as $u = a^2 - x^2$, $u = a^2 + x^2$ or $u = x^2 - a^2$. We might also consider the use of a hyperbolic substitution, based on the hyperbolic identities, $\tanh^2 x = 1 - \operatorname{sech}^2 x$, $\cosh^2 x = 1 + \sinh^2 x$ and $\sinh^2 x = \cosh^2 x - 1$.