A Table For Trigonometric Substitution

Recall the Pythagorean trigonometric identities:	So if an integrand involves: *I(x)*	We may make the inverse substitution:	Along with the differential	In this case	Also note	So that the integrand now involves: *I(x)*
$\cos^2 q = 1 - \sin^2 q$	$\sqrt{a^2 - x^2}$	$x = a\sin q$	$dx = a\cos q \cdot dq$	$q = \sin^{-1} \frac{x}{a}$	$q \in \frac{-p}{2}, \frac{p}{2}$ $q\,\hat{I}$ QI or QIV	$\sqrt{a^2 - (a\sin q)^2}$ $= \sqrt{a^2 - a^2 \sin^2 q}$ $= \sqrt{a^2(1 - \sin^2 q)}$ $= \sqrt{a^2 \cos^2 q}$ $= a\cos q$
$\sec^2 q = 1 + \tan^2 q$	$\sqrt{a^2 + x^2}$	$x = a\tan q$	$dx = a\sec^2 q \cdot dq$	$q = \tan^{-1} \frac{x}{a}$	$q \in \frac{-p}{2}, \frac{p}{2}$ $q\,\hat{I}$ QI or QIV	$\sqrt{a^2 + (a\tan q)^2}$ $= \sqrt{a^2 + a^2 \tan^2 q}$ $= \sqrt{a^2(1 + \tan^2 q)}$ $= \sqrt{a^2 \sec^2 q}$ $= a\sec q$
$\tan^2 q = \sec^2 q - 1$	$\sqrt{x^2 - a^2}$	$x = a\sec q$	$dx = a\sec q \cdot \tan q \cdot dq$	$q = \sec^{-1} \frac{x}{a}$	$q \in 0, \frac{p}{2}$ $\cup\ p, \frac{3p}{2}$ $q\,\hat{I}$ QI or QIII	$\sqrt{(a\sec q)^2 - a^2}$ $= \sqrt{a^2 \sec^2 q - a^2}$ $= \sqrt{a^2(\sec^2 q - 1)}$ $= \sqrt{a^2 \tan^2 q}$ $= a\tan q$

Here we assume $a>0$ is a positive, real constant. Furthermore, we should first consider the effectiveness of a straight forward substitution, such as $u = a^2 - x^2$, $u = a^2 + x^2$ or $u = x^2 - a^2$. We might also consider the use of a hyperbolic substitution, based on the hyperbolic identities, $\tanh^2 x = 1 - \operatorname{sech}^2 x$, $\cosh^2 x = 1 + \sinh^2 x$ and $\sinh^2 x = \cosh^2 x - 1$.

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