2.2: Synthetic Division:
The Remainder and Factor Theorems

Starter 2.2

Find the quotient and the remainder when the first polynomial is divided by the second.

1) $2x^3 + x^2 + 3x + 7\ ;\quad x + 2$

2) $x^5 - 2x^3 + x^2 - 4\ ;\quad x - 1$

The Remainder Theorem

When a polynomial $P(x)$ is divided by $x – a$, the remainder is $P(a)$.

$$x-a\overline{)\;P(x)}^{\;Q(x)} \quad \textit{remainder } R$$

Dividend = Divisor × Quotient + Remainder

$$P(x) = (x - a) \times Q(x) + R$$

Substituting *a* for *x*,

$$P(a) = (a - a) \times Q(a) + R$$

$$= 0 \times Q(a) + R$$

$$P(a) = R$$

Example 1

Divide $P(x) = x^3 + 5x^2 + 5x – 2$ by $x + 2$

***Solution*:**

After rewriting $x + 2$ as $x – (–2)$, use synthetic division:

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 5 & -2 \\ & & -2 & -6 & 2 \\ \hline & 1 & 3 & -1 & 0 \\ & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

The quotient is: $1x^2 + 3x - 1$

The remainder is: **0**

Example 1

Divide $P(x) = x^3 + 5x^2 + 5x – 2$ by $x + 2$

***Solution*:**

Using the Remainder Theorem, $x = -2$

The Factor Theorem

For a polynomial $P(x)$, $x – a$ is a *factor* if and only if $P(a) = 0$.

Example 2

If $P(x) = 2x^4 + 5x^3 – 8x^2 – 17x – 6$, determine whether each of the following is a factor of $P(x)$.

a) $x – 1$ **b) $x – 2$**

***Example* 2**

a) $x - 1$

b) $x - 2$

***Example* 2**

b) $x - 2$