2.1: Polynomial Functions

GOAL 1 EVALUATING POLYNOMIAL FUNCTIONS

A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where $a_n \neq 0$ and the exponents are all whole numbers.

For this polynomial function, a_n is the **leading coefficient**, a_0 is the **constant term** and n is the **degree**

A polynomial function is in **standard form** if its terms are written in descending order of exponents from left to right.

GOAL 1 EVALUATING POLYNOMIAL FUNCTIONS

You are already familiar with some types of polynomial functions. Here is a summary of common types of polynomial functions.

Degree	Type	Standard Form
0	Constant	$f(x) = a_0$
1	Linear	$f(x) = a_1x + a_0$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

EXAMPLE Identifying Polynomial Functions

Decide whether the function is a polynomial function. If it is, write the function in standard form and state its degree, type and leading coefficient.

$$f(x) = \frac{1}{2}x^2 - 3x^4 - 7$$

EXAMPLE Identifying Polynomial Functions

Decide whether the function is a polynomial function. If it is, write the function in standard form and state its degree, type and leading coefficient.

$$f(x) = x^3 + 3^x$$

EXAMPLE Identifying Polynomial Functions

Decide whether the function is a polynomial function. If it is, write the function in standard form and state its degree, type and leading coefficient.

$$f(x) = 6x^2 + 2x^{-1} + x$$

EXAMPLE **Identifying Polynomial Functions**

Decide whether the function is a polynomial function. If it is, write the function in standard form and state its degree, type and leading coefficient.

$f(x) = -0.5x + \pi x^2 - \sqrt{2}$

EXAMPLE **Identifying Polynomial Functions**

Polynomial function?

$f(x) = \frac{1}{2}x^2 - 3x^4 - 7$

$f(x) = x^3 + 3^x$

$f(x) = 6x^2 + 2x^{-1} + x$

$f(x) = -0.5x + \pi x^2 - \sqrt{2}$

GOAL 1 **Roots and Zeros of Polynomials**

How the roots, solutions, zeros, *x*-intercepts and factors of a polynomial function are related.

GOAL 1 **Roots and Zeros of Polynomials**

A ***polynomial expression*** can be a monomial or a sum of monomials. The polynomial expressions that we are discussing today are in terms of one variable.

In a ***polynomial equation***, two polynomials are set equal to each other.

GOAL 1 **Factoring Polynomials**

Terms are ***factors*** of a polynomial if, when they are multiplied, they equal that polynomial:

$$x^2 + 2x - 15 = (x-3)(x+5)$$

- **(*x* - 3)** and **(*x* + 5)** are ***factors*** of the polynomial

$$x^2 + 2x - 15$$

GOAL 1 **Factoring Polynomials**

Since factors are a Product...

…and the only way a ***product*** can equal zero is if one or more of the factors are zero…

…then the only way the ***polynomial*** can equal zero is if one or more of the factors are zero.

GOAL 1 Solving a Polynomial Equation

Rearrange the terms to have zero on one side:

$$x^2 + 2x = 15 \quad \Rightarrow \quad x^2 + 2x - 15 = 0$$

Factor:

$$(x+5)(x-3) = 0$$

Set each factor equal to zero and solve:

$$(x+5) = 0 \quad \text{and} \quad (x-3) = 0$$

$$x = -5 \qquad\qquad x = 3$$

The only way that $x^2 + 2x - 15$ can $= 0$ is if $x = -5$ or $x = 3$

GOAL 1 Solutions / Roots of Polynomials

Setting the ***factors*** of a *polynomial expression* ***equal to zero*** gives the ***solutions*** to the *equation* when the polynomial expression equals zero.

Another name for the solutions of a polynomial is the ***roots*** of a *polynomial* !

GOAL 1 Zeros of a Polynomial Function

A ***polynomial function*** is usually written in function notation or in terms of *x* and *y*.

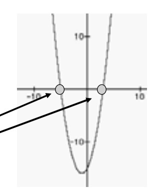
$$f(x) = x^2 + 2x - 15 \quad \text{or} \quad y = x^2 + 2x - 15$$

The ***zeros*** of a ***polynomial function*** are the ***solutions*** to the equation you get when you set the polynomial equal to zero.

GOAL 1 Zeros of a Polynomial Function

Here is the graph of our polynomial function:

$$y = x^2 + 2x - 15$$

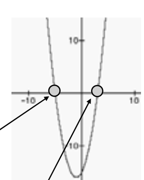


The ***zeros*** of the polynomial are the values of *x* when the polynomial equals zero. In other words, the ***zeros*** are the *x*-values where ***y equals zero***.

GOAL 1 *x*-intercepts of a Polynomial function

The points where $y = 0$ are called the ***x*-intercepts** of the graph.

The ***x*-intercepts** for our graph are the points...



$y = x^2 + 2x - 15$

(-5, 0) and **(3, 0)**

GOAL 1 *x*-intercepts of a Polynomial function

When the *factors* of a *polynomial expression* are set equal to zero, we get the ***solutions*** or ***roots*** of the *polynomial equation*.

The ***solutions/roots*** of the *polynomial equation* are the *x*-coordinates for the *x-intercepts* of the *polynomial graph*!

GOAL 1 Factors, Roots, and Zeros

For the ***polynomial function***:

$$y = x^2 + 2x - 15$$

The ***factors*** are: $(x + 5)$ & $(x - 3)$

The ***roots/solutions*** are: $x = -5$ and 3

The ***zeros*** are at: $(-5, 0)$ and $(3, 0)$

EXAMPLE **State whether each function is a polynomial function. Give the zeros of each function, if they exist.**

a) $f(x) = 2x^3 - 32x$

b) $g(x) = \dfrac{x+1}{x-1}$

EXAMPLE **State whether each function is a polynomial function. Give the zeros of each function, if they exist.**

a) $f(x) = 2x^3 - 32x$

EXAMPLE **State whether each function is a polynomial function. Give the zeros of each function, if they exist.**

b) $g(x) = \dfrac{x+1}{x-1}$

EXAMPLE **Evaluating Polynomials**

Given a polynomial $P(x)$, we can substitute not only numbers but also variable expressions for x.

Example:

If $P(x) = 3x^4 - 7x^3 - 5x^2 + 9x + 10$, find:

a) $P(2)$ *b)* $P(-3n)$

EXAMPLE **Evaluating Polynomials**

a) $P(2)$ $P(x) = 3x^4 - 7x^3 - 5x^2 + 9x + 10$

EXAMPLE **Evaluating Polynomials**

b) ***P*(-3*n*)** $P(x) = 3x^4 - 7x^3 - 5x^2 + 9x + 10$

EXAMPLE **Evaluating Polynomials:** ***Synthetic Substitution***

1. Start with the leading coefficient.
2. Repeat the following until the last coefficient is reach.
   - Multiply by the value of ***x***.
   - Add the next coefficient.

EXAMPLE **Evaluating Polynomials:** ***Synthetic Substitution***

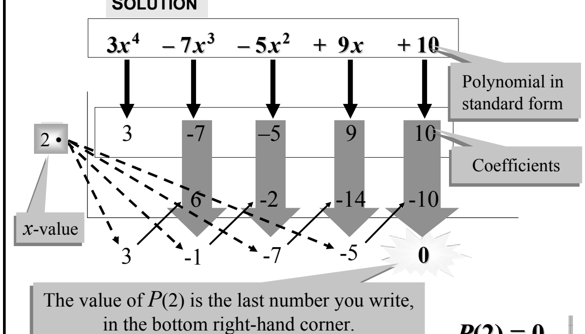
Examples:

a) If $\boldsymbol{P(x) = 3x^4 - 7x^3 - 5x^2 + 9x + 10}$, find $\boldsymbol{P(2)}$.

b) If $\boldsymbol{S(x) = 3x^4 - 5x^2 + 9x + 10}$, find $\boldsymbol{S(-2)}$.

EXAMPLE **Using Synthetic Substitution**

SOLUTION



$\boldsymbol{P(2) = 0}$

It's your Turn…

CW 2.1

Even#s **2-12, 16-30, pp. 56-57**