

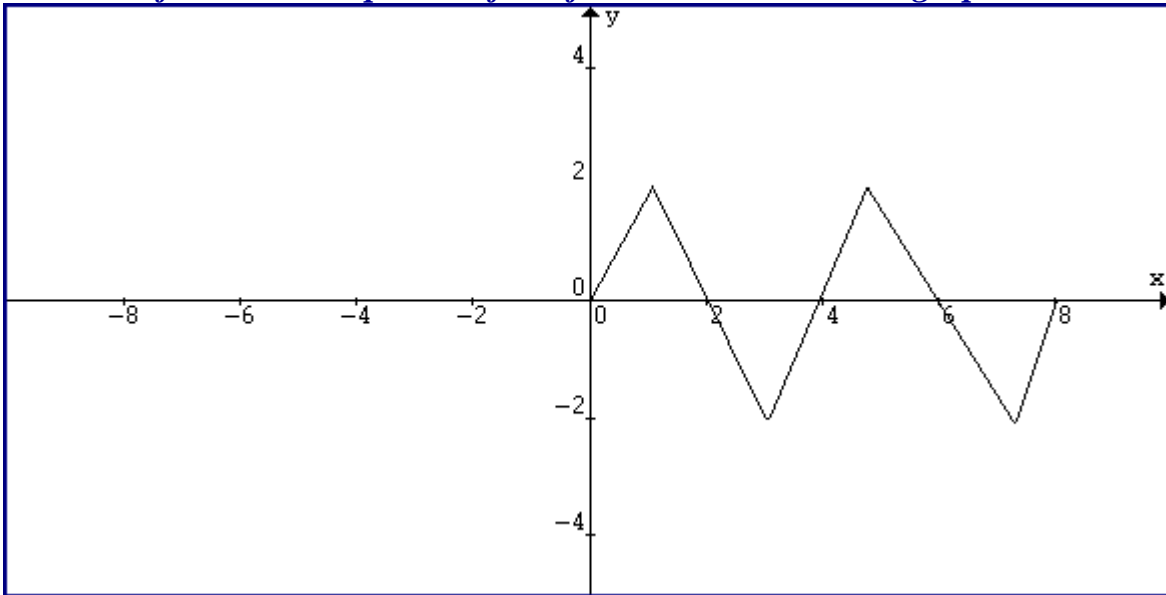
Section 4-4: Periodic Functions - Stretching and Translating

A function f is periodic if there is a positive number p such that:

$$f(x + p) = f(x)$$

for all x in the domain of f .

The definition means that the y values will repeat over some p value called the fundamental period of the function. Look at the graph:



The graph starts at 0, goes up to 2, back down to 0, down to -2 and back to 0. At this point the graph starts repeating. Look at the x value to find the period length. The period length $p = 4$, because it takes 4 units for this graph to repeat the y values. You can tell where the graph will be at larger values by knowing the repeat. $f(0) = 0$, $f(1) = 2$, $f(2) = 0$, $f(3) = -2$ and $f(4) = 0$. What is $f(21)$? Divide 21 by 4 and use the remainder. The remainder is 1. Thus $f(21) = f(1) = 2$. What is $f(82)$? Divide 82 by 4. The remainder is 2. Thus $f(82) = f(2) = 0$.

If a periodic graph has a maximum value M and a minimum value m , then the amplitude A of the function is:

$$A = (M - m)/2$$

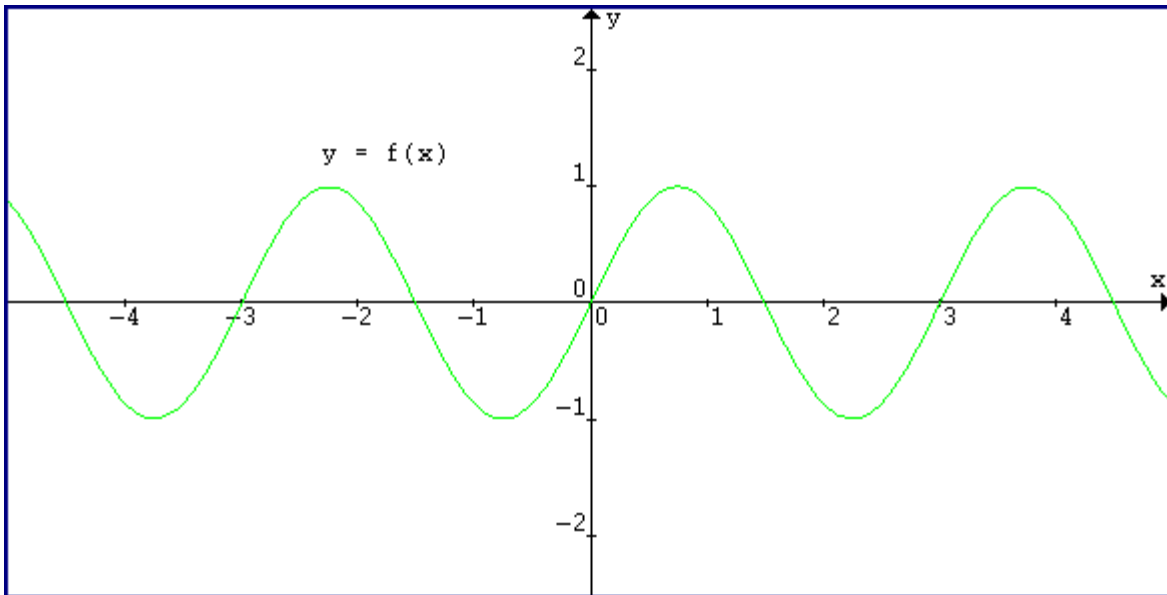
The amplitude of the above graph is:

$$A = (2 - (-2))/2 = 4/2 = 2$$

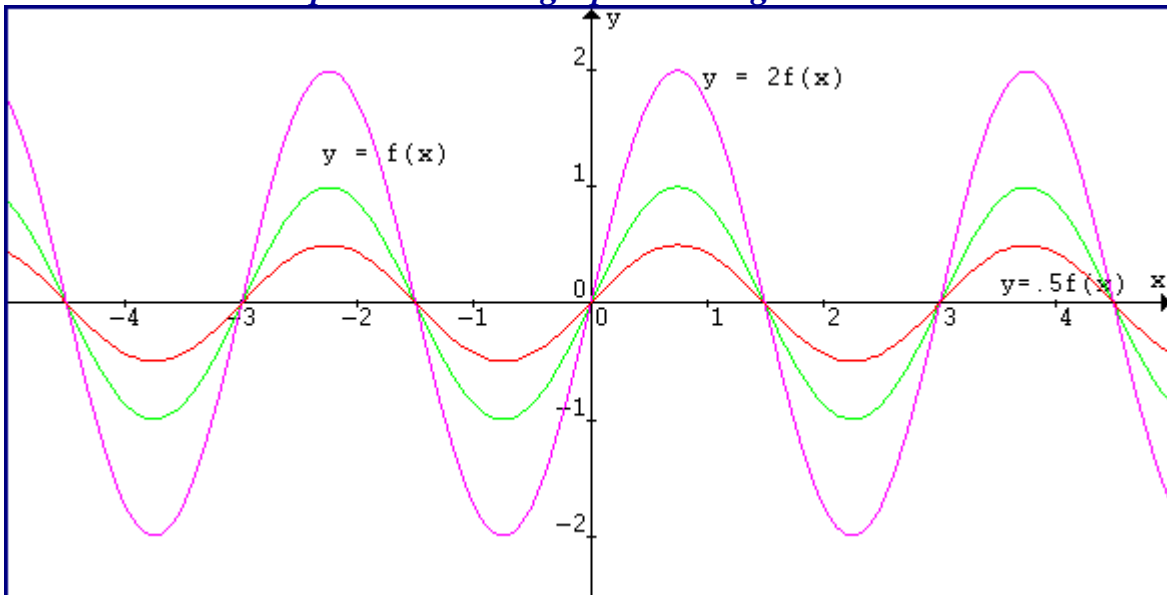
Stretching a graph vertically

The graph of $y = cf(x)$ where c is a positive number not equal to 1, is obtained by vertically stretching or shrinking the graph of $y = f(x)$.

Let $f(x)$ be the following graph:



Now compare these two graphs to the green one above.

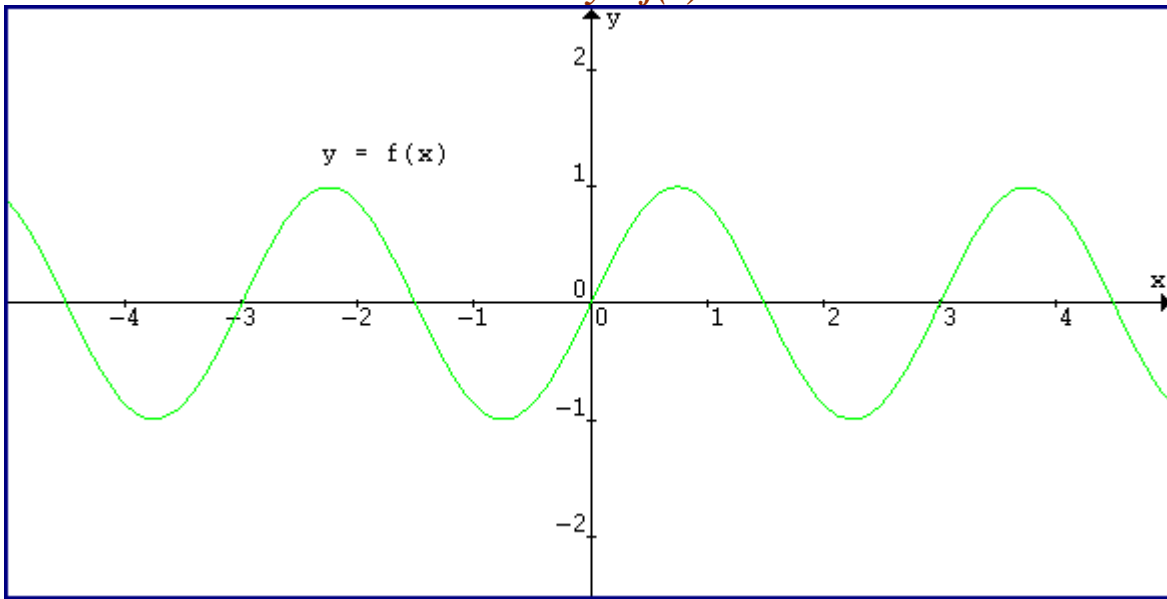


The purple graph doubled the green graph. Notice, all that changed was the high and low points of the graph. In other words, stretched vertically. Look at the red graph. It is the green graph multiplied by $1/2$. All that has changed is the high and low points. In other words, shrunk vertically. The period length has not changed. In all three graphs, the period length is 3.

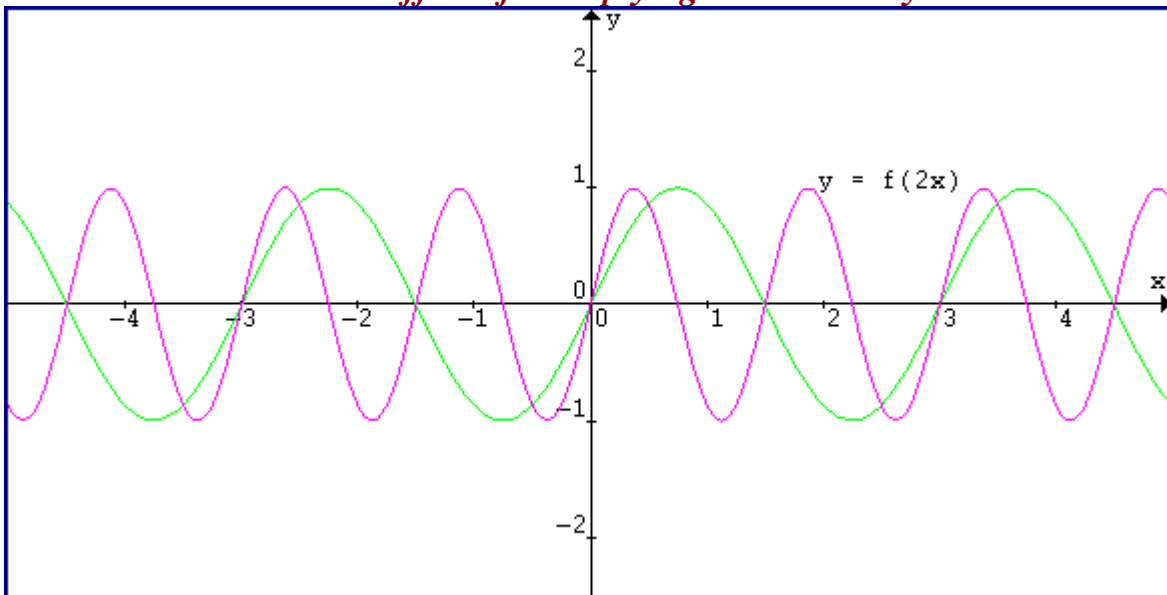
Horizontally stretching or shrinking

The graph of $y = f(cx)$ where c is positive and not equal to one is obtained by horizontally stretching or shrinking the graph of $y = f(x)$. If $c > 1$ it is a horizontal shrink. If $0 < c < 1$, it is a horizontal stretch.

Here is $y = f(x)$:

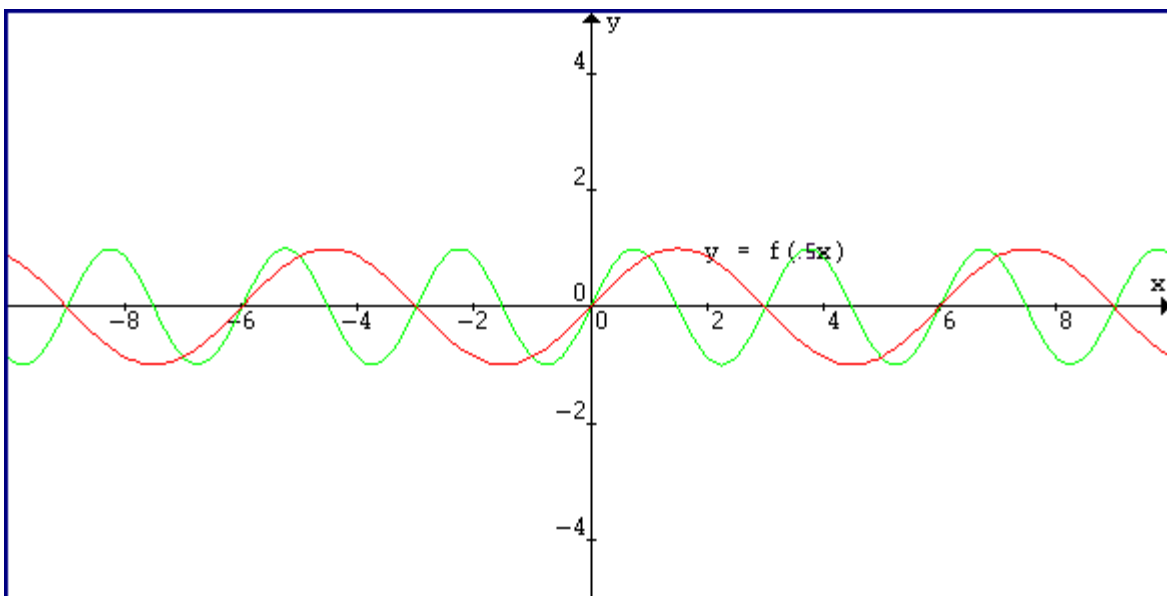


Watch the effect of multiplying the x value by 2.



*Notice the amplitude didn't change. The graph high and low points are the same. But look at the **purple** graph. Notice it has gone through two complete cycles by the time the **green** graph has gone through one cycle. It is like compressing a spring.*

Now watch the effect of multiplying by $1/2$.



The effect this time is to stretch the graph. Look at the red graph. At $x = 3$, the red graph is only half way through the cycle. It takes 6 units for the red graph to repeat instead of three for the green graph. It is like pulling out on a spring.

Summary of above

If a periodic function f has a period p and amplitude A , then:



$y = cf(x)$ has period p and amplitude cA



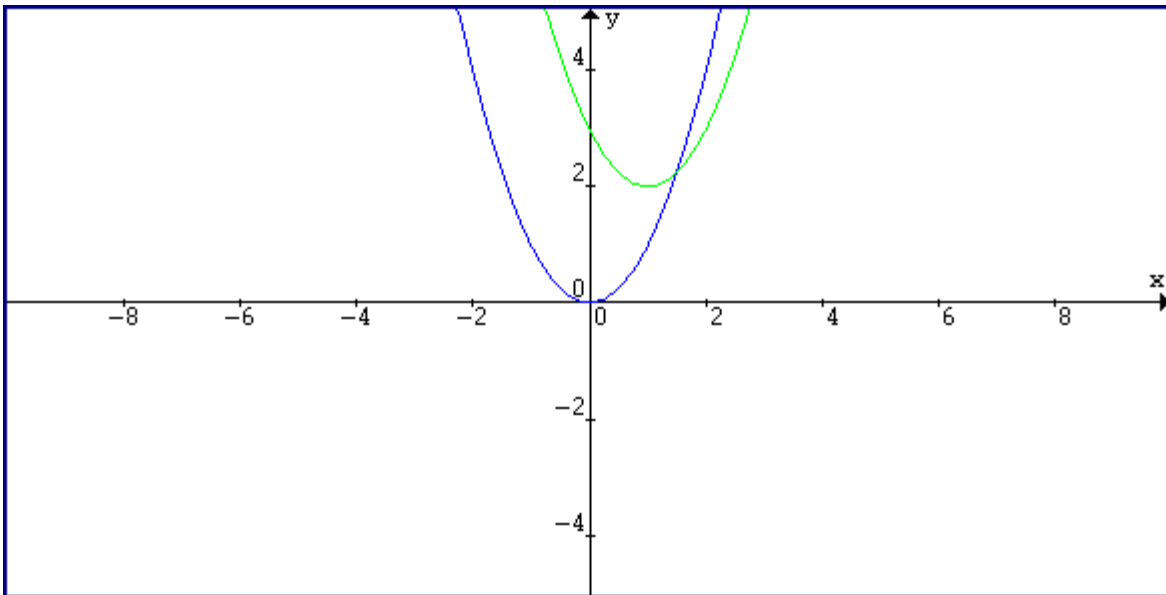
$y = f(cx)$ has period p/c and amplitude A

Translating Graphs

A translation is simply moving the exact same graph to another location. The size and shape does not change from the original graph, only the placement of the graph changes. Your knowledge of basic graphs is very helpful when doing translations. Here is how to translate:

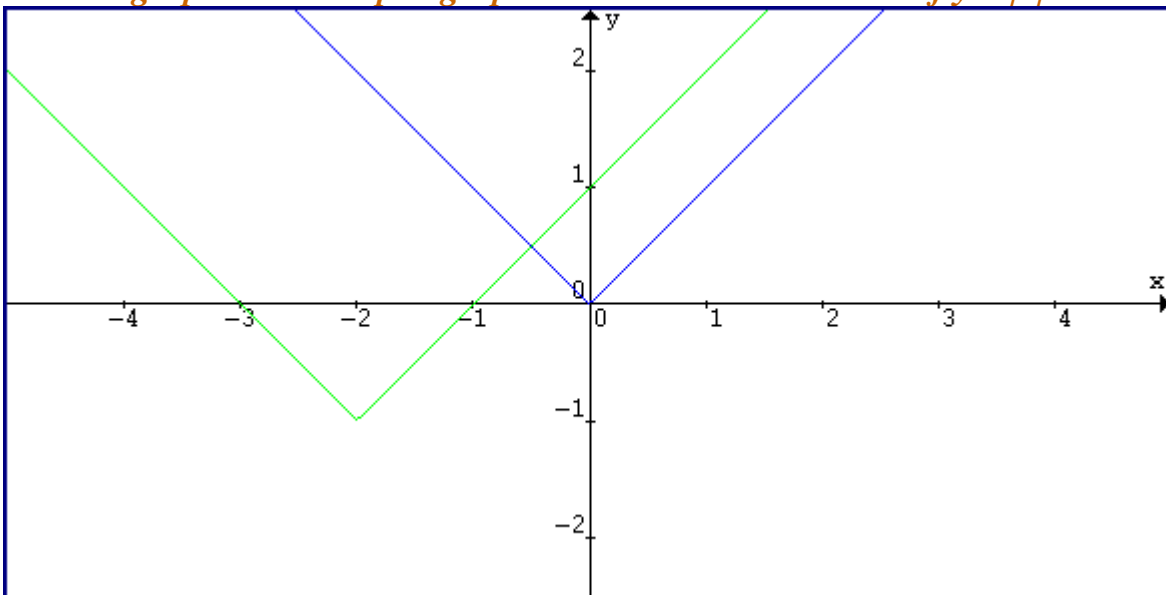
$y - k = f(x - h)$ is obtained by shifting the graph of $y = f(x)$, k units up/down and h units right/left.

You already know what $y = x^2$ is. How does $y - 2 = (x - 1)^2$ compare?



Notice the **green** graph is the same size and shape of the **blue** graph. It is shifted one unit right and two units up.

Now graph $y + 1 = |x + 2|$. This depends on you knowing that the absolute value graph is a v-shaped graph. So this is a translation of $y = |x|$



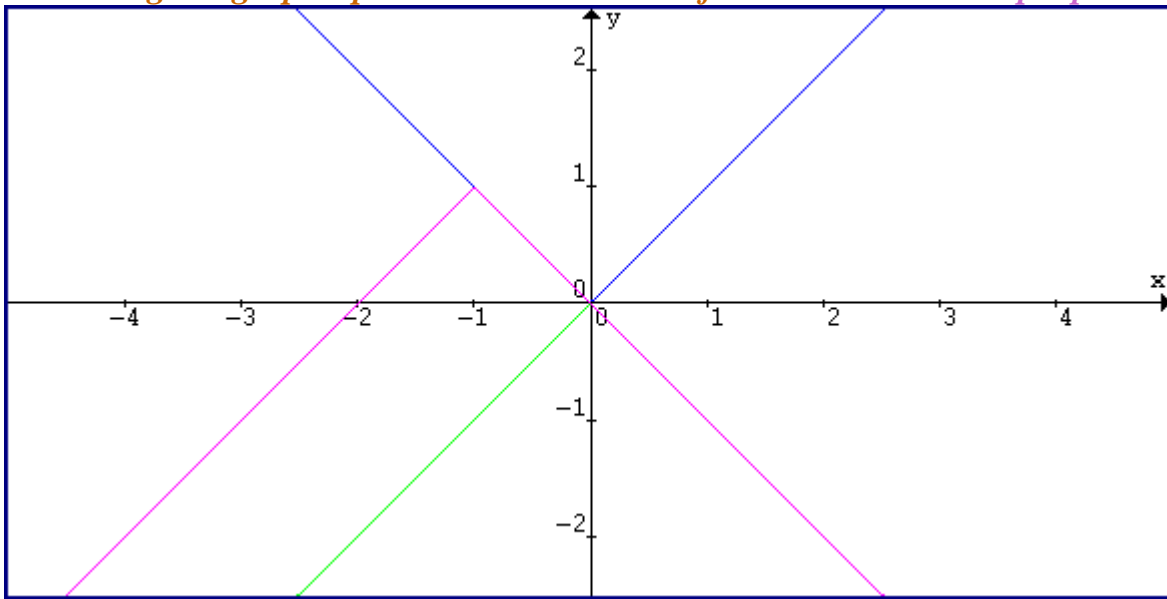
The **green** graph is the graph of the function we want. It is a translation of the **blue** graph moved one unit down and 2 units left. Notice in this problem and the last problem what causes the graph to be shifted right vs. left and up vs. down.

Combining reflections and translations

When combining reflections and translations, remember to reflect first then translate. Failure to work the problem in this order may result in the wrong answer.

Graph the function $y - 1 = -|x + 1|$

The basic graph is $y = |x|$, a v-shaped graph. The negative sign in front makes this a reflection about the x-axis. Do this first. Then translate the result by moving the graph up one and one to the left. Our answer is in purple.



Remember to make a copy of the chart on page 142 in your notebook. You must know that chart!! It helps a great deal in future chapters!!



Section 4-5: Inverse Functions



An inverse is the operation that takes you back to where you started. The inverse of multiplication is division, adding and subtracting, square and square root, etc.

For functions, there are two conditions for a function to be the inverse function:

- ☛ 1) $g(f(x)) = x$ for all x in the domain of f*
- ☛ 2) $f(g(x)) = x$ for all x in the domain of g*

Notice in both cases you will get back to the element that you started with, namely, x .

*The notation used to indicate an inverse function is: $f^{-1}(x)$ pronounced "f inverse". **This notation does not mean $1/f(x)$.***

Example

1) If $f(x) = 3x - 1$ and $g(x) = (x + 1)/3$, show that f and g are inverses to each other.

To show that they are inverses, we must prove both of the above parts.

$$\begin{aligned}g(f(x)) &= g(3x - 1) = (3x - 1 + 1)/3 = 3x/3 = x \\f(g(x)) &= f((x + 1)/3) = 3[(x + 1)/3] - 1 = x + 1 - 1 = \\&\quad x\end{aligned}$$

Since both parts work, they are indeed inverses of each other.

To find a rule for the inverse function

Find the inverse function for $y = 5x + 2$
To find the inverse, interchange x and y .

$$x = 5y + 2$$

Now isolate for y !!

$$x - 2 = 5y$$

$$(x - 2)/5 = y$$

We now have the inverse!!

Notice, that this inverse make sense. The original problem had adding by two and the inverse is subtracting two. The original function had multiplying by five and the inverse has division by five.

Graphs of inverse functions

We have to make sure that the inverse is indeed a function. Not all functions will have inverses that are also functions. In order for a function to have an inverse, it must pass the horizontal line test!!

Horizontal line test

If the graph of a function $y = f(x)$ is such that no horizontal line intersects the graph in more than one point, then f has an inverse function.

This will make sense when we discover how to graph the inverse function. To graph the inverse function, it is simply the reflection about the line $y = x$. Makes sense, because in order to get the graph, we interchange x and y . Recall from previous sections what the reflection about the line $y = x$ looks like. Any two points on the same horizontal line when reflected will be on the same vertical line. Can't have this

because it wouldn't be a function. That's why the horizontal line test works.

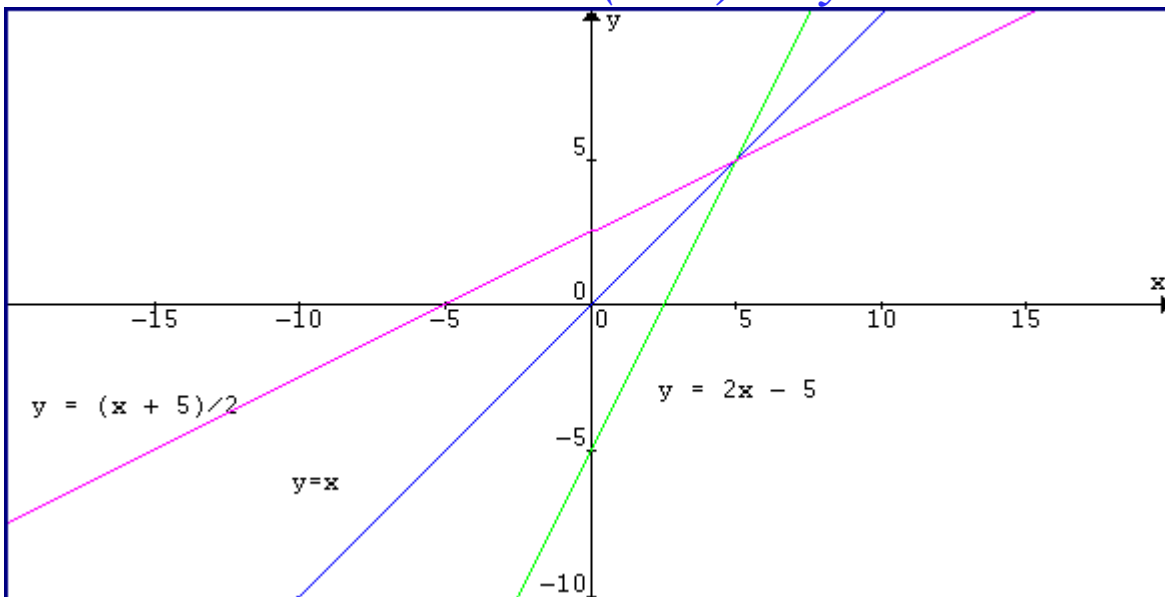
Example

1) Find the equation of f^{-1} and graph f , f^{-1} , and $y = x$ for $f(x) = 2x - 5$.

First, $f(x)$ is a line and it passes the horizontal line test.

Find the inverse:

$$y = 2x - 5$$
$$x = 2y - 5$$
$$x + 5 = 2y$$
$$(x + 5)/2 = y$$



2) Let $f(x) = 9 - x^2$ for $x \geq 0$

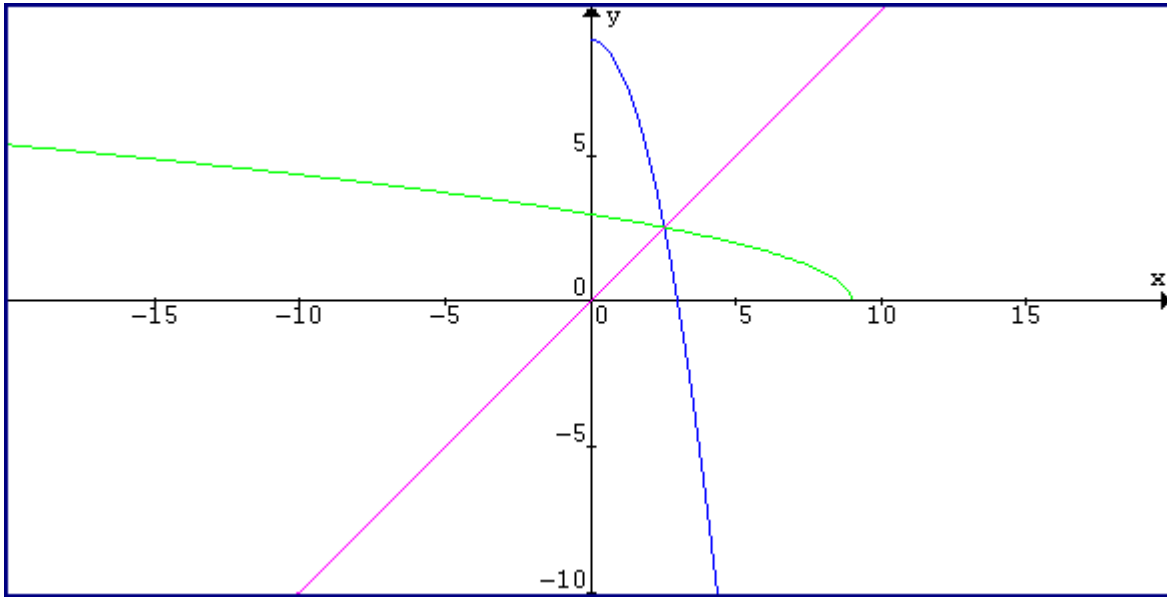
Find the equation for $f^{-1}(x)$

Sketch the graph of f , f^{-1} , and $y = x$.

Notice that the equation is half of a parabola. Only the side to the right of zero. If we tried to use the entire parabola, it wouldn't pass the horizontal line test.

To find the equation: $y = 9 - x^2$
 $x = 9 - y^2$
 $x - 9 = -y^2$

$$\sqrt{9-x} = y, x \leq 9$$





Section 4-6: Functions of Two Variables



Suppose a company makes two products. Let x represent the number of the first product made and y represent the number of the second product made. If the first product makes \$2 each and the second \$3 each, the profit function would be:

$$P(x, y) = 2x + 3y$$

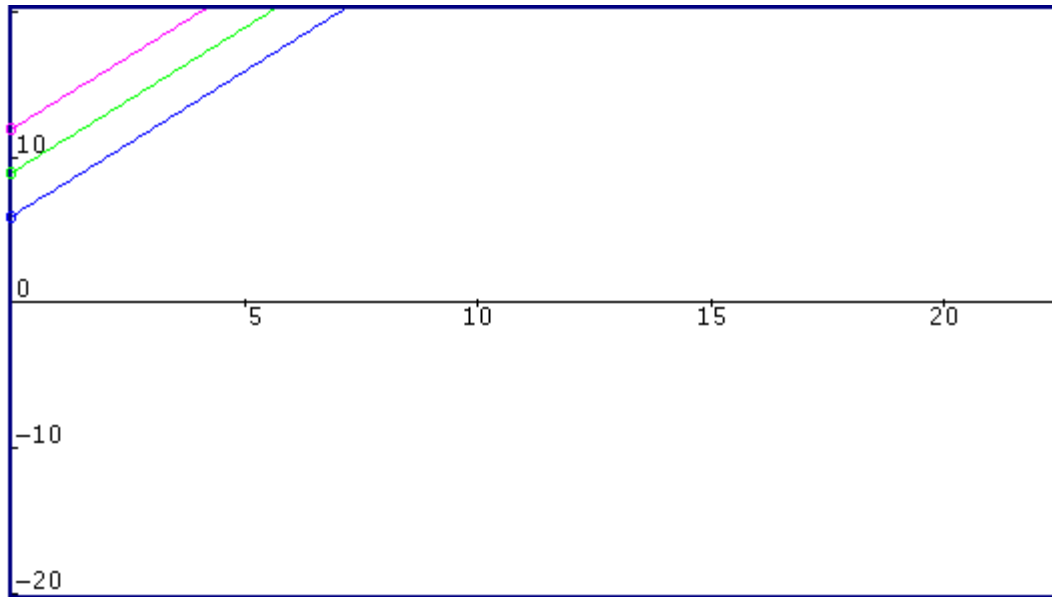
Notice that profit is a function of two variables. There are a few ways we could graph this function. One way is to graph in a three dimensional plane. We will talk about this later in the year. Another way to graph this function is to hold one of the variables constant. For example, we could hold y constant like: $y = 2$, or $y = 3$ or $y = 4$, etc. The equations would look like:

$$P(x, 2) = 2x + 6$$

$$P(x, 3) = 2x + 9$$

$$P(x, 4) = 2x + 12$$

These graphs all have the same slope but different y -intercepts. These make a family of curves. They will be graphed in the xP plane.



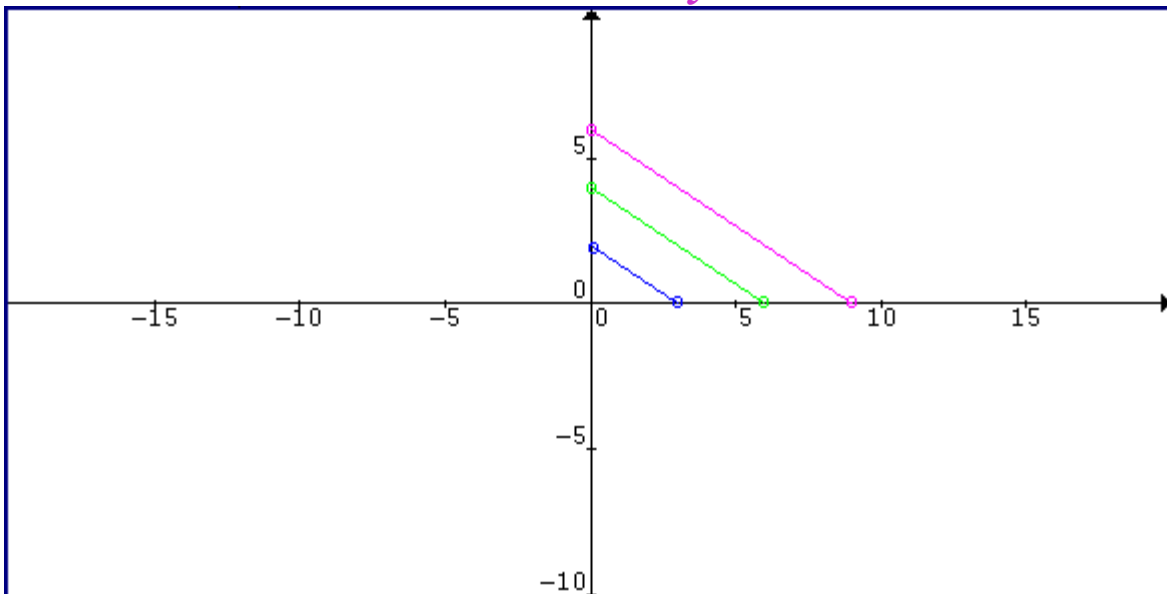
Another way we can graph this is by holding the profit constant. For example, we could let $P = 6$ or $P = 12$ or $P = 18$.

Then the equations would be:

$$6 = 2x + 3y$$

$$12 = 2x + 3y$$

$$18 = 2x + 3y$$



Another equation we could use is:

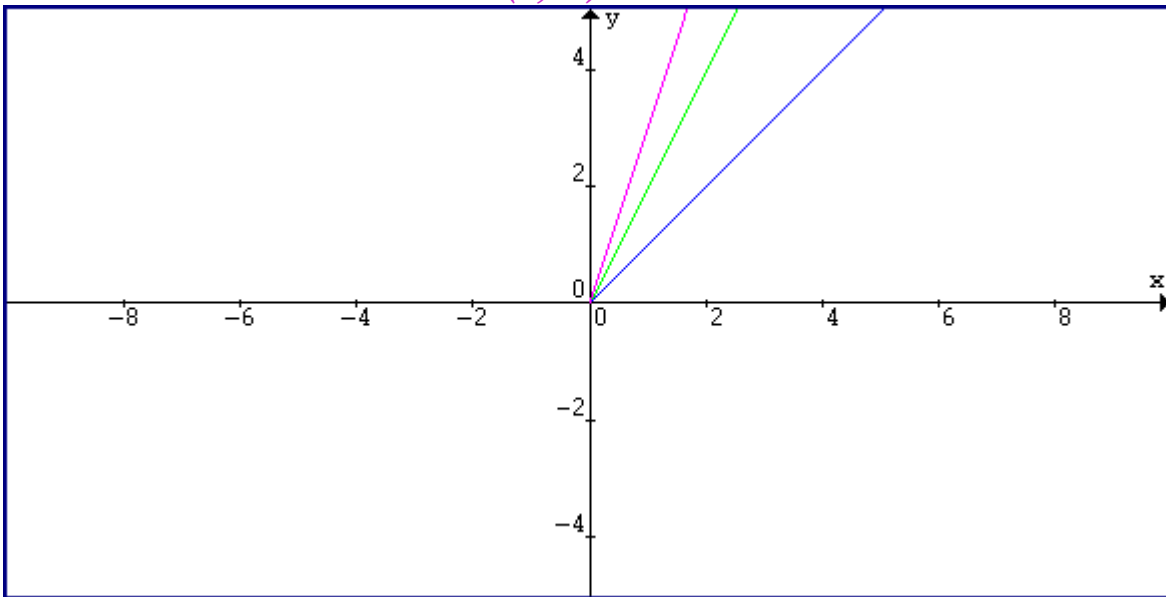
$$d(r, t) = rt$$

Suppose we hold t constant at $t = 1$, $t = 2$ and $t = 3$. Make a graph of these three equations:

$$d(r, 1) = r$$

$$d(r, 2) = 2r$$

$$d(r, 3) = 3r$$



Now graph the same function holding the distance constant at $d = 10$, $d = 20$, and $d = 30$. The equations are

$$10 = rt$$

$$20 = rt$$

$$30 = rt$$

