CHAPTER 1. INTRODUCTION

1.1. BACKGROUND

Poroelasticity is a continuum theory for the analysis of a porous media consisting of an elastic matrix containing interconnected fluid-saturated pores. In physical terms the theory postulates that when a porous material is subjected to stress, the resulting matrix deformation leads to volumetric changes in the pores. Since the pores are fluid-filled, the presence of the fluid not only acts as a stiffener of the material, but also results in the flow of the pore fluid (diffusion) between regions of higher and lower pore pressure. If the fluid is viscous the behavior of the material system becomes time dependent. The phenomenological model for such a material generally adopted today was proposed by Biot (Biot, 1941a, 1955, 1956a, 1962, 1964; Biot and Willis, 1957). His motivation, and the application of the theory over the years, has generally concerned soil consolidation (quasi-static) and wave propagation (dynamic) problems in geomechanics. As such, much of the research done has centered on massive three dimensional structural formations; sub-three dimensional structures such as bars, plates and shells have received much less attention.

Solutions for poroelasticity have been found for massive structures in different situations. Early efforts considered quasi-static problems involving consolidation, e.g. Biot (1941b, 1956b) and McNamee and Gibson (1960b). Finding solutions for dynamic poroelasticity involving wave propagation is still of interests today, e.g. Kaynia and Banerjee (1993), Chen (1994a,b), Senjuntichai and Rajapakse (1994) and Rajapakse and Senjuntichai (1995a). Chandrasekharai and Cowin (1993) presented a complete solution for a unified system of thermoelasticity and poroelasticity. Rajapakse and Senjuntichai (1995a) and Senjuntichai and Rajapakse (1995) found solutions for a multilayered poroelastic medium. Yue et al. (1994) considered a seabed as a poroelastic layer resting on a rigid impermeable base and extracted solutions for the problem. Other fundamental solutions were also reported, e.g. Rice and Cleary (1976), Cleary (1977), Simon et al. (1984), Cheng et al. (1993), Rajapakse (1993), Rajapakse and Senjuntichai (1993).

Analytical solution methods for partial differential equations have been extensively investigated for poroelastic problems. These include, for example, the integral equation method (e.g. Cheng and
Liggett, 1984; Dominguez, 1991; Kurashige and Clifton, 1996), using special mathematical functions (e.g. Senjuntichai and Rajapakse, 1994) and integral transform techniques (e.g. McNamee and Gibson, 1960a,b; Rajapakse, 1993 and Rajapakse and Senjuntichai, 1993). Classical numerical techniques are also employed, e.g. Rajapakse and Senjuntichai (1995b). These investigations, when the methods used are analytical, are applied only to problems with certain geometry and material properties, as well as specific combinations of the physical boundary conditions.

Advanced computer methods are required in order to consider general complicated cases. The finite element method has found its applications to the time dependent problem. The pioneer work of application of finite element methods and investigation of the corresponding variational principles is due to Sandhu and Wilson (1969), Christian and Boehmer (1970), Hwang et al. (1971) and Ghaboussi and Wilson (1972, 1973). Recent work reported in the literature includes Huang et al. (1990), Atkinson and Appleby (1994), Gajo et al. (1994), Appleby and Atkinson (1995) and Cui et al. (1996). Recent studies have often focused on cases which can include very general material models or applications to specific engineering problems. For instance, the dual variational principles proposed by Appleby and Atkinson (Atkinson and Appleby 1994; Appleby and Atkinson, 1995) are justified for anisotropic and spatially inhomogeneous materials; Cui et al. (1996) developed a nonlinear anisotropic model to solve problems in rock mechanics as of interest in the petroleum industry.

The boundary element method has also been adapted to the poroelastic problems, e.g., Dargush and Banerjee (1989), Dominguez (1992), Badmus et al. (1993), Chiou and Chi (1994), Chen and Dargush (1995), Chopra and Dargush (1995) and Dargush and Chopra (1996). The fact that the method does not require discretizing the domain provides some advantages for problems with infinite domains and this is often the case for consolidation or similar problems in poroelasticity.

Most investigations have been based on Biot's theory, whether analytical or numerical. Parallel formulations have also been proposed for problems in porous elastic media. For example, Burridge and Keller (1981) derived the governing equations from the point of view of the material's microscopic mechanical behaviour and geometry. Their equations are the same as Biot's if the dimensionless viscosity of the fluid is small. Crochet and Naghdhi (1966), Okuno and Kingsbury
(1989), Thompson and Willis (1991), Pariseau (1993), Li (1994) and Berge and Berryman (1995) are among other papers which discussed the constitutive equations or material parameters. Nonlinear theories have also been developed, e.g. Crochet and Naghdi (1966), Norris and Grinfeld (1995), Cui et al. (1996). The continuum theory of mixtures has also been applied to porous elastic medium, e.g. Drumheller (1978), Bowen (1980, 1982), Bedford and Drumheller (1983) and de Boer and Ehlers (1988). However, the application of the poroelastic theories based on the theory of mixtures in engineering has not been extensively reported.

Other than massive structures, poroelasticity mainly found applications in biological tissue mechanics, for which the use of poroelastic models began much later than the applications which originally motivated the material model. It has been reported that such poroelastic models are helpful in studying the mechanical response of biological tissues and organs such as bone, hearts and bladders. Nowinski and Davis (1972) modelled bones as poroelastic. Spines were extensively investigated by poroelastic models in Simon and Gaballa (1988), Simon et al. (1985), Simon et al. (1996) and Wu and Chen (1996). A poroelastic heart model was proposed by Yang et al. (1994). Other poroelastic models include models for tumors (Netti et al., 1995) and for general soft tissues (Laible et al., 1994). See also Jensen et al. (1994), Sachs et al. (1994), Roth et al. (1995), Argoubi and Shiraziadl (1996) and Vankan et al. (1996). Nonlinear poroelastic models have also been adopted in this field, e.g. Jensen et al. (1994), Vankan et al. (1996), Yang et al. (1994), Argoubi and Shiraziadl (1996). Finite element methods are found to be very effective in extracting solutions, e.g. Laible et al. (1991), Laible et al. (1994), Argoubi and Shiraziadl (1996), Vankan et al. (1996) and Wu and Chen (1996). Among these papers devoted to biomechanical problems, Vankan et al. (1996) used a continuum theory of mixtures.

In short, poroelasticity has found many applications in geomechanical engineering during the last decades. It has dealt with large structural formations. The applications in biological tissue mechanics in recent years, though dealing with small structures, are essentially three dimensional. Very few papers have thus far investigated the poroelastic beams or plates, the light structures, for which the boundary conditions and the type of loadings, and thus the behaviour of the structure, are quite different from those for large formations. The few applications of poroelastic theory to beams, plates and shells in the literature have generally been motivated by problems in biomechanics. When such
elements are subjected to bending, the stress gradients would generally be expected to be much
greater in the perpendicular direction than in axial or in-plane directions. Thus if the bulk material is
considered to be isotropic, the diffusion in the transverse direction is dominant. Hence, in the studies
reported in the literature the diffusion in the axial or in-plane directions is, justifiably, considered
negligible and the fluid movement in the perpendicular direction has been taken as the prevailing
diffusion effect. Among such papers available is that of Nowinski and Davis (1972) who modeled a
beam as an anisotropic poroelastic body and developed a procedure for solutions. The paper gives
the solutions for cases of beams subjected to uniform bending moment or uniform torsional moment,
the situation when no diffusion occurs in the direction of the beam axis since the stress gradients are
zero in the direction of the beam axis. Tajuddin and Sarma (1980) and Tajuddin (1982) also
considered torsion problems. Taber (1992) and Theodorakopoulos and Beskos (1993, 1994)
formulated the Kirchhoff plate assuming that the fluid-velocity gradients within the plate plane relative
to the solid are negligible. Zhang and Cowin (1994) considered combinations of pure bending and
axial compression for the rectangular beams, again for no diffusion in the direction of the beam axis.
Biot (1964) discussed the buckling problem for a plate with special deformation: in one direction
within the plate, the normal strain is zero; and in the perpendicular direction within the plate, the fluid
displacement relative to the solid is zero. Thus the fluid can flow only in the transverse direction.

No work has previously been done for cases with diffusion along the beam axis or in the plane of
the plate.

1.2. MOTIVATION

The present work is carried out for fluid-saturated poroelastic rods and plates made of a material for
which diffusion in the longitudinal direction(s) is facilitated while in the perpendicular direction
the flow can be considered negligible, because of the microgeometry of the solid skeletal material. It will
be found that diffusion patterns versus position are strongly dependent on the nature of the
boundaries and on the loading. Note that the geometric boundary conditions and the diffusion
boundary conditions can be coupled since in some cases a geometric boundary is also a diffusion
boundary.
We are motivated by consideration of plant stems and petioles. These elements of plants serve
the dual functions of providing structural strength and stiffness, and also contain the vascular tissue
which conducts water from the root system to transpiring leaves. Living herbaceous stems and
woody stem tissue are water saturated, the former often containing as much as 85% free water by
weight, the latter as much as 60%. Such structural plant material is highly anisotropic. Axial
stiffnesses are some 20 times greater than transverse stiffnesses for woody tissue and for other plant
tissue the anisotropy is probably much greater (Schulgasser and Witztum, 1992). The crucial
attribute of such plant elements is that their microstructure is designed to transport water axially.
Plants native to nondesert areas transport enormous quantities of water daily from the root systems
through stems which are transpired from the leaves (Weier et al., 1982). We are thus led to model a
living plant stem as a beam consisting of a poroelastic material for which water movement in the axial
direction is dominant, in order to help understand the time dependent response of such elements to
various loadings. The material is taken to be transversely isotropic in the cross-sectional plane.

We emphasize that our material model is inspired by the situation existing in plant stems, but we
deal with it in a broad sense. It would not be difficult to fabricate such a material, and structural
elements constructed from such material would exhibit interesting behaviour patterns and might be
helpful for some special purposes. Thus we also extend the formulations for transversely isotropic
plates with in-plane diffusion.

Biot's theory is employed in this work since it has been well accepted in the scientific community
and is generally adopted in publications today.

1.3. OUTLINES OF THE PRESENT WORK

The present work investigates the following situations in order to understand various aspects of the
diffusion problem.

1) QUASI-STATIC BEAM PROBLEMS

a) THE GOVERNING EQUATIONS

The governing equations for a transversely isotropic poroelastic rod (isotropic in the cross-
section) subjected to axial or transverse loads with the small deflection theory are presented in
Chapter 2, including the inertia of the bulk material. Biot’s consolidation theory, with relative motion between the solid and fluid governed by Darcy’s law, is adapted for the special case considered here. The governing differential equations can be broken into two groups, one for bending and another for extension; since they are not coupled they can be solved independently. Each group includes three equations for three unknown time-dependent functions: the total stress resultant, the pore pressure resultant and the displacement. The conditions for determination of solutions include the geometrical boundary conditions, the load boundary conditions and the diffusion boundary conditions, as well as the initial conditions.

The quasi-static problem of beams is presented by deleting the inertia term from the partial differential equation governing equilibrium of the beam.

b) THE ANALYTICAL SOLUTIONS

The quasi-static problem of beam bending is analyzed. The elastic solutions, i.e. the solutions for the corresponding drained beams, are introduced to simplify the solution procedure so that various closed form solutions for the poroelastic beams can be found. Series solutions are found for normal loadings with various mechanical and diffusion boundary conditions. The major results are included in Chapter 3.

c) THE FINITE ELEMENT METHOD

Due to the complexity of the boundaries and the governing differential equations, it is often difficult to get analytical solutions for general cases, especially when the boundary conditions are not homogenous or they can not be decoupled. Therefore, finding suitable numerical methods for respective problems is an important part of the present work. The finite element method is employed for the quasi-static rods under small deflection (Chapter 4), which method has apparently so far not been reported in poroelasticity on light structures such as beams and plates.

Variational principles are developed first for our situation. The variational functional is expressed as integrals of the unknown time-dependent functions with respect to position of the beam and convolution integrals with respect to time, following the approach of Sandhu and Wilson (1969) for seepage flow problems. Two types of variables, the displacements and pore pressure resultants, are involved in the time-dependent functionals. The method of Lagrange multipliers is employed in order
to include the flow equations (generalized Darcy’s law) into the Euler-Lagrange equations of the functionals. Two functionals are given, of which one includes the initial values of the unknown functions and is more convenient for the interpolation of the speeds (the derivatives of the displacements with respect to time); another functional is more convenient for the interpolation of the displacements. Both functionals are found to be equivalent to each other in terms of their stationary conditions, which give the governing differential equations and boundary conditions.

A mixed finite element scheme is then presented based on one of the variational functionals obtained. Numerical solution examples for both types of variables are presented in order to test the finite element model, and good coincidence with the existing analytical solutions is found. The results also demonstrate some features of the poroelastic beam model which can not be shown by the analytical solutions.
2) VIBRATIONS OF BEAMS

Flexural vibration solutions are found for free and forced vibration situations. The nature of the behavior patterns found is considered and comparisons are made with the types of behavior exhibited by damped elastic beams. It is found that the behavior of the present system can diverge qualitatively from that found in damped elastic beams (Chapter 5).

Closed form solutions of the initial value problems are obtained for simply-supported beams with general loading, by use of Laplace transformation. It turns out that the fluid works like a damper. Similar to the classic vibration theory, the responses to initial deviations can be classified into three kinds: light damping, critical damping and over damping. The vibration patterns are also dependent on the nature of the initial conditions; observed behaviour of this sort can not be explained by the classical vibration theory. Computations for the harmonic vibrations are carried out for different boundaries. The amplitude response versus the frequency of the loading, and the resonance areas, are investigated.

The vibration of poroelastic plates was studied in Theodorakopoulos and Beskos (1993, 1994), but the recognition of the dominance of the movement of the fluid in the direction perpendicular to the plate surface (valid for the type of material they considered) was adopted there.

3) BUCKLING OF BEAMS

The buckling problem for the poroelastic beam is investigated. The critical loads and their history are investigated for different boundaries. Buckling can be avoided during a loading procedure by properly choosing the loading path, even when the load at finite time is greater than the lower limit of the critical load.

Biot (1964) has studied the buckling problem for a poroelastic plate with special deformation as mentioned earlier; there transverse diffusion is dominant. Our situation here is exactly the opposite. Consequently, some unique features in our case are to be expected. The time dependent behaviours of the critical loads and deflections are considered for various diffusion and geometrical boundaries (Chapter 6).
4) LARGE DEFLECTION OF BEAMS

In the previous studies, the deflection is considered to be small and thus the linear theories are sufficient when the constitutive law adopted is linear. For some situations it may be necessary to employ a large deflection theory in order to correctly describe the behavior of the porous elements. On the other hand, the deformation can be still small and the skeletal material yet behaves elastically; the large deflection is made possible by the slenderness of the beam. Therefore, they are modeled as geometrically nonlinear and constitutively linear. Biot's constitutive law and Darcy's law are adopted as in the linear theory, while new geometrical relations and equilibrium equations are necessarily introduced.

The governing equations are presented for the large deflection of fluid-saturated poroelastic beams in Chapter 7. The stretching and bending are now coupled. The nonlinear boundary value problem is solved numerically by using the finite difference method with respect to the spatial coordinate and using a simple successive implicit formula (the trapezoid formula) to deal with the time variable (Chapter 8). Several types of geometrical and diffusion boundary conditions are investigated by means of the numerical solutions. Results are presented for which observations are made, and some interesting features are found which do not occur when the problem is modeled as linear (i.e. small deflections).

5) FORMULATIONS AND SOLUTIONS FOR PLATES

Formulations are found in Chapter 9 for fluid-saturated poroelastic plates consisting of a material for which the diffusion is possible in the in-plane directions only, both for bending and for in-plane loading. The plates considered are isotropic in the plate plane and the Kirchhoff hypotheses are assumed. Again Biot's constitutive law is adopted and Darcy's law is used to describe the fluid flow in pores. The basic equations are so derived that they could be easily extended for the situation of an orthotropic poroelastic plate. Closed form solutions are extracted for quasi-static problems and for vibrations; these are given in Chapter 10. Observations are made on the types of deflection/vibration patterns which are obtained.
1.4. PUBLICATIONS OF THE PRESENT WORK

Much of the present research has been published (or submitted for publication) as follows:


