Minimum Probability of Error Demodulation for Uplink MC-CDMA

P.M Dayal,A. Mahanta
Department of ECE
IIT Guwahati
&
U.B. Desai
Department of EE
IIT Bombay

MC-CDMA: Why ?

- Combination of OFDM and CDMA [1] [2]
- Benefits derived from OFDM
  - Attractive Spectral Characteristics
  - Therefore easier system coexistence
  - ISI can be mitigated with the use of Cyclic Prefix
- Benefits derived from CDMA
  - Achieves frequency diversity unlike conventional OFDM
- Simple implementation using FFT and IFFT

MC-CDMA: How ? (Signal Model)

- Consider a MC-CDMA system with N subcarrier and N users. The low pass transmitted signal of the m^{th} user can be written as

$$s_m(t) = \sum_{k=-\infty}^{k=+\infty} \sum_{n=0}^{n=N-1} a_m^{[k]} c_{m,n} exp^{\frac{2\pi n}{T_b}} g(t) \tag{1}$$

- $a_m^{[k]} \epsilon \{+1, -1\}$if user is active during the k^{th} symboling duration else 0.
- $c_{m,n} \epsilon \{\frac{1}{\sqrt{N}}, -\frac{1}{\sqrt{N}}\}$ corresponds to the spreading code element of m^{th} user for the n^{th} subcarrier.

$$C = \begin{bmatrix} c_{0,0} & c_{1,0} & \cdots & c_{N-1,0} \\ c_{0,1} & c_{1,1} & \cdots & c_{N-1,1} \\ \cdots & \cdots & \cdots & \cdots \\ c_{0,N-1} & c_{1,N-1} & \cdots & c_{N-1,N-1} \end{bmatrix} \tag{2}$$

- The function g(t) takes care of the cyclic prefix. It is defined as

$$g(t) = \begin{cases} \text{t + T}_b - kT_s & \text{-T}_g + kT_s \le t < kT_s \\ \text{t - kT}_s & \text{kT}_s \le t < kT_s + T_b \end{cases} \tag{3}$$

- T_b is the bit duration, T_g is the symbol duration and $T_s = T_b + T_g$ is the symboling duration

Effect of the Channel

- The cylic prefix is used to overcome the effect of the ISI
- If the channel impulse response of the m^{th} path is $h_m(t)$, the channel output is

$$y_m(t) = h_m(t) * s_m(t) \tag{4}$$

- If the guard interval period T_g is greater than or equal to the delay spread of the channel, the ISI is limited only to the guard interval period and the rest of the received symbol is ISI free.
- The received signal at the receiver is the sum of signals from all users added with an additive white Gaussian noise (AWGN) component.

$$y(t) = \sum_{m=0}^{N-1} y_m(t - \tau_m) + n(t) \tag{5}$$

- $\tau_m's$ are the transmission delays of the different users and n(t) is the complex awgn.
- Let us assume the case where $\tau_0 = \tau_1 = \ldots = \tau_{N-1}$ and let us drop the subscript k.

Effect of the Channel ...

- To demodulate the bit, we sample at $t = kT_s + \frac{pT_b}{N}$, where $p = 0, 1, \ldots, N-1$ to get $y[p]$

$$y[p] = \sum_{m=0}^{N-1} h_m[p] \otimes y_m[p] + n[p] \tag{6}$$

- $\otimes$ denotes circular convolution.
- In frequency domain, ie., after performing the DFT

$$Y = H_{eff} A + N \tag{7}$$

- $A = [a_0 a_1 \ldots a_{N-1}]^T$ is the vector of transmitted user symbols, N is the resulting noise vector after the DFT
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$$H_{eff} = \begin{bmatrix} c_{0,0}H_{0,0} & \ldots & c_{N-1,0}H_{N-1,0} \\ c_{0,1}H_{0,1} & \ldots & c_{N-1,1}H_{N-1,1} \\ \ldots & \ldots & \ldots \\ c_{0,N-1}H_{0,N-1} & \ldots & c_{N-1,N-1}H_{N-1,N-1} \end{bmatrix} \tag{8}$$

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$$\begin{bmatrix} H_{m,0} \\ H_{m,1} \\ \ldots \\ H_{m,N-1} \end{bmatrix} = W \begin{bmatrix} h_m[0] \\ h_m[1] \\ \ldots \\ h_m[N-1] \end{bmatrix} \tag{9}$$

Effect of the Channel ...

- W is the N point DFT matrix
- $h_m[p]$ is the p^{th} sample of the low pass channel response $h_m(t)$ sampled at a sampling rate of $\frac{1}{T_b}$.
- Thus $H_{m,n}$ is the complex n^{th} subcarrier gain for the m^{th} user with its amplitude Rayleigh distributed and phase uniformly distributed in $[0, 2\pi]$.
- N is the Gaussian noise vector with each component having a variance σ^2.
- This means
  - At the mobile station end, the users are separated by orthogonal codes.
  - This orthogonality is lost at the base station end.
  - Thus we cannot simply multiply with the spreading sequence and form the decision variable.
  - We need a single tap equalizer.

Equalizer

- Conventionally adaptive filtering has been based on the principle of Mean Square Error (MSE) [3]
- MMSE equalizer minimizes the MSE, not necessarily the Bit Error Rate (BER)
- As it is the BER and not the MSE that is the deciding criterion in a digital communication system, a receiver that directly minimizes the BER is desired. [4]
- Minimum Probability of Error Detector does just that !!! [5] [6] [7]

Probability of Error

- Let $w_k = [w_0 \; w_1 \; \ldots \; w_{N-1}]^H$ be the filter vector acting on the vector Y to demodulate the k^{th} user
- The filter output $x_k = w_k^H H_{eff} A + w_k^H N$
- As the original modulation was BPSK, real part of x_k is sufficient and can be used to demodulate the user's bit

$$\Re(x_k) = \Re(w_k^H H_{eff} A) + \Re(w_k^H N) \tag{10}$$

- Conditioned on the transmitted bit vector A, $\Re(x_k)$ is Gaussian with mean $\mu_k = \Re(w_m^H H_{eff} A)$ and variance $\sigma_k^2 = \frac{\sigma^2 \|w_k\|^2}{2}$.
- Demodulated symbol is $sgn\left(\Re(x_k)\right)$, $sgn(x)$ being the signum function.
- An error in demodulation occurs whenever $\Re(x_k|a_k = -1) > 0$ or $\Re(x_k|a_k = +1) < 0$
- Using the standard Q function, the probability of error in demodulation is given by

$$P_e(x_k|A) = Q\left(\frac{a_k \mu_k}{\sigma_k}\right) \tag{11}$$

- Average probability of error can be written as

$$P_e = \frac{1}{2^N} \sum_{\forall A} Q\left(\frac{a_k \mu_k}{\sigma_k}\right) \tag{12}$$

MPOE detector

- The filter vector that minimizes the average probability of error is the MPOE detector
- We will use gradient descent algorithm to find the probability of error
  - Start with an initial guess for the filter vector
  - During the $i+1^{th}$ iteration

$$w_k^{i+1} = w_k^i - \mu \bigtriangledown (P_e(w_k)) \tag{13}$$

  - If $J(w)$ is a real valued cost function, then the gradient $\bigtriangledown J$ is given by [8]

$$\bigtriangledown J = 2\frac{\partial J}{\partial w^*} \tag{14}$$

$$\frac{\partial P_e(w_k)}{\partial w_k^*} = -\frac{1}{\sqrt{2\pi}2^N}\sum_{\forall A}\left(e^{-\frac{\mu_k^2}{2\sigma_k^2}}\frac{\partial}{\partial w_k^*}\left(\frac{\mu_k a_k}{\sigma_k}\right)\right) \tag{15}$$

  - After some steps, we can write

$$\frac{\partial P_e(w_k)}{\partial w_k^*} = -\frac{1}{\sqrt{2\pi}2^N}\sum_{\forall A}\left(a_k e^{-\frac{\mu_k^2}{2\sigma_k^2}}\phi\right) \tag{16}$$

  - ϕ is given by

$$\phi = \frac{1}{\sigma_k}\left(H_{eff}A - \frac{\mu_k w_k}{\|w_k\|^2}\right) \tag{17}$$

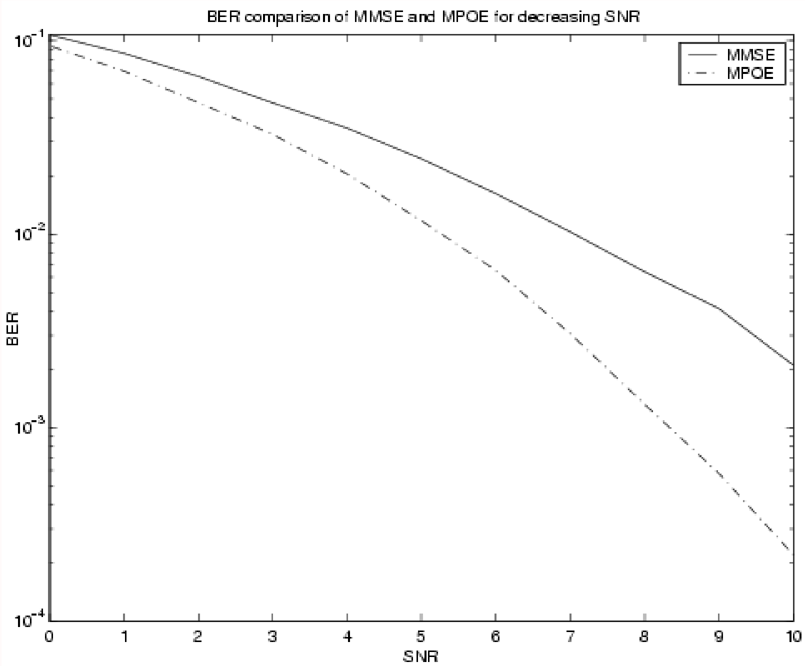
Reducing the Computational Complexity

- w can be normalized to unit length after every step, but that makes the algorithm more sensitive to step size.
- We can reduce the computational complexity by half if we note that the expression for the average probability remains unchanged if we replace A by -A.
- Thus we need to sum up only 2^{N-1} elements in place of 2^N elements.
- Another approach is to minimize the conditional probaility of error instead of average probability of error
  - This detector is known as Minimum Conditional Probability of Error Detector [7] [9]
  - Needs a training sequence
  - Linear Complexity
  - Huge savings compared to MMSE (polynomial) and MPOE (exponential)

Simulations

- Three sets of simulations
  - Study of BER performance of MPOE v/s MMSE detector
  - Study of the effect of load on the system performance
  - Study of the effect of Near Far Problem on system performance
- Simulation Parameters
  - Number of subcarriers were 8
  - Maximum no. of active users were 8
  - Walsh Hadamard Codes were used to spread the data
  - If the local mean power of the k^{th} user is is p_k, then we define Signal to Interference Ratio, SIR_m as $\frac{p_k}{p_m}$.
  - By definition SIR_k = 0 dB

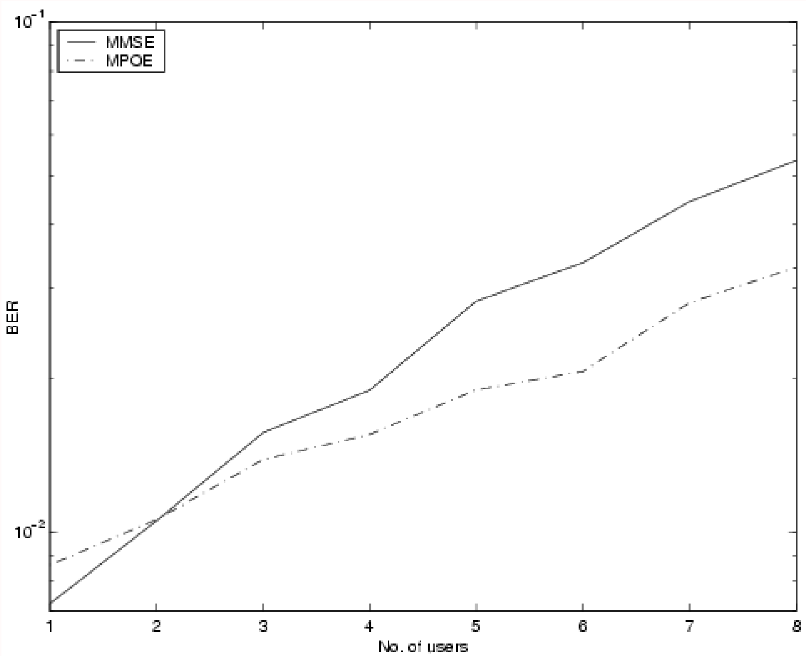
Simulation Results



- BER of MPOE v/s MMSE for increasing SNR

- All users received with the same average power
- All users active

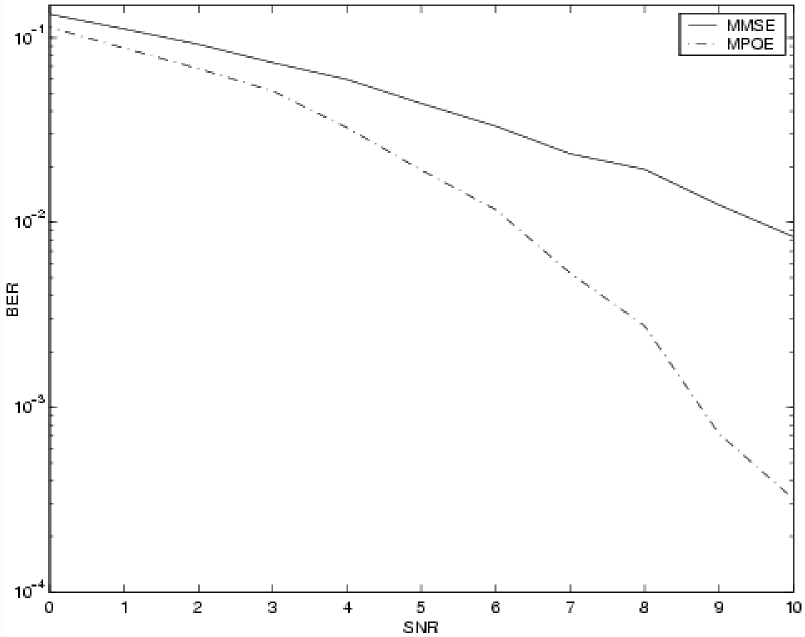
Simulation Results ...

- 

BER of MPOE v/s MMSE for increasing load

- All users received with the same average power

Simulation Results ...



- BER of MPOE v/s MMSE for increasing SNR when SIR_7 = -6 dB
- All users active

Conclusions

- At a BER of 10^{-3}, MPOE detector gains about 3 dB over MMSE detector when no near far problem is present.
- MPOE detector has been experimentally shown to be more near far resistant than MMSE detector.
- Compuational complexity of the MPOE detector can be reduced by following the MCPOE approach with marginal decrease in performance.

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Prateek Mohan Dayal
Dept. of Electronics and Communication Engineering
Indian Institute of Technology - Guwahati
Guwahati - 781039