Minimum Probability of Error Detection for Asynchronous Uplink MC-CDMA

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***Abstract*— Multi-Carrier Code Division Multiple Access (MC-CDMA) is an attractive choice for high data rate wireless communication as it mitigates the problem of severe Intersymbol Interference (ISI) associated with CDMA techniques. The data stream in MC-CDMA is spread using a user specific spreading sequence in the frequency domain and thus every chip travels on a different subcarrier. Like OFDM, the various subcarriers in MC-CDMA are orthogonal to each other. Therefore MC-CDMA systems minimize ISI and at the same time achieve frequency diversity. To combat multipath fading, the different sub-carriers have to be combined properly to form the decision variable at the receiver. Minimum Mean Square Error (MMSE) detector is a popular choice for this, but as it tries to minimize the Mean Square Error (MSE) and not the Bit Error Rate (BER), the BER it attains is not optimum. In this paper, we propose a Minimum Probability of Error (MPOE) based linear multiuser detector for uplink MC-CDMA, that tries to minimize the probability of error by exploiting the structure of the probability density function (pdf) of the filter's output. Theoretical and simulation results are presented for the MPOE detector and it can be seen that the MPOE detector outperforms the MMSE detector in terms of BER. It has also been shown in the paper that the MPOE detector has superior near far resistance compared to the MMSE detector.**

I. Introduction

Multi-Carrier Code Division Multiple Access (MC-CDMA) is a fairly recent multiple access technique that has attracted much research during the past decade. A combination of Orthogonal Frequency Division Multiplexing (OFDM) and CDMA [1] [2], MC-CDMA combines the advantages of both the schemes. Like OFDM, MC-CDMA signals have attractive spectral characteristics. Their spectral energy is confined almost to the allocated bandwidth. Owing to this and other spectral characteristics, MC-CDMA allows for easier system coexistence. Moreover, by transmitting the same data bit over different sub-carriers, MC-CDMA exploits frequency diversity better than OFDM and also reduces the symbol rate on each of the sub-carrier. Due to this reduced symbol rate, the effect of Intersymbol Interference (ISI) on the transmitted signal is also reduced. The residual ISI can be eliminated by using sufficient guard interval.

This paper focuses on the uplink scenario, where the mobile sets communicate with the base station. The base station receives a composite signal from all the users active in that cell. Unlike the downlink case, the various user's signals can be received with different power levels if there is no central power regulation scheme. This problem is known as the near far problem and in certain cases, the stronger users can overwhelm the weaker users. Another characteristic unique to the uplink communication is that of relative user asynchronism due to clock misalignments. Even in the case where the users try to align to a central clock, the different user's clocks might be skewed with respect to each other by a few chips. Multiuser detection [3] deals with the problem of the reliable demodulation of the user's bits in such a scenario. Various detectors, such as MRC, ERC and MMSE [4] have already been proposed for the uplink case. Traditionally, adaptive filtering in communication has been based on the Mean Square Error (MSE) criterion [5]. Recently, [6] [7] [8] [9], several linear detectors have been suggested for CDMA and OFDM based on the principle of Minimum Probability of Error (MPOE). As these detectors try to minimize the BER and not the MSE, these detectors achieve a better BER than the MMSE detectors.

A detector for downlink MC-CDMA has already been proposed based on the MPOE criterion [10] [11]. In this paper we propose a detector for the Asynchronous Uplink MC-CDMA system based on the MPOE criterion. The theoretical and simulation results for the same have been presented. It can be seen that the MPOE detector outperforms the MMSE detector in terms of BER albeit at a slightly higher computational complexity when in weight adaptation mode. Once in the decision directed mode, both MMSE and MPOE detectors have linear complexity. Ways to reduce the computational complexity of the MPOE detector in the weight adaptation mode have also been proposed in the paper. Also another advantage of the MPOE detector over MMSE detector is its improved resistance to the near far problem widely prevalent in mobile communication.

The organization of the paper is as follows. Section II describes the signal model assumed. Section III derives the expression for the probability of error for MC-CDMA and using this expression, arrives at the MPOE detector. Simulation results are presented in Section IV. The conclusions are presented in Section V.

II. Signal Model

Consider a MC-CDMA systems with N subcarriers and N users. The low pass transmitted signal of the m^{th} user during the k^{th} symbol duration is given by

$$s_m^k(t) = \sum_{n=0}^{N-1} a_m^k c_{m,n} p(t - kT_s) e^{j\left(\frac{2\pi n}{T_b} g(t)\right)} \quad (1)$$

where, a_m^k is the transmitted symbol of the m^{th} user during the k^{th} symboling interval and belongs to the set $\{+1, -1\}$ if the m^{th} user is active and is 0 if the user is inactive. The matrix

$$C = \begin{bmatrix} c_{0,0} & c_{1,0} & \cdots & c_{N-1,0} \\ c_{0,1} & c_{1,1} & \cdots & c_{N-1,1} \\ \cdots & \cdots & \cdots & \cdots \\ c_{0,N-1} & c_{1,N-1} & \cdots & c_{N-1,N-1} \end{bmatrix} \quad (2)$$

where $c_{m,n} \epsilon \{\frac{1}{\sqrt{N}}, -\frac{1}{\sqrt{N}}\}$ corresponds to the spreading code element of m^{th} user for the n^{th} subcarrier. The function g(t) is defined as below

$$g(t) = \begin{cases} t + T_b - kT_s & -T_g + kT_s \le t < kT_s \\ t - kT_s & kT_s \le t < kT_s + T_b \end{cases} \quad (3)$$

where T_b, T_g are the bit period, guard interval period and $T_s = T_g + T_b$ is the symbol period. Function $p(t - kT_s)$ is a unit pulse over the duration $[-T_g, T_b)$. During the guard interval period, the transmitted signal is a replica of the signal transmitted during the last T_g time interval of the entire symboling period T_s for that bit. The channel output is a linear convolution of the transmitted signal and the low pass channel impulse response $h_m(t)$.

$$y_m(t) = h_m(t) * s_m(t) \quad (4)$$

where * denotes the linear convolution operation. Using the tap delay model, the low pass channel impulse response can be written as

$$h_m(t) = \sum_{l=0}^{L-1} g_{m,l} \delta \left(t - \frac{lT_b}{N}\right) \quad (5)$$

where, $g_m' s$ are the complex path gains, with their amplitudes Rayleigh distributed and phase uniformly distributed in $[0.2\pi]$. As a result of the linear convolution with the channel impulse response, the received signal for the k^{th} symbol will have ISI from the $(k-1)^{th}$ symbol. If the guard interval period T_g is greater than or equal to the delay spread of the channel, the ISI is limited only to the guard interval period and the rest of the received symbol is ISI free. The received signal at the receiver is the sum of the signals received from the user and corrupted by an additive white Gaussian noise (AWGN) component.

$$y = \sum_{m=0}^{N-1} y_m(t - \tau_m) + n(t) \quad (6)$$

where $\tau_m' s$ are the transmission delays of the different users, uniformly distributed between $[0, T_s]$ and n(t) is the complex awgn. The composite received signal can therefore be written as

$$y = \sum_{m=0}^{N-1} \sum_{l=0}^{L-1} g_{m,l} s_m \left(t - \tau_m - \frac{lT_b}{N}\right) + n(t) \quad (7)$$

Let us assume that the users have been arranged in the order of increasing time delays, i.e.,

$$\tau_0 < \tau_1 < \ldots < \tau_{N-1} \quad (8)$$

Also let us assume that our user of interest is the m^{th} user and align the sampling clock with this user's clock, ie., to demodulate the k^{th} bit of the m^{th} user, we collect N samples as

$$y[p] = y\left(kT_s + \tau_m + \frac{pT_b}{N}\right) + n[p] \quad (9)$$

The above samples shall contain contributions from the i^{th} user's k^{th} and $k+1^{th}$ bits, if $i < m$ and from the $(k-1)^{th}$ and k^{th} bits if the $i > m$. Therefore the sampled signal can be written as

$$\begin{aligned} y[p] = & \sum_{l=0}^{L_{m,k}} g_{m,l} \sum_{n=0}^{N-1} a_m^k c_{m,n} e^{j\left(\frac{2\pi n}{T_b} g(kT_s + \frac{pT_b}{N} - \frac{lT_b}{N})\right)} \\ & + \sum_{i=0}^{m-1} \left(\sum_{l=L_{i,k}}^{L-1} g_{i,l} \sum_{n=0}^{N-1} a_i^k c_{i,n} e^{j\left(\frac{2\pi n}{T_b} g(kT_s + \tau_m - \tau_i + \frac{(p-l)T_b}{N})\right)} \right. \\ & \left. + \sum_{l=0}^{L_{i,k+1}} g_{i,l} \sum_{n=0}^{N-1} a_i^{k+1} c_{i,n} e^{j\left(\frac{2\pi n}{T_b} g(kT_s + \tau_m - \tau_i + \frac{(p-l)T_b}{N})\right)} \right) \\ & + \sum_{q=m+1}^{N-1} \left(\sum_{l=L_{q,k-1}}^{L-1} g_{q,l} \sum_{n=0}^{N-1} a_q^{k-1} c_{q,n} e^{j\left(\frac{2\pi n}{T_b} g(kT_s + \tau_m - \tau_q + \frac{(p-l)T_b}{N})\right)} \right. \\ & \left. + \sum_{l=0}^{L_{q,k}} g_{q,l} \sum_{n=0}^{N-1} a_q^k c_{q,n} e^{j\left(\frac{2\pi n}{T_b} g(kT_s + \tau_m - \tau_q + \frac{(p-l)T_b}{N})\right)} \right) \\ & + n[p] \end{aligned} \quad (10)$$

The first term in the above sum represents the contribution from the user of interest. In case of a sufficiently long guard interval, the contribution will only be from the k^{th} bit. The upper limit $L_{m,k}$ depends upon the sampling instant, delays etc and removes the dependence on the pulse term $p(t)$ and determines which multipaths will contribute to the samples at a given sampling instant. The second term in the sum represents the contribution from the users $0, 1, \ldots, m-1$, arriving before the m^{th} user. Again the upper and lower limits determine which paths will contribute to the sum. The final term represents the contribution from the users arriving later than the m^{th} user. The various limits are given as

$$
\begin{aligned}
L_{m,k} &= \left\lfloor \frac{N}{T_b}\left(\frac{pT_b}{N} + T_g\right)\right\rfloor \\
L_{i,k} &= \left\lceil \frac{N}{T_b}\left((\tau_m - \tau_i) + \frac{(p-N)T_b}{N}\right)\right\rceil \\
L_{i,k+1} &= \left\lfloor \frac{N}{T_b}\left((\tau_m - \tau_i) + \frac{(p-N)T_b}{N}\right)\right\rfloor \\
L_{q,k-1} &= \left\lceil \frac{N}{T_b}\left(T_g - (\tau_i - \tau_m) + \frac{pT_b}{N}\right)\right\rceil \\
L_{q,k} &= \left\lfloor \frac{N}{T_b}\left(\frac{pT_b}{N} - (\tau_i - \tau_m) + T_g\right)\right\rfloor
\end{aligned} \tag{11}
$$

where $\lceil x \rceil$ denotes the smallest integer greater than x and $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. A special case of (10) is observed when $\tau_0 = \tau_1 = \ldots = \tau_{N-1}$. This case, known as the synchronous case is observed when all the users reach the base station with the same delay. In this case, there is no ISI from the $k-1^{th}$ or the $k+1^{th}$ bits if the condition for the sufficient guard interval is met. As can be seen from the simulation results, this translates into better BER performance, but asynchronous reception is more general and practical scenario.

III. Minimum Probability of Error Detector

The components of the received signal vector y have to be combined properly to form the decision variable at the receiver. MMSE detector tries to minimize the mean square between the filter output and the transmitted symbol. As it is the BER and not the MSE that is the deciding factor in a digital communication system, a detector that tries to minimize the BER is desired. The MPOE detector possesses this desired characteristic and hence achieves a lower BER.

A. Successive Demodulation Approach

At the receiver, we have the estimates of the previously tranmitted bits of all the users available to us. Therefore, if we are demodulating the k^{th} bit, we can subtract from the samples, the contribution of the $k-1^{th}$ bits of all the users. To demodulate k^{th} bits, we start with the 0^{th} user's samples. The samples for the 0^{th} user after ISI cancellation will contain, apart from the desired term and noise, the contribution from the k^{th} bits of the other users. Therefore, by applying a single tap filter, calculated on the MPOE/MMSE criterion, we can demodulate the k^{th} bit of the 0^{th} user. To demodulate the 1^{st} user, we do the ISI cancellation using the $k-1$ bits of all the users and the k^{th} bit of the 0^{th} user that we have just demodulated. The samples now contain contribution from the 0^{th} users $k+1^{th}$ bit and the other users k^{th} bits. Again, this can be demodulated by applying MPOE/MMSE criterion. As the users are demodulated one after the other, this approach is called Successive Demodulation approach. The approach is similar in implementation to the Successive Interference Cancellation (SIC) approach, applied in case of severe near far problem. The difference is that instead of subtracting the interference based on the received power levels, the interference cancellation order is based on the user delays.

B. Probability of Error

Let us assume that we wish to demodulate the m^{th} user's k^{th} bit. The estimates for the $k-1^{th}$ bits of all the users and the k^{th} bits of user $0, 1, \ldots, m-1$ are available with us. From the samples obtained by sampling the received signal, given by (7), we subtract the contribution from the $k-1^{th}$ bits of users $m+1, \ldots, N-1$ and the recently demodulated k^{th} bits of the users $0, 1, \ldots, m-1$. The resulting samples can be written as

$$
\begin{aligned}
y[p] = &\sum_{l=0}^{L_{m,k}} g_{m,l} \sum_{n=0}^{N-1} a_m^k c_{m,n} e^{j\left(\frac{2\pi n}{T_b} g(kT_s + \frac{pT_b}{N} - \frac{lT_b}{N})\right)} \\
&+ \sum_{i=0}^{m-1} \sum_{l=0}^{L_{i,k+1}} g_{i,l} \sum_{n=0}^{N-1} a_i^{k+1} c_{i,n} e^{j\left(\frac{2\pi n}{T_b} g(kT_s + \tau_m - \tau_i + \frac{(p-l)T_b}{N})\right)} \\
&+ \sum_{q=m+1}^{N-1} \sum_{l=0}^{L_{q,k}} g_{q,l} \sum_{n=0}^{N-1} a_q^k c_{q,n} e^{j\left(\frac{2\pi n}{T_b} g(kT_s + \tau_m - \tau_q + \frac{(p-l)T_b}{N})\right)} \\
&+ n[p]
\end{aligned} \tag{12}
$$

The above expression can be written in matrix and vector notation as

$$
Y = H_{eff} A + N \tag{13}
$$

where $Y = [y[0]\ y[1]\ \ldots\ y[N-1]]^T$, $A = [a_0^{k+1}\ \ a_1^{k+1}\ \ \ldots\ \ a_{m-1}^{k+1}\ \ a_m^k\ \ a_{m+1}^k\ \ \ldots\ \ a_{N-1}^k\ \ ]^T$, $N = [n[0]\ \ n[1]\ \ \ldots\ \ n[N-1]\ \ ]^T$ and H_{eff} is the effective channel matrix derived from the coefficients of the components of A vector, in equation (12). All the symbols represent time domain quantities.

Let $w_m = [w_0\ w_1\ \ldots\ w_{N-1}]^H$ be the filter vector used to demodulate the bits of the desired user (user m in our case). The filter output x_m can be written as

$$
\begin{aligned}
x_m &= w_m^H Y \\
x_m &= w_m^H H_{eff} A + w_m^H N
\end{aligned} \tag{14}
$$

The real part of x_m is sufficient and can be used to demodulate the user's bit

$$
\Re(x_m) = \Re(w_m^H H_{eff} A) + \Re(w_m^H N) \tag{15}
$$

Conditioned on the transmitted bit vector A, $\Re(x_m)$ is Gaussian with mean

$$
\mu_k = \Re(w_m^H H_{eff} A) \tag{16}
$$

and variance

$$
\sigma_m^2 = \frac{\sigma^2 \|w_m\|^2}{2} \tag{17}
$$

where σ^2 is the variance of each component of the noise vector N. Since the demodulated bit is $sgn\left(\Re(x_m)\right)$, $sgn(x)$ being the signum function, an error in demodulation occurs whenever

$$\Re(x_m|a_m^k = -1) > 0 \text{ or } \Re(x_m|a_m^k = +1) < 0 \quad (18)$$

Since $\Re(x_m)$ is Gaussian, with mean μ_k and variance σ_m^2, the pdf of $\Re(x_m)$ is

$$f_{\Re(x_m|A)}(u) = \frac{1}{\sigma_m\sqrt{2\pi}} exp\left(-\frac{(u-\mu_m)^2}{2\sigma_m^2}\right) \quad (19)$$

Conditioned on $a_m^k = +1$ the probability of error in demodulation is

$$P_e(x_m|a_m^k = +1) = \int_{-\infty}^{0} \frac{1}{\sigma_m\sqrt{2\pi}} exp\left(-\frac{(u-\mu_m)^2}{2\sigma_m^2}\right) du \quad (20)$$

and conditioned on $a_m^k = -1$, the probability of error in demodulation is

$$P_e(x_m|a_m^k = -1) = \int_{0}^{-\infty} \frac{1}{\sigma_m\sqrt{2\pi}} exp\left(-\frac{(u-\mu_m)^2}{2\sigma_m^2}\right) du \quad (21)$$

Using the standard Q function, the probability of error can be written as

$$P_e(x_m|A) = Q\left(\frac{a_m^k \mu_m}{\sigma_m}\right) \quad (22)$$

Since the vector A can take any of the 2^N possible values with equal probability, the average probability of error in demodulating the user can be written as

$$P_e = \frac{1}{2^N} \sum_{\forall A} Q\left(\frac{a_m^k \mu_m}{\sigma_m}\right) \quad (23)$$

The filter vector that minimizes the probability of error given by the above equation is the MPOE detector.

C. MPOE Algorithm

The filter vector that minimizes the probability of error given by expression (23) can be found adaptively by two different approaches. The first approach requires computation of the gradient of the function to be minimized. The gradient may be computed analytically or numerically. Algorithms like conjugate gradient method, variable metric method fall in this category. The other approach requires function and not gradient evaluation. Algorithms that fall in this approach are downhill simplex method and Powell's method.

We have used the method of *g*radient descent, as the gradient of (23) can be easily calculated. Starting from an initial guess, the filter weights are updated iteratively along the negative gradient of the probability of error surface. During the i^{th} iteration,

$$w_m^{i+1} = w_m^i - \mu \bigtriangledown (P_e(w_m)) \quad (24)$$

where μ is a suitably chosen step size. To compute the gradient of (23) we note that if $J(w)$ is a real valued cost function, then the gradient $\bigtriangledown J$ is given by [12]

$$\bigtriangledown J = 2\frac{\partial J}{\partial w^*} \quad (25)$$

$\frac{\partial J}{\partial w^*}$ is the conjugate derivative of the function $J(w)$. The conjugate derivative of (23) is given by

$$\frac{\partial P_e(w_m)}{\partial w_m^*} = -\frac{1}{\sqrt{2\pi}2^N} \sum_{\forall A} \left(e^{-\frac{\mu_m^2}{2\sigma_m^2}} \frac{\partial}{\partial w_m^*} \left(\frac{\mu_m a_m^k}{\sigma_m}\right)\right) \quad (26)$$

The term $\frac{\partial}{\partial w_m^*}\left(\frac{\mu_x a_0}{\sigma_x}\right)$ can be written as

$$\begin{aligned}
\frac{\partial}{\partial w_m^*}\left(\frac{\mu_m a_m^k}{\sigma_m}\right) &= \frac{\sigma_m a_m \frac{\partial}{\partial w_m^*}\mu_m - \mu_m a_m^k \frac{\partial}{\partial w_m^*}\sigma_m}{\sigma_m^2} \\
&= \frac{a_m^k}{\sigma_m}\left(\frac{\partial}{\partial w_m^*}\Re(w_m^H H_{eff} A) - \frac{\mu_m}{\sigma_m}\frac{\partial}{\partial w_m^*}\sigma\|w_m\|\right) \\
&= \frac{a_m^k}{\sigma_m}\left(H_{eff}A - \frac{\mu_m}{\|w_m\|}\frac{\partial}{\partial w_m^*}(w_m^H w)^{1/2}\right) \\
&= \frac{a_m^k}{\sigma_m}\left(H_{eff}A - \frac{\mu_m w_m}{\|w_m\|^2}\right)
\end{aligned} \quad (27)$$

Using the above, the gradient of (23) can be written as

$$\frac{\partial P_e(w_m)}{\partial w_m^*} = -\frac{1}{\sqrt{2\pi}2^N} \sum_{\forall A} \left(a_m^k e^{-\frac{\mu_m^2}{2\sigma_m^2}} \phi \right) \quad (28)$$

where ϕ is

$$\phi = \frac{1}{\sigma_m}\left(H_{eff}A - \frac{\mu_m w_m}{\|w_m\|^2}\right) \quad (29)$$

w can be normalized to unit length for computational advantage, but that makes the algorithm more sensitive to the step size μ. The computational complexity of the above algorithm can be reduced by noting that the expression (23) for the probability of error is a sum of 2^N terms, one for each bit vector A. The sum remains unaffected if A is replaced by -A, i.e., all $+1s$ are changed to $-1s$ and vice versa. Therefore we need to sum up only 2^{N-1} terms now instead of 2^N terms. Another way to reduce the complexity of the algorithm is to use a training sequence and minimize the probability of error conditioned on the transmitted bit vectors instead of the minimizing the average probability of error given by equation (23). This approach, known as the Minimum Conditional Probability of Error approach (MCPOE) has marginally inferior results compared to the MPOE approach [9] [11], but has a computational complexity that is linear in the number of users when in weight adaptation mode. This is a significant improvement over MPOE which is exponential and MMSE, which is polynomial complexity.

IV. SIMULATIONS

Four sets of simulations were carried out to test the performance of the MPOE detector. The first three sets involved synchronous reception and the last set considered asynchronous reception. In the first set of simulation the BER of MPOE

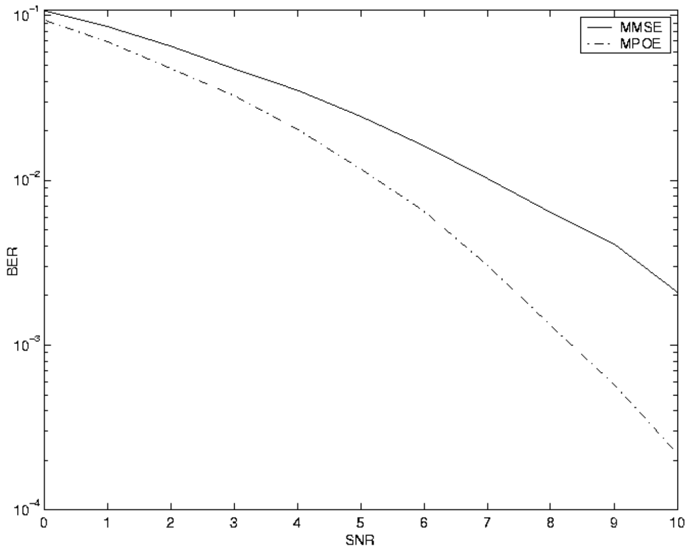

Fig. 1. BER of MPOE v/s MMSE for Synchronous MC-CDMA

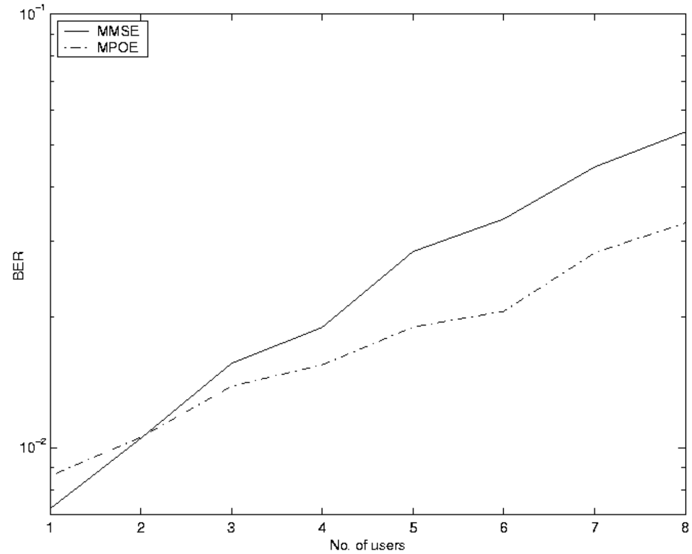

Fig. 2. BER of MPOE v/s MMSE for Synchronous MC-CDMA with increasing load at 5 dB SNR

detector is compared with the MMSE detector at different SNR levels, for a full load system (all users active). In the second set, the effect of increasing the load on the system at a fixed SNR is studied. In the third set, the effect of the near far problem is studied on the detectors by increasing the received power level of an interfering user. In the final set, the performance of the MPOE detector is again compared to the MMSE detector, but this time taking into account unequal user delays.

A. Simulation Parameters

The number of subcarriers in the simulations was 8 and the maximum number of active users was also 8. Walsh Hadamard codes of length 8 were used to separate the users at the mobile station end. Flat independent Rayleigh fading was assumed for each user and each subcarrier. Estimate of the channel matrix H_{eff} was assumed to be known at the base station.

To calculate the signal to noise ratio, we need to calculate the local mean power of the desired user's received signal. Since the contribution to the samples obtained as in (10) from the desired user involve a circular convolution, the mean power calculation in time domain is a tedious job. To circumvent this calculation, we can exploit the fact that circular convolution in time domain is equivalent to a point wise multiplication of the corresponding Discrete Fourier Transform (DFT). Therefore it is much more simpler to compute the power of the signal in the Fourier domain and then compute the equivalent time domain signal energy by using Parseval's relationship. Following this approach, the frequency domain power of the desired user's signal is calculated to be $\sigma^2_{ch,m}$ [13], where $\sigma^2_{ch,m}$ is the variance of the N random variables obtained by taking the N point DFT of the low pass channel impulse response coefficients $g_{m,l}$ introduced in (5). As the channel impulse response coefficients are I.I.D distributed rv's with each having Rayleigh distributed amplitude and independently and uniformly distributed phase, the resulting N random variables after the DFT are zero mean with variance equal to the sum of the mean power of the original Rayleigh random variables. Applying Parseval's relationship, we get the time domain signal power as $\frac{\sigma^2_{ch,m}}{N}$ and SNR as $\frac{\sigma^2_{ch,m}}{N\sigma^2}$, where σ^2 is defined in (17). Also let us define another quantity, Signal to Interference Ratio (SIR) as $SIR_i = \frac{\sigma_{ch,m}}{\sigma_{ch,i}}$. This quantity, which is a ratio of the mean power of the user of interest and mean power of the i^{th} user gives us a rough quantitative measure of the amount of the near far problem. Note that by definition $SIR_m = 0dB$ and that $SIR_i = 0dB$ shows that both the i^{th} and m^{th} users are being received with the same average power.

B. Simulation Results

Fig. 1. shows the results of the first set of simulations. It can be seen that for a full load system, MPOE detector outperforms the MMSE detector by more than 3 dB SNR for a BER of 10^{-3}. This translates to more than 100% saving in the transmitted power. Fig. 2. shows the effect of increasing load on MPOE and MMSE detector. Again it can be seen that as the number of active users in the system increases, the MPOE detector starts to outperform the MMSE detector. For these two sets of simulations if was assumed that all the users are being received with the same average power and with the same relative delay.

Fig. 3. shows the effect of increasing the received power of one interfering user. In this graph, the user of interest is the 0^{th} user and $SIR_7 = -6dB$, ie., the 7^{th} user is being received with four times more power than the user of interest. Also the other users are still being received at the same average power level as the user of interest, i.e., $SIR_i = 0$ for $i < 7$. It can be seen that although the performance of both the detector degrades, the MMSE detector suffers much more performance

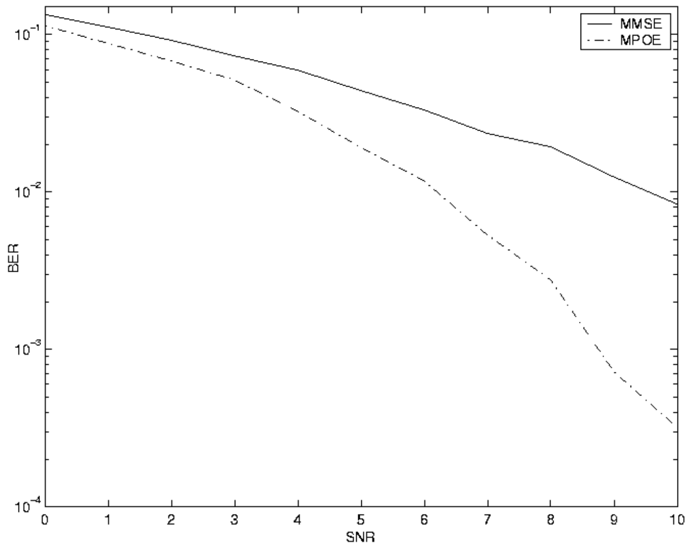

Fig. 3. BER of MPOE v/s MMSE for synchronous MC-CDMA with a stronger interfering user

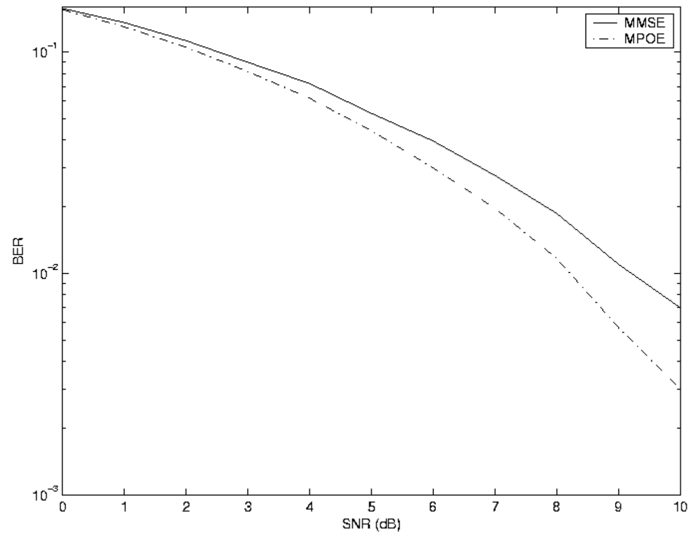

Fig. 4. BER of MPOE v/s MMSE for Asynchronous MC-CDMA

loss than the MPOE detector. For a BER of 10^{-2}, the MPOE detector gains about 1.5 dB over the MMSE detector when there is no such near far problem, but in case of stronger interfering user, the gain is about 3.5 dB.

Fig. 4. shows BER comparison of MPOE and MMSE detectors in the asynchronous reception case. As can be seen, the performance of both the detectors degrades, but still MPOE detector outperforms the MMSE detector by about a dB at a BER of 10^{-2}. The performance gain increases at lower BER.

V. Conclusions

A MPOE detector has been proposed in this paper for uplink MC-CDMA system. Performance of this detector has been compared with the MMSE detector both for the synchronous and asynchronous reception. In case of synchronous reception, the MPOE detector gain more than 3 dB over MMSE detector at a BER of 10^{-3} and full load assuming no near far problem. In presence of a stronger interfering user, the MPOE detector outperforms the MMSE detector with an even greater margin and gains about 3.5 dB at a BER of 10^{-2}. In case of no near far problem, but asynchronous reception, performance of both the detectors degrades, but the MPOE detector still gains about 1 dB over the MMSE detector at a BER of 10^{-2}. In case of synchronous reception this difference is about 1.5 dB at the same BER. On the downside, the MPOE detector has slightly higher computational complexity compared to the MMSE detector when in the weight adaptation mode. Two approaches to reduce the computational complexity of the detector have also been proposed in the paper.

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