

# What Works for Children with Mathematical Difficulties?

Ann Dowker  
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with Mathematical Difficulties?*

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The views expressed in this report are the author's and do not necessarily reflect those of the Department for Education and Skills.

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## CONTENTS

1.	EXECUTIVE SUMMARY .....	i
2.	THE SCOPE OF THE REVIEW .....	1
3.	METHODS .....	1
4.	THE PROBLEM OF MATHEMATICAL DIFFICULTIES .....	2
4.1	Individual differences in mathematics .....	2
4.2	Children's mathematical difficulties .....	3
4.2.1	The incidence and characteristics of arithmetical difficulties .....	3
4.2.2	Conclusion .....	5
4.3	Arithmetical ability is made up of many components .....	5
4.4	Which aspects of arithmetic are most likely to create problems? .....	7
4.5	Arithmetical difficulties in relation to verbal and spatial ability .....	9
4.6	Arithmetical difficulties in children with dyslexia and children with speech, language and communications needs .....	11
4.7	Arithmetic in people with general learning difficulties .....	13
4.8	Arithmetical difficulties: specific disorder or low end of a continuum? .....	14
4.9	Severe specific difficulties with arithmetic .....	15
5.	IMPLICATIONS FOR INTERVENTION .....	16
5.1	The problem of mathematics anxiety .....	16
5.2	The need to take account of the componential nature of arithmetic .....	16
6.	WHAT DOES WORK FOR MATHEMATICAL DIFFICULTIES? .....	17
6.1	Why schools need to take account of mathematical difficulties .....	17
6.1.1	Ability grouping .....	18
6.1.2	Individualized and small group work within a class .....	19

6.1.3	The role of Teaching Assistants .....	23
7.	INTERVENTION PROGRAMMES .....	23
7.1	Preschool intervention programmes to prevent the later development of mathematical difficulties .....	24
7.2	Interventions with school-age children with arithmetical difficulties .....	27
7.2.1	Peer tuition and group collaboration .....	27
7.2.2	Training children in Piagetian operations .....	28
7.2.3	Training in metacognition .....	29
7.2.4	Group interventions more specific to arithmetic .....	30
8.	INDIVIDUALIZED INTERVENTION IN ARITHMETIC .....	31
8.1	Assessments for targeted intervention .....	31
8.2	Some of the history of individualized remedial work .....	32
8.3	Recognizing and avoiding potential problems with individualized instruction and remediation .....	34
8.4	Individualized intervention programmes with primary school children .....	35
8.4.1	Mathematics Recovery .....	38
8.4.2	Numeracy Recovery .....	39
8.5	Computer programs for individual instruction and intervention .....	41
9.	INTERVENTIONS CURRENTLY USED WITHIN THE NATIONAL NUMERACY STRATEGY .....	43
10.	CHILDREN WITH GENERAL LEARNING DIFFICULTIES: THE SPIRAL MATHEMATICS PROGRAMME .....	44
11.	CONCLUSIONS .....	45
11.1	Need for further development and investigation of intervention programmes ..	45
11.2	Implications for educational practice .....	46
	BIBLIOGRAPHY .....	49

## 1. EXECUTIVE SUMMARY

### *Rationale*

There has long been some concern over children with numeracy difficulties, and research on methods of helping such children. Recently, there has been an increased emphasis on numeracy in educational policy and practice in Britain as well as in some other countries, resulting in an increased number of studies of the nature of such difficulties, and of intervention techniques. However, many of the studies that have been carried out on these topics have not received wide publicity, and the research base is quite fragmented. The aims of the review included bringing together the results of such studies, and discussing implications for further research and for teaching.

### *Methods*

The research methodology for preparing this review has involved consulting and reviewing books, papers and research reports dealing with children's mathematical difficulties, and with the intervention techniques used to address them. The materials consulted included books and journals in Oxford University libraries; major Education and Social Science databases; the proceedings of conferences of major relevant organization; and a few yet unpublished documents involving action research in schools and local authorities, and work in progress within the DfES.

### *The problem of mathematical difficulties*

Several studies have investigated the prevalence of learning difficulties in mathematics. The results vary according to the nature of the samples, and criteria used. The main conclusion one can gain from most studies is that many children have difficulties with mathematics, and a significant number have relatively *specific* difficulties with mathematics. Such difficulties appear to be equally common in boys and girls, in contrast to language and literacy difficulties which are more common in boys.

Ultimately, the criteria for describing children as having 'mathematical difficulties' must involve not only test scores, but the children's educational and practical functioning in mathematics. Within the present British educational system, it may be considered that children who are working at Level 1 at age 7, or Level 3 at age 11, have some degree of mathematical difficulty; and that those who have failed to reach the above levels at these ages have quite marked mathematical difficulties.

## *Arithmetical ability is made up of many components*

In order to study the *nature* of the arithmetical difficulties that children experience, and thus to understand the best ways to intervene to help them, it is important to remember one crucial thing: arithmetic is not a single entity, but is made up of many components. These include knowledge of arithmetical facts; ability to carry out arithmetical procedures; understanding and using arithmetical principles such as commutativity and associativity; estimation; knowledge of mathematical knowledge; applying arithmetic to the solution of word problems and practical problems; etc.

Experimental and educational findings with typically developing children, adults with brain damage, and children with mathematical difficulties have shown that it is possible for individuals to show marked discrepancies between almost any two possible components of arithmetic.

### *Common types of mathematical difficulty: what aspects of arithmetical thinking tend to cause most trouble?*

Despite such variable patterns of strengths and weaknesses, some areas of arithmetic do appear to create more problems than others for children. One of the areas most commonly found to create difficulties is *memory for arithmetical facts*. For some children, this is a specific, localized problem; for children with more severe mathematical difficulties it may be associated with exclusive reliance on cumbersome counting strategies. Other common areas of difficulty include *word problem solving*, *representation of place value* and the ability to solve *multi-step* arithmetic problems.

### *Arithmetical difficulties in relation to other cognitive difficulties*

Mathematical difficulties often (by no means always) co-occur with *dyslexia* and other forms of language difficulty. In particular, people with dyslexia usually experience at least some difficulty in learning number facts such as multiplication tables.

Overall, however, people with *general learning difficulties* tend to show similar arithmetical performance and strategies to typically developing individuals *of the same mental age*.

### *Severe specific difficulties with arithmetic*

The general evidence is that arithmetical difficulties are part of a continuum of ability. However, a few individuals do have specific difficulties with arithmetic, which do not resemble anything observed in the general population: for example, they may be unable to recognize small quantities of objects (even as low as two or three) without counting them. The term 'dyscalculia' is sometimes reserved for such individuals, though it is sometimes used for any individual with relatively specific mathematical difficulties.

### *Implications for intervention*

Interventions can take place successfully at any time. However, it is desirable that interventions should take place at an early stage, partly because mathematical difficulties can affect performance in other aspects of the curriculum, and partly to prevent the development of negative attitudes and *mathematics anxiety*.

Crucially when planning interventions, it is important to take account of the overwhelming evidence that arithmetical ability is not unitary. It is made up of many components, ranging from knowledge of the counting sequence to estimation to solving word problems. Weaknesses in any one of them can occur relatively independently of weaknesses in the others. Thus, interventions that focus on the particular components with which an individual child has difficulty are likely to be most effective.

### *Strategies for dealing with mathematical difficulties within a class*

There are several guides for teachers, influenced both by research findings and by teachers' reported experience, regarding strategies for dealing with individual differences within a class, and including children with mathematical difficulties. Common approaches include provision of a variety of differentiated activities for different pupils working on the same topics; and including revision and consolidation sessions for all pupils. Streaming and setting are sometimes used. However, some studies suggest that they may have deleterious effects on low achievers' performance, both because they may lead to self-fulfilling prophecies, and because they may give the teachers false expectations of homogeneity of ability and performance, and lead to taking too *little* account of individual differences.

### *Teaching Assistants*

One method of assisting children with mathematical difficulties within a classroom is to employ Teaching Assistants to provide additional support for such pupils. This is not a panacea in itself, but may have beneficial effects if carried out appropriately; and, in particular, if the Teaching Assistants are suitably trained for this role.

### *Intervention projects with preschoolers.*

Some intervention programmes target children from low socio-economic groups, who show an increased incidence of educational difficulties, and are aimed at preventing such difficulties from arising in the first place. They often involve preschool children and are not always restricted to numeracy. They expose such children to mathematical activities and games, and often involve the parents in the interventions.

### *Individualized intervention with school-age children's arithmetic*

There are several individualized component-based intervention projects that take into account individual children's strengths and weaknesses in specific components of arithmetic, and the ways in which they use mathematical rules and strategies, including misapplications of rules. Some of these projects are totally individual; some include small-group work at least in part, but include individualized assessments.

Such targeted intervention techniques have a surprisingly long history; several such techniques were developed and used in North America in the early twentieth century. Recent targeted interventions have included Denvir and Brown's intervention programme; and currently Mathematics Recovery and Numeracy Recovery.

### *Computer programs for individual instruction*

A number of computer programs have been developed for individualized instruction and intervention. Most studies of computer-based intervention with children have yielded positive results, but computer-based interventions alone appear in general result in less progress than interventions carried out by teachers.

### *Interventions currently used within the National Numeracy Strategy*

The National Numeracy Strategy is developing intervention techniques for children who are struggling with arithmetic. The Numeracy Strategy incorporates three 'waves' or levels of intervention: Wave 1, or whole-class teaching for all children (e.g. the Daily Mathematics Lesson); Wave 2, or interventions in small groups with children who are experiencing mild or moderate difficulties in the subject; and Wave 3, or targeted interventions for children with special educational needs.

The main Wave 2 intervention programme is the Springboard programme. Wave 3 materials have been piloted since June 2003, and are still in draft form, and awaiting full evaluation. They emphasize individualized diagnosis of the errors and misconceptions shown by children with significant difficulties in mathematics (usually at least one National Curriculum level below age-related expectations). They are given diagnostic interviews to determine their difficulties, and materials are being developed to correct the errors and misconceptions that are observed. Many of the materials involve presenting children with a variety of words, symbols, models and images to represent the same concept or process.

### *Conclusions*

Research strongly suggests that:

**Children's arithmetical difficulties are highly susceptible to intervention.**

**Individualized work with children who are falling behind in arithmetic has a significant impact on their performance. The amount of time given to such individualized work does not, in many cases, need to be very large to be effective.**

Future goals should include further development and investigation of individualized and small-group interventions, especially with younger children than those who have been most frequently studied so far.

## 2. *The scope of the review*

In recent years, there has been an increased emphasis on numeracy in educational policy and practice, at least in Britain (DfEE, 1999; also see Mertsens, 1999; Straker, 1999; Thompson, 2000; Askew and Brown, 2001). There is, however, still a considerably smaller research base on numeracy difficulties and interventions than, for example, on corresponding issues within literacy. There is agreement that far more research is needed on mathematical development and mathematical difficulties and that policy needs to be more informed by research (Thompson, 2000; Askew and Brown, 2001).

Moreover, many of the studies that *have* been carried out on these topics have not received wide publicity, and the research base is quite fragmented. This is because the research has been carried out over a long period; in different countries; and within different disciplines (education, psychology and neuroscience). People are often unaware of relevant research that has been carried out in other countries, other disciplines, or at an earlier time. This situation limits the scope and application of such research. Communication between researchers, teachers and other practitioners, and policy-makers has been still more limited, and is crucial if the existing research is to have appropriate application, and if suitable future projects are to be devised.

This review will discuss findings with regard to the incidence of mathematical difficulties; and their common characteristics. It will discuss the important fact that arithmetical ability is not a single entity, but is made up of many components; and therefore arithmetical difficulties are varied and heterogeneous. It will discuss their relationships to other abilities: i.e. general cognitive abilities, language, reading and spatial abilities. It will then discuss some of the methods of intervention that have been used over the years. It will conclude with some recommendations for further educational research, and for practice in schools.

Although mathematics includes much more than arithmetic (e.g. geometry; measurement; algebra), most studies of mathematical difficulties have focused on problems with number and arithmetic. This review will therefore focus predominantly on this topic.

## 3. *Methods*

The research methodology for preparing this review has involved consulting and reviewing books, papers and research reports dealing with children's mathematical difficulties, and with the intervention techniques used to address them. Although the central focus is on work carried out relatively recently within the United Kingdom, the research has also examined relevant studies from the United States, Australia, Europe, Japan and elsewhere. Important studies from the first half of the twentieth century, which provide an important background to current research, have been reviewed.

Material was obtained by consulting books and journals in the Oxford University Bodleian Library and the libraries of the Departments of Experimental Psychology and Educational Studies; and by consulting major Education and Social Science databases. These have included the British Educational Index; ERIC; the International Bibliography of the Social Sciences; the Social Science Citation Index; PsychInfo; and the Mathematics Didactics

Database. The research also consulted the proceedings of conferences of major relevant organizations including the British Educational Research Association, the American Educational Research Association; and the international Psychology of Mathematics Education group. It also made use of international statistical surveys of pupils' achievement and progress in mathematics: especially the TIMSS (Third International Mathematics and Science Survey). The Department for Education and Skills gave the researcher access to a few as yet unpublished documents, especially those involving action research in schools and local authorities, and work in progress on Wave 3 interventions.

The research has been aimed at reviewing the studies; at comparing them where possible (though studies in such areas have tended up to the present to be too diverse for true 'meta-analyses' to be used); and at drawing conclusions both about the nature of mathematical difficulties, and best practice in preventing and remediating them.

#### 4. *The problem of mathematical difficulties*

##### 4.1 *Individual differences in mathematics*

It is well known that individual differences in arithmetical performance are very marked in both children and adults. For example, Cockcroft (1982) reported that an average British class of eleven-year-olds is likely to contain the equivalent of a seven-year range in arithmetical ability. Despite many changes in UK education since then, including the introduction of a standard National Curriculum and a National Numeracy Strategy, almost identical results were obtained by Brown, Askew, Rhodes et al (2002). They found that the gap between the 5th and 95th percentiles on standardized mathematics tests by children in Year 6 (10 to 11-year-olds) corresponded to a gap of about 7 chronological years in 'mathematics ages'.

Individual differences in arithmetic among pupils of the same age are also very great in most other countries, as was found, for example, in the TIMSS (1996) survey of the performance of 14-year-olds in over 30 countries. It is sometimes suggested that these individual differences are less pronounced in Pacific Rim countries (TIMSS, 1996), and that the incidence of serious mathematical difficulties is lower in these countries.

However, careful analysis of the TIMSS results (Schmidt, McKnight, Cogan, Jackwerth and Houang, 1999) and some within-country studies (e.g. Tsuge, 2001 in Japan) has shown that individual differences are in fact also great in these countries, and that a significant number of pupils do have difficulty in keeping up with their peers.

##### 4.2 *Children's mathematical difficulties*

Children's numeracy difficulties can take several forms. Some children have difficulties with many academic subjects, of which arithmetic is merely one; some have specific *delays* in arithmetic, which will eventually be resolved; and some have persisting, specific problems with arithmetic. The *causes* for such difficulties are also varied, though they tend to overlap: they include individual characteristics (e.g. unusual patterns of brain development); inadequate or inappropriate teaching; and lack of preschool home experience with mathematical activities and language. The type and extent of intervention needed to address

arithmetical difficulties will depend in part on the nature and causes of these difficulties. For example, intervention is more necessary for children with persisting arithmetical difficulties than for those with temporary delays.

#### 4.2.1 *The incidence and characteristics of arithmetical difficulties*

There have been several studies of arithmetical difficulties and their characteristics. As will be seen in the section on the history of individualized interventions, research has been carried out on arithmetical errors and faulty procedures since at least the 1920s.

Many people experience difficulties with mathematics. For example, Bynner and Parsons (1997) gave some Basic Skills Agency literacy and numeracy tests to a sample of 37-year-olds from the National Child Development Study cohort (which had included all individuals born in Britain in a single week in 1958). The numeracy tests included such tasks as working out change, calculating area, using charts and bus and train timetables, and working out percentages in practical contexts. According to the standards laid down by the Basic Skills Agency, *nearly one-quarter* of the cohort had 'very low' numeracy skills that would make everyday tasks difficult to complete successfully. This proportion *was about four times as great* as that classed as having very low literacy skills. Most of the adults with numeracy difficulties had already been experiencing difficulties with school mathematics at the age of 7.

The reasons for these people's mathematical difficulties are undoubtedly various. Some would have had insufficient or inappropriate instruction in mathematics. Some would have had learning difficulties affecting many subjects. There is no doubt, however, that some individuals experience learning difficulties that are relatively specific to mathematics.

Several studies have investigated the prevalence of learning difficulties in mathematics.

For example, Lewis, Hitch and Walker (1994) studied 1056 unselected 9-to 10-year-olds English children (the entire age group within a particular, socially highly heterogeneous, local education authority; excluding only those assessed as having severe general learning difficulties). They were given the Ravens Matrices IQ test; Young's Group Mathematics Test; and Young's Spelling and Reading Test. 1.3% of the sample had specific arithmetical difficulties, defined as an arithmetic scaled score of 85 or below despite a Ravens IQ score of 90 or above. A further 2.3% had difficulties in both reading and arithmetic (scaled scores of 85 or below in both the reading and arithmetic tests) despite a Ravens IQ score of 90 or above. Thus, the prevalence of arithmetic difficulties in children of at least average cognitive ability was 3.6%. The children with arithmetical difficulties were equally divided as to gender; which contrasts with the general finding that boys are far more likely than girls to have language and literacy difficulties.

Gross-Tsur, Manor and Shalev (1996) assessed the incidence of dyscalculia in a cohort of 3029 Israeli 11- to- 12-year-olds. The 600 children who scored in the lowest 20% on a standardized city-wide arithmetic test were selected for further testing. 555 were located and given an individualized arithmetic test battery previously constructed and standardized by the authors. This included reading, writing and comparing numbers; comparing quantities; simple calculations; and more complex (multi-digit) calculations. 188 children or 6.2% of the total were classified as having dyscalculia, using the criterion of a score equal or below the mean for children two years younger. 143 of these children were located and received

parental consent for further testing. This included the WISC-R IQ test, and reading and spelling tests standardized on 70 age-matched typically developing children. 3 children were excluded from the 'dyscalculic' group because they obtained IQ tests below 80. Of the 140 dyscalculic children, 75 were girls and 65 were boys, once again indicating an approximately equal gender distribution. Their IQs ranged from 80 to 129, with a mean of 98.2. They were assessed for symptoms of other learning problems. The researchers diagnosed 17% as dyslexic, and 26% as having symptoms of attention deficit hyperactivity disorder. They came from significantly lower socio-economic backgrounds than the children without dyscalculia. 42% had first-degree relatives with specific learning disabilities.

Bzufka, Hein and Neumarker (2000) studied 181 urban and 182 rural German third-grade pupils. They were given standardized school achievement tests of arithmetic and spelling. 12 children in each sample (about 6.6% of the whole population) performed above the 50th percentile in spelling, but below the 25th percentile in mathematics. When the urban and rural children were compared, they showed little difference in incidence of specific spelling or mathematics difficulties, but the urban children [who were on the whole of lower socio-economic background] were far more likely than the rural children to have difficulty with both (48.6% versus 3.3%).

Thus, the incidence of mathematical learning difficulties depends widely between studies, depending on the methods and criteria used. For example, they have used different IQ tests; different mathematics tests –which may be emphasizing quite different components; and different cut-off points for establishing deficit in both IQ and mathematics. Moreover, it has been pointed out (Mazzocco and Myers, 2002; Desoete, Roeyers and DeClercq, 2004) that findings about the incidence, nature and outcomes of mathematical difficulties may vary considerably, depending on whether one uses criteria of *discrepancy* between mathematical performance and IQ, *severity* of mathematical weaknesses, or *persistence* of mathematical weaknesses. Given the marked differences in criteria between studies, it is perhaps inappropriate to focus too much on the exact numbers obtained. The main conclusion one can gain from most studies is that many children have difficulties with mathematics, and a significant number have relatively *specific* difficulties with mathematics.

#### 4.2.2 Conclusion

Ultimately, the criteria for describing children as having 'mathematical difficulties' must involve not only test scores, but the children's educational and practical functioning in mathematics. Within the present British educational system, it may be considered that children who are working at Level 1 at age 7, or Level 3 at age 11, have some degree of mathematical difficulty; and that those who have failed to reach the above levels at these ages have quite marked mathematical difficulties.

#### 4.3 *Arithmetical ability is made up of many components*

In order to study the *nature* of the arithmetical difficulties that children experience, and thus to understand the best ways to intervene to help them, it is important to remember one crucial thing: arithmetic is not a single entity: it is made up of many components, including knowledge of arithmetical facts; ability to carry out arithmetical procedures; understanding and using arithmetical principles such as commutativity and associativity; estimation;

knowledge of mathematical knowledge; applying arithmetic to the solution of word problems and practical problems; etc.

Experimental and educational findings with typically developing children (Ginsburg, 1977; Dowker, 1998) and adults (Geary and Widaman, 1992) have shown that it is possible for individuals to show marked discrepancies between almost any two possible components of arithmetic. For example, Dowker (1998) studied calculation and arithmetical reasoning in 213 unselected children between the ages of 6 and 9. She reported (p. 300) that “(1) individual differences in arithmetic are relatively marked; (2) that arithmetic is indeed not unitary and that it is relatively easy to find children with marked discrepancies [in either direction] between [almost any two] different components; and that (3) in particular it is risky to assume that a child “does not understand maths” because he or she performs poorly in some calculation tasks”.

Studies of adults who have arithmetical difficulties as a result of brain damage (Dehaene, 1997; Butterworth, 1999) show that almost any component of arithmetic can be selectively impaired: e.g. patients can show double dissociations between estimation and calculation; memory for facts and following procedures; written versus oral arithmetic; different arithmetical operations such as subtraction versus multiplication; etc. Detailed case studies of children with mathematical difficulties (usually not associated with obvious brain damage) have also shown extreme discrepancies between different types of mathematical ability. For example, Temple (1991) reports one child who could carry out arithmetical calculation procedures correctly but could not remember number facts, and another child who could remember the facts but not carry out the procedures.

Macaruso and Sokol (1998) studied 20 adolescents with both dyslexia and arithmetical difficulties, and found that the arithmetical difficulties were very heterogeneous, and that factual, procedural and conceptual difficulties were all represented.

Desoete, Roeyers and De Clercq (2004) studied 37 Belgian third grade pupils (8 to 9-year-olds) with mathematical difficulties, as demonstrated in both their school performance, and scores of at least two standard deviations below the mean on at least one mathematics test. Many showed discrepancies in their performance on tests of *number knowledge and mental arithmetic*; *memory for number facts*; and *word problem solving*. Only when children were given *all three* of the tests, were all 37 identified as having mathematical difficulties.

Children, with and without mathematical difficulties can indeed have strengths and weaknesses in almost *any* area of arithmetic.

Ginsburg (1972, 1977) and his colleagues carried out several individual case studies of children who were failing in school mathematics. Such children typically combined significant strengths with specific weaknesses. Some had a good informal understanding of number concepts, but had trouble in using written symbolism and standard school methods. Some had particular difficulties with the *language* of mathematics. Some children appeared to have very limited number understanding at first sight, but still had a good understanding of counting techniques and principles. Though some patterns of strengths and weaknesses were more common than others, some children showed unusual and distinctive patterns.

In view of this variability of patterns of strengths and weaknesses, Ginsburg recommends the use of clinical interview techniques as part of assessments, in order to understand a child's

specific strengths and weaknesses, and the reasons for their errors: "Standard tests...usually provide only vague characterizations of a child's performance. They show perhaps that he or she does well or poorly. But usually you already know that...(C)hildren's mathematical thinking is complex. You need to understand their intuitions, their errors, their invented strategies."

Further evidence for variable patterns of strengths and weaknesses comes from the work of Denvir and Brown (1986a), who worked with 7-to 9-year-old children who were low attainers in arithmetic. 41 children from four classes, who were identified by their teachers as low attainers, were given the NFER Basic Maths Tests, Maths Tests, and also a diagnostic assessment test battery, developed by Denvir and Brown.

The major aspects included in the diagnostic tests included: (i) strategies for adding and subtracting small numbers in 'sums' and word problems; (ii) commutativity of addition; (iii) enumerating grouped collections; (iv) strategies for adding larger numbers, and dealing with place value; and (v) Piagetian tasks including number conservation and class inclusion. Each aspect included numerous specific items. There was an approximate order of difficulty of items, ranging from "makes 1:1 correspondence" to "mentally carries out two-digit 'take away' with regrouping".

When each of the skills was ordered according to facility and each of the pupils ordered by overall raw score, it was possible to group the skills into 'levels' defined by a particular range of facility so that every pupil who had succeeded in 2/3 of the skills at any level had succeeded in 2/3 of the skills at every preceding level. However, it was not possible to establish an exact hierarchy of skills, such that the more advanced skills invariably followed on from the easier skills. Some skills did indeed seem to be prerequisite to other skills (e.g. counting collections grouped in tens for tasks that involved adding tens and units) but many seemed almost independent of each other, and some tended to develop in one order, but could occur in the reversed order (e.g. most children who could carry out 2-digit addition without regrouping mentally could also carry out 2-digit addition with regrouping using base ten apparatus; but there were some who could do the former without the latter.) Thus, although the study does establish *approximate* hierarchies of skills, it also study supports the componential theory of arithmetic, with the possibility of double dissociations/discrepancies between at least some of the components.

#### 4.4 Which aspects of arithmetic are most likely to create problems?

Despite the variable patterns discussed above, here are particular areas of arithmetic that do appear to create more problems than others for children. One of the areas most commonly found to create difficulties is *memory for arithmetical facts*.

Studies of children with mathematical difficulties show them to be more consistently weak at retrieving arithmetical facts from memory than at other aspects at arithmetic. They often rely on counting strategies in arithmetic at ages when their age-mates are relying much more on fact retrieval (Russell and Ginsburg, 1984; Siegler, 1988; Geary and Brown, 1991; Ostad, 1997, 1998; Cumming and Elkins, 1999; Fei, 2000).

Jordan, Hanich and Kaplan (2003) tested American second grade children for knowledge of addition and multiplication facts. 45 children with poor arithmetic fact mastery were compared with 60 children with good arithmetic fact mastery. They were followed up longitudinally

through second and third grade. The children with poor fact mastery showed little improvement on timed number fact tests in over a year, but showed normal progress in other aspects of mathematics. When IQ was held constant, the children with poor fact mastery performed similarly with good fact mastery in tests of reading and mathematics word problem solving at the end of third grade.

Thus, difficulties in memory for arithmetic facts tend to be persistent. They appear to be independent of reading skills, and did not affect performance on other aspects of arithmetic.

Part of the reason for the associations between number fact retrieval and more general arithmetical performance lies in the ways in which arithmetical performance is often assessed. If arithmetical tests and assessments emphasize fact retrieval, then those who are poor at fact retrieval are likely to do badly in the tests, and be classed as having arithmetical difficulties. If arithmetical fact retrieval is emphasized in the school curriculum, then those who are weak at this aspect of arithmetic will struggle with their school arithmetic lessons and assignments, even if they have no difficulty with other aspects of arithmetic.

It does, however, seem that arithmetic fact retrieval difficulties have effects beyond tasks that emphasize such facts, and spill over into other areas of mathematics. If people have trouble in remembering basic arithmetic facts, then they will have to calculate these facts by alternative and usually more time-consuming strategies. Even if they are able to do so accurately, this means that they must devote time and attention to obtaining facts that someone else might retrieve automatically; and this will divert time and attention from other aspects of arithmetical problem-solving, resulting in lower efficiency.

There do seem to be different degrees and forms of difficulty with number facts and resulting reliance on counting strategies. Some children, such as those in the studies by Russell and Ginsburg (1984) and Jordan et al (2003), seem to have specific, localized difficulties with fact retrieval, and to be able to use a wide variety of alternative strategies. Other children (Gray 1997; Ostad, 1997, 1998) seem to rely narrowly and exclusively on counting strategies, and fail to use any other form of strategy, including derived fact strategies. To add to their difficulties, such children are often less efficient than others at using the very counting strategies on which they rely the most.

For example, Ostad (1997) studied Norwegian children with mathematical difficulties. The study included 32 children with and 32 children without mathematical difficulties in Grade 1; 33 children with and 33 children without mathematical difficulties in Grade 3; and 36 children with and 36 children without mathematical difficulties in Grade 5. The children with mathematical difficulties were those who scored below the 14th percentile on a Norwegian standardized mathematics achievement test. The pupils were asked to solve 28 single-digit addition problems on two different occasions separated by a period of two years. Their strategies on each problem were recorded. The children with mathematical difficulties used almost exclusively counting-based strategies, while those without such difficulties children were more likely to use retrieval or derived fact strategies. Moreover, children without mathematical difficulties increased their use of retrieval and decreased their use of counting-based strategies as they grew older, while the strategies of the children with mathematical difficulties did not change with age. At all ages, children without mathematical difficulties used a far wider variety of strategies than those with mathematical difficulties, and the differences increased with age.

It should, however, be noted that, while difficulty in remembering number facts is a very common component of arithmetical difficulties, not all children with arithmetical difficulties have this problem. Most of the children in Dowker's intervention study (Dowker, 2004, in press) certainly did experience problems with number facts; but not all did. Some children could remember many number facts, but seemed to lack strategies (including suitable counting strategies) for working out sums when they did not know the answer. Some other children could deal with single-digit arithmetic but had serious difficulty in achieving even limited understanding of tens, units and place value.

Russell and Ginsburg (1984) found that difficulties with *word problem solving*, as well as with memory for facts, characterized 9-year-old children who were described by their teachers as weak at arithmetic. In this study, most were not significantly worse than others at tasks involving estimation, derived fact strategy use, or understanding the relationships between tens and units.

Bryant, Bryant and Hammill (2000) found that several difficulties were common in children with mathematical weaknesses, but that the commonest problem was a difficulty in *carrying out multi-step* arithmetic.

They carried out a large-scale study of the characteristics of children and young people with mathematical difficulties. The participants were 1724 American pupils from 8;0 to 18;11, diagnosed as learning disabled and receiving special education services. 870 were rated by their teachers as having mathematical weaknesses; 854 were not.

The researchers constructed a list of 33 mathematical behaviours derived from consulting the literature on developmental and acquired dyscalculia. Items included "difficulty with word problems"; "difficulty with multi-step problems"; "does not recognize operator signs"; "does not verify numbers, and settles for first answer"; etc. Teachers were asked to check the items on the list that applied to each pupil.

Stepwise multiple regression showed that just under 31% of the variance between the groups with and without mathematical weaknesses was caused by a single item: "Has difficulty with multi-step problems and makes borrowing errors". 7 other items contributed significantly to group membership (though to a much lesser extent, only accounting in total to just under 5% of the variance). These were: "Cannot recall number facts automatically"; "Misspells number words"; "Reaches unreasonable answers"; "Calculates poorly when order of digits is altered"; "Cannot copy numbers accurately"; "Orders and spaces numbers inaccurately in multiplication and division"; and "Doesn't remember number words."

The discussion so far has concerned arithmetical processes, concepts and skills that are important from the earliest stages of arithmetical learning. Weaknesses in these areas are noticeable in primary school children, though they often persist into adolescence and adulthood. Weaknesses can, and frequently do, arise in various components of secondary school mathematics. Hart (1981) and her team found that secondary school pupils have many difficulties, both procedural and conceptual, with many mathematical topics, including ratio and proportion; fractions and decimals; algebra; and problems involving area and volume. She concluded (p. 209) that "mathematics is a *very difficult* subject for most [secondary school] children" and that "understanding improves only slightly as the child gets older". Since difficulties that are specific to secondary school mathematical topics are indeed very frequent, they are not usually seen as special difficulties in mathematics, and are not the subject of this

review. It must be remembered, however, that if early difficulties are not remediated, they are likely to result in very severe difficulties with those more advanced topics that tend to present problems for many people.

#### *4.5 Arithmetical difficulties in relation to verbal and spatial ability*

Reasoning, including arithmetical reasoning, can be carried out in many ways. Two broad categories that are often discussed with regard to individual differences are verbal and spatial reasoning. Information can be represented, manipulated and analysed in words; it can also be represented, manipulated and analysed in terms of visual-spatial imagery.

A number of researchers have investigated the issue of whether arithmetical skills are particularly associated with verbal or spatial reasoning and/or with discrepancies between the two. The factor analytic studies used to construct some commonly used IQ scales have consistently placed the Arithmetic subtest (one which emphasizes word problem solving) within the Verbal scale. However it has sometimes been suggested that spatial difficulties are particularly associated with difficulties in arithmetical reasoning. Rourke (1983, 1993) proposed that verbal weaknesses lead to memory difficulties and that nonverbal weaknesses lead to logical difficulties.

He proposed two basically different groups of children with arithmetical learning difficulties. Children in the first group have difficulties in retrieval of number facts and in working memory, but have a reasonably good understanding of number concepts. They have higher nonverbal than verbal IQs and often have difficulties with reading as well as mathematics. Children in the second group do not have memory problems but do have conceptual problems; they have higher verbal than nonverbal IQs; are less likely to have reading or language difficulties, but more likely to have spatial and social difficulties associated with right hemisphere deficits.

A few studies by Rourke and others (e.g. Robinson, Menchetti and Torgesen, 2002) have supported the view that children with both reading and mathematical deficits tend to have more memory difficulties but fewer conceptual difficulties than those with just mathematical deficits.

However, there has been no consistent support for the view that 'left hemisphere'-type verbal deficits are associated with procedural and factual memory difficulties in arithmetic, while 'right-hemisphere'-type nonverbal deficits are associated with conceptual difficulties in arithmetic. Shalev, Manor, Amir, Weirman and Gross-Tsur (1997) found no differences in the types of mathematical difficulty demonstrated by dyscalculic children with higher verbal versus higher non-verbal IQ.

Jordan and Hanich (2000) studied 76 American second-grade children. They were divided into four achievement groups: 20 children with normal achievement in reading and mathematics; 10 children with difficulties in both reading and mathematics (MD-RD), 36 children with difficulties in reading only (RD) and 10 children with difficulties in mathematics only (MD). They were given tests of four areas of mathematical thinking: number facts, story problems, place value and written calculation. Children with MD/RD performed worse than NA children on all aspects of mathematics; those with MD performed worse than NA children only on story problems.

Hanich, Jordan, Kaplan and Dick (2001) similarly divided 210 second-graders into four achievement groups: children with normal achievement in reading and mathematics; children with difficulties in both reading and mathematics (MD-RD), children with difficulties in reading only (RD) and those with difficulties in mathematics only (MD). Both MD groups performed worse than the other groups in most areas of arithmetic. The MD-only group outperformed the MD-RD group in both exact mental calculation and problem solving. The two MD groups performed similarly on written calculation, place value understanding, and approximate arithmetic.

Geary, Hoard and Hamson (1999) studied 90 first-grade children in the average IQ range. They included 35 children with normal achievement in reading and mathematics (N); 15 children with mathematical difficulties (MD; as shown by scores below the 30th percentile on the Mathematical Reasoning subtest of the Wechsler Individual Achievement Test); 15 children with reading difficulties (RD; as shown by scores below the 30th percentile on the Word Attack subtest of the Woodcock Johnson Psycho-Educational Battery; and 25 children with both mathematical and reading difficulties (MD/RD). Both MD groups showed problems in fact retrieval and in using counting strategies correctly in arithmetic. Children who had difficulties with both mathematics and reading tended to show problems in understanding counting principles and detecting counting errors; those with only MD or RD did not. However, about half of the MD children made double-counting errors. The MD/RD children, and those MD children who made double-counting errors, had lower backward digit spans than the other children.

Thus, the studies by Jordan and her colleagues and by Geary et al (1999) suggest that children with combined mathematical and reading disabilities tend to perform badly on more aspects of mathematics than children who only have mathematical difficulties; but do not support the type of dichotomy suggested by Rourke.

Thus, while signs of verbal or spatial weaknesses should serve as a warning signal that a child *may* experience mathematical difficulties, they cannot be used as definite predictors of either the existence or type of mathematical difficulty that a child may have.

There is still less evidence that, within the general population, verbal and nonverbal ability are associated with consistently different forms of strengths and weaknesses within arithmetic. (This is not to say that there might not be such patterns within the broader domain of mathematics; e.g. geometry is likely to be more specifically associated with spatial ability than is arithmetic). Dowker (1995, 1998) looked at WISC IQ scores, calculation and derived fact strategy use in 213 children between the ages of 6 and 9. Both Verbal and Performance I.Q. predicted performance on tasks of both arithmetical calculation and derived fact strategy use. Verbal I.Q. was a stronger predictor than Performance I.Q. of both types of arithmetical task. Children who showed a strong discrepancy between verbal and nonverbal I.Q. in *either* direction tended to do well at tasks that involve the use of derived fact strategies; such discrepancies did not predict calculation performance.

#### *4.6 Arithmetical difficulties in children with dyslexia and children with speech, language and communication needs*

Although there is no clear association between relative verbal versus spatial strengths and particular types of mathematical difficulty, there is no doubt that mathematical difficulties often co-occur with dyslexia and other forms of language difficulty.

People with dyslexia usually experience at least some difficulty in learning number facts such as multiplication tables. Miles (1993) found that 96% of a sample of 80 nine-to-twelve-year-old dyslexics had been unable to recite the 6x, 7x and 8x tables without stumbling.

Miles, Haslum and Wheeler (2001) used data from the British Births Cohort Study of 12,131 children born in England, Wales and Scotland between April 5th and 11th, 1970. The children were given a word recognition test, the Edinburgh Reading Test of reading comprehension, the British Abilities Scales spelling test, and the Similarities and Matrices 'intelligence' subtests of the British Abilities Scales. The children were categorized as normal achievers (49% of the sample; IQ scores of at least 90, and no significant mismatch between IQ, reading and spelling); low ability children (25% of the sample; IQ scores below 90); moderate underachievers (13% of the sample; reading and/or spelling score 1 to 1.5 standard deviations below the prediction); and severe underachievers (7% of the sample; reading and/or spelling score more than 1.5 standard deviations below the predictions). 6% were excluded due to insufficient data. 269 of the 907 severe underachievers were considered as probable dyslexics, on the grounds of poor performance on a digit span test, and on the Left-Right, Months Forwards and Months Reversed subtests of the Bangor Dyslexia Test. These dyslexic children performed less well on average on a calculation task, the Friendly Maths Test, than the normal achievers, and even than underachievers who did not meet full criteria for dyslexia. Items that were particularly difficult for the dyslexics were those which involved several steps (e.g. borrowing from two columns, and thus placed a heavy load on working memory; and those which involved fractions and decimals.

Yeo (2001) is a teacher at Emerson House, a school for dyslexic and dyspraxic primary school children, and has written extensively about the mathematical difficulties of some dyslexic children. She reports that while many dyslexic children have difficulties only with those aspects of arithmetic that involve verbal memory, some dyslexic children have more fundamental difficulties with 'number sense'. They comprehend numbers solely in terms of quantities to be counted and do not understand them in more abstract ways, or perceive the relationships between different numbers. Yeo suggests that the counting sequence presents so much difficulty for this group that it absorbs their attention and prevents them from considering other aspects of number. This sort of difficulty occurs in some children who are not dyslexic (see section above on "Common types of arithmetical difficulty"); and at present the extent to which it characterizes dyslexics more than others is not clear.

Children with spoken language and communication difficulties usually have some weaknesses in arithmetic, but once again some components tend to be affected much more than others. Fazio (1994) compared 20 5-year-olds with diagnosed specific language impairments with 20 age-matched controls and 20 language-matched younger children.

The children with language difficulties children resembled the younger children in the range and accuracy of their counting, but the age-matched controls in their understanding of counting-related concepts, such as the fact that the last item in a count sequence indicates the

number of items in the set. Two years later, Fazio (1996) followed up 16 of the children with language difficulties, 15 of the age-matched controls and 16 of the language-matched controls. The children with language difficulties were still poor at verbal counting, but resembled their age-matched controls in counting objects, and in reading numerals. They were worse at calculation than the age-matched controls, but no worse than the language-matched controls.

Grauberg, E. (1998) has written a book based on her experiences of teaching mathematics to pupils with language difficulties.

She notes that pupils with language difficulties tend to have difficulties in particular with:

- (1) Symbolic understanding. This includes difficulty in understanding how one item can ‘stand for’ another item or items, and effects can range from difficulties in understanding how a numeral can represent a quantity to difficulties in understanding how a coin of one denomination may be equivalent to a set of coins of a smaller denomination. Typically developing children under the age of 4 may have problems in distinguishing the cardinal use of numbers to represent quantities from their use as labels (“I am four”; “I live at number 63”). For children with language difficulties, such problems can persist for far longer. Place value – the use of the position of a digit to represent its value – can present problems for any child, but such problems are likely to be far greater for those with language difficulties
- (2) Organization. Children with language difficulties often have difficulties with organizing items in space or time, which may, for example, affect their ability to arrange quantities in order; to organize digits spatially on a page; and to ‘talk through’ a problem, especially a word problem.
- (3) Memory. Poor short-term and long-term verbal memory are frequent characteristics of individuals with language difficulties (see studies quoted above) and will affect learning to count, remembering number facts, and keeping track of one step in an arithmetic problem while carrying out subsequent steps.

In addition, language difficulties will directly affect the child’s ability to benefit from oral or written instruction, and to understand the language of mathematics.

#### 4.7 *Arithmetic in people with general learning difficulties*

There are certain forms of brain damage and of genetic disorder (e.g. Williams syndrome) which not only lead to general intellectual impairment, but to disproportionate difficulties in arithmetic. There are also some people with general intellectual impairments who nevertheless perform well at arithmetic: extreme examples are savant calculators (Heavey, 2003). In general, however, even people with severe intellectual impairments tend to show similar arithmetical performance and strategies to typically developing individuals *of the same mental age* (Baroody, 1988; Fletcher, Huffman, Bray and Grupe, 1998).

Severe general learning difficulties account for only 2.5% of children with statements of special needs or a status of “School Action Plus” (DfES, 2004), and are not the major topic of this review. However,, many of the children in mainstream schools who have difficulty with

arithmetic do have below-average general cognitive ability, rather than difficulties specific to arithmetic.

Hoard, Geary and Hamson (1999) compared 19 American first grade children with low IQs (mean 78; s.d. 5.6) with 43 children with average or above-average IQ (mean 108; s.d. 11.3). The low-IQ group showed lower backward digit span and slower articulation rates for familiar words, which may suggest working memory deficits. They were less good at number naming and number writing and magnitude comparisons. They performed worse than their peers at detecting counting errors, especially when set sizes increased beyond 5.

They made more errors in simple addition, but used a similar range of strategies: an interesting point when one remembers that they were being compared with children of similar *chronological age*.

We have established that arithmetical difficulties often but not always occur in people who either have generally low cognitive abilities or relatively specific reading difficulties (dyslexia). We have also established that many people have arithmetical difficulties that are not associated with either low IQ or dyslexia. Does the nature or severity of the arithmetical difficulties actually differ according to their level of specificity?

A few studies have suggested that the level of specificity may not in fact be important in predicting the nature of the arithmetical difficulties. Gonzalez and Espinel (1999) found that children whose arithmetical achievement was much worse than would be predicted from their IQ did not differ much in their arithmetic performance from those whose poor arithmetic performance was consistent with below-average IQs. The two performed similarly on addition and subtraction word problem solving tasks and on some working memory tasks. Similar results were obtained by Jimenez and Garcia (2002). If there *are* differences between specific and non-specific mathematical difficulties, they are probably in the direction of specific difficulties being milder and less pervasive than non-specific ones (Jordan and Montani, 1997).

Thus, it appears that distinguishing specific arithmetical difficulties from difficulties associated with low IQ is important from the point of view of understanding a child's *general* educational needs, but may not be crucial to planning arithmetical intervention as such. [Of course, good general reasoning abilities may be used in helping children to develop compensatory arithmetical strategies; but many children develop such strategies even when their I.Q.s are relatively low.] The issue is further discussed in the next section.

#### 4.8 *Arithmetical difficulties: specific disorder or low end of a continuum?*

One question which may be asked about people who have difficulties in any area of functioning is whether they are fundamentally different from individuals without such difficulties, or whether they represent the lower end of a continuum. Do people with arithmetical difficulties lack some function which everyone else has, or are they doing the same things as others, but less well or less efficiently?

It is difficult to make generalizations about *all* people with arithmetical difficulties, because, as we have seen, such difficulties take very heterogeneous forms. However, the evidence is that the vast majority of such difficulties can be seen as representing the lower end of a continuum. Studies of unselected groups of individuals have indicated that individual variations are

considerable even for adults in even such apparently basic numerical skills as counting accuracy and speed (Deloche et al, 1994) and single-digit addition and multiplication (Levevre, 2003). (See also the chapter on "Counting and after"). One study by Szanto (1998) found that adults with arithmetical difficulties performed in a similar fashion on both computational skills and arithmetical reasoning to normally achieving adolescents and children who were at a similar overall arithmetical level.

This does not mean that people with arithmetical difficulties do not have a genuine problem. Many characteristics vary continuously in the population, and yet may pose serious problems at the extremes. Body weight varies continuously, but being very overweight or underweight may both indicate and cause serious health problems. I.Q. varies continuously, but those with at the low end of this continuum are likely to experience major educational difficulties, even when no specific pathological cause can be identified. However, it does mean that people with arithmetical difficulties are not sharply distinct from other individuals, and that most of the difficulties that they show can also be seen to varying degrees in the general population.

Jordan and Montani (1996) drew the distinction between developmental *delays* in arithmetic, where children simply take longer than others to acquire certain skills but eventually do acquire them, and developmental *deficits*, where there is a partial or total failure to acquire certain components of arithmetic. As with so many aspects of arithmetical development, it is difficult to tell to what extent the distinction between developmental deficits and delays reflects children's intrinsic characteristics, and to what extent it is reflects environmental influences: perhaps in some cases a delay may be eventually overcome through appropriate teaching, while inappropriate teaching, or labelling the child as 'no good at maths', may lead to its remaining as a deficit.

#### 4.9 Severe specific difficulties with arithmetic

The term 'developmental dyscalculia', implying a specific disorder of mathematical learning, appears to have been popularised by Kosci (1974, 1981); though there was some earlier research on related problems (Kinsbourne and Warrington, 1963).

There are undoubtedly some exceptions to the general evidence that arithmetical difficulties are part of a continuum of ability. A few individuals do have severe specific difficulties with arithmetic which do not resemble anything observed in the general population. For example, Butterworth (1999) described "Charles", a university graduate, who had no problems with literacy or general reasoning, but who could only solve even single-digit sums by counting slowly on his fingers. He could not subtract or divide at all, or carry out any sort of multi-digit arithmetic. He was extremely slow even at comparing numbers: for example, saying which was bigger, 9 or 3. He could only give the answer to such comparison problems after counting on his fingers from the smaller number to the larger number. He took about ten times as long as most people even to state whether two numerals were the same or different. He seemed to lack even the most basic numerical abilities that usually seem to be present in babies: he could not even recognize two dots as two without counting them.

Such difficulties are rare, even among people with diagnoses of dyscalculia, However, screening for such difficulties may be helpful. It is important that people like Charles be identified as having difficulties early on, so as to reduce the risks of intellectual confusion and emotional

frustration and possibly humiliation if they are expected to cope with the typical school arithmetic curriculum without special help.

## 5. *Implications for intervention*

The findings indicate that individual differences are large. They also suggest that simple explanations in terms of nature alone or nurture alone are misleading and counter-productive. Children with apparently similar environments can perform very differently in arithmetic. A child's poor performance in arithmetic is not necessarily due to the child's lack of effort, or to failings on the part of parents or teachers. On the other hand, difficulties in arithmetic are not purely innate or immutable. There are very large differences in performance between children in different countries, suggesting a very important influence of environment. Such findings suggest that successful interventions are *possible*.

It is desirable that interventions should take place at an early stage. This is not because of any 'critical period' or rigid timescale for learning. Age of starting formal education has little impact on the final outcome (TIMSS, 1996). People who, to varying degrees, lacked opportunity for or interest in learning arithmetic in school, may learn later as adults (Evans, 2000).

### 5.1 *The problem of mathematics anxiety*

Nonetheless, there is one important potential constraint on the timescale for learning arithmetic and other aspects of mathematics (apart, of course, from the practical constraints imposed by school curricula and the timing of public examinations). Many people develop *anxiety* about mathematics, which can be a distressing problem in itself, and also inhibits further progress in the subject (Fennema, 1989; Hembree, 1990; Ashcraft, Kirk and Hopko, 1998). This is rare in young children (Wigfield and Meece, 1988) and becomes much more common in adolescence. Intervening to improve arithmetical difficulties in young children may reduce the risk of later development of mathematics anxiety. In any case, interventions are easier and less painful if they take place before mathematics anxiety has set in. Therefore, while it is *never too late* to intervene to help people with their arithmetical difficulties, interventions may be particularly effective if they are *early*.

### 5.2 *The need to take account of the componential nature of arithmetic*

Crucially when planning interventions, there is by now overwhelming evidence that arithmetical ability is not unitary: it is made up of many components, ranging from knowledge of the counting sequence to estimation to solving word problems. Moreover, though the different components often correlate with one another, weaknesses in any one of them can occur relatively independently of weaknesses in the others. Several studies have suggested that it is not possible to establish a strict hierarchy whereby any one component invariably precedes another component.

The componential nature of arithmetic is important in planning and formulating interventions with children who are experiencing arithmetical difficulties. Any extra help in arithmetic is likely to give some benefit. However, *interventions that focus on the particular components*

*with which an individual child has difficulty are likely to be more effective than those which assume that all children's arithmetical difficulties are similar* (Weaver, 1954; Keogh, Major, Omari, Gandara and Reid, 1980).

## 6. *What does work for mathematical difficulties?*

There is much research that indicates that the school environment and teaching methods are important influences on the mathematical performance of children throughout the ability range. Appropriate teaching may prevent some mathematical 'difficulties' from ever becoming apparent; and many mathematical difficulties are undoubtedly mainly the result of limited or inappropriate teaching (or, worldwide, to a complete or near-complete lack of schooling). This report will not, however, deal with general educational influences, which have been extensively discussed elsewhere. Rather, it will focus on targeted interventions with children who demonstrate, or are at risk for, mathematical difficulties that are considerably greater than those typically experienced by others within the same educational system.

It is often important to distinguish between tools of *access* and tools of *intervention*. Tools of access are means of circumventing a difficulty which does not affect mathematics learning directly, but which may interfere with a child's benefiting from standard forms of mathematics teaching or mathematical activities. Examples may include the provision of a sign language interpreter for a child with hearing impairment; oral presentation of material to a visually impaired or dyslexic pupil; or allowing a dyslexic or dyspraxic child to use a word processor instead of writing by hand. Tools of intervention involve remediating, or in some case preventing, difficulties with mathematical learning itself. The focus of this report is on tools of intervention, though the two may not always be sharply distinguishable.

There have been far fewer intervention programmes for children with arithmetical difficulties than, for example, for children with literacy difficulties (Brooks, 2003). Nevertheless, there have been more such programme, and over a longer historical period, than is often realized.

### 6.1 *Why schools need to take account of mathematical difficulties*

Individual differences cannot be ignored or disregarded. Ignoring the existence of individual differences (whatever their sources) is not going to make them disappear. Individual differences are not solely created by the school environment, though they may be exaggerated by it. Unless expectations are reduced for *all* pupils, some pupils will struggle and become discouraged, if not 'math phobic', if their difficulties are not taken into account. Unless expectations are raised to the point of creating difficulty for a large number of pupils, some other pupils will find the tasks too easy and become bored. Pupils who are much better at some components of arithmetic than at others will not be helped to use their strengths to overcome their weaknesses. [None of this is to say that mathematical weaknesses are innate, fixed or immutable: merely to say that ignoring or disregarding them, or demanding that pupils perform at a higher level of ability, is not necessarily going to make them overcome their weaknesses.]

The needs of children with mathematical difficulties are being increasingly recognized in many other countries, including the United States (Ginsburg, 1977; Muller and Mercer, 1997; Woodward, 2004); Australia (Wright, Martland and Stafford, 2000; Wright, Martland, Stafford

and Stanger, 2002; Van Krayenoord and Elkins, 2004); Germany (Bzufka, Hein and Neumarker, 2001); Belgium (Desoete, Roeyers and DeClercq, 2004), Spain (Casas and Garcia Castellar, 2004), Italy (Cornoldi and Lucangeli, 2004) and Japan (Tsuge, 2001; Woodward and Ono, 2004).

The need to take mathematical difficulties into account, even while attempting to raise overall standards, has been discussed from an American perspective by Muller and Mercer (1997). They discuss the teaching of pupils with mathematical difficulties (specific or otherwise) in mainstream classrooms in the face of pressures to increase the rigour of mathematics education courses for all pupils, and to increase the mathematics requirements for high school graduation. The authors emphasize the importance of accommodating diversity, and adapting teaching to individual strengths and weaknesses, "Once the [individual's] strengths and weaknesses have been assessed, his or her mathematical needs should be explored, related to both short-term and long-term goals... When considering the diversity among all students with and without disabilities, it is unrealistic to assume that one curriculum or one set of standards will suit the math needs of everyone...Most individuals with disabilities are going to need accommodations or modifications in textbooks, assignments, teaching methods, tests and homework". Like a number of other researchers on intervention, the authors also emphasize the importance of teachers and researchers collaborating in devising and selecting effective methods of teaching pupils with and without disabilities.

The current statutory National Curriculum Inclusion Statement in Britain requires schools to set suitable learning challenges, respond to pupils' diverse learning needs, and overcome potential barriers to learning and assessment for individuals and groups of pupils.

Given that mathematical difficulties need to be taken into account, how can this best be done?

### 6.1.1 Ability grouping

One possible approach, which has often been taken in practice, is to divide children into ability groups, thus reducing the level of individual variation in any given class. This may involve children being placed in separate schools or classes according to supposed overall ability; or in 'sets' specifically for mathematics. This avoids some of the problems that occur when individual differences are totally ignored (but see discussion below of Boaler's (1997) study), but can create its own problems. Some children may be labelled as 'failures' or 'no good at maths', and live down to expectations. Resources may be concentrated on pupils who are regarded as more able, and the 'less able' classes or sets may be assigned to weaker or less experienced teachers.

Research on the effects of streaming and setting has yielded somewhat complex and equivocal results (Harlen and Malcolm, 1999; Ireson and Hallam, 2001). Overall, streaming and setting do not influence school performance overall, or in most subjects. However, several studies suggest that they do have a deleterious effect specifically on the performance of low achievers in *mathematics*, who perform less well than similar pupils in mixed ability groups (Hoffer, 1992; Boaler, 1997; Ireson and Hallam, 2001).

Boaler (1997) carried out interviews with pupils in schools which used either mixed ability groups or sets for mathematics. The setted pupils of all ability levels expressed greater dissatisfaction. Their major source of concern was that they were required to follow a standard pace, and that individual differences were not taken into consideration as they had been in

primary school mixed-ability classes. Thus, setting may sometimes increase the very problems that it seeks to avoid, because teachers and curriculum planners assume that the classes are homogeneous and that individual differences can be ignored whereas in fact individual differences are still marked even within a set. Streaming and setting, as commonly used, may have deleterious effects on low achievers' performance, not because they take too much account of individual differences, but because they lead to taking too *little* account of individual differences.

In view of the componential nature of arithmetical thinking, the construction of homogeneous 'sets' may not even be possible: where, for example, do we place a child who is excellent at mental arithmetic but poor at using written symbolism, or one who has difficulty in remembering arithmetical facts but is an accomplished user of derived fact strategies?

### 6.1.2 *Individualized and small-group work within a class*

Another approach is to enable some independent individualized or small-group work within a class. Individualized work within a class usually involves progressing through a textbook at one's own pace; the use of individualized worksheets; and/or (in recent years) the individualized use of educational computer software. Such approaches are very variable, but include both component-based approaches and approaches which treat arithmetical ability as a more global unitary ability. Small group approaches may take a similar form, or may involve group projects where several pupils work together on the solution or solutions to a problem.

Lou, Abrami, Spence, Poulsen, Chambers and d'Appolonia (1996) carried out a meta-analysis of ability grouping and small group work *within* classes, in a variety of school subjects. In contrast to most findings with regard to setting, this study indicated that *within*-class grouping had a positive effect on the performance of low achievers; but only if it was accompanied by provision of appropriate materials and activities.

There are by now several guides for teachers, influenced both by research findings and by teachers' reported experience, regarding strategies for dealing with individual differences within a class, and including children with mathematical difficulties. Examples include El-Naggar (1996) and Poustie (2001) on mathematical difficulties in general; Kay and Yeo (2003) on mathematical difficulties associated with dyslexia; and publications from the National Numeracy Strategy such as *Guidance to Support Pupils with Special Educational Needs in the Daily Mathematics Lesson* (DfES 05451/2001) and *Including All Children in the Literacy Hour and Daily Mathematics Lesson* (DfES 0465/2002).

El-Naggar's (1996) book is based on the experience and views of members of the NASEN Mathematics Working Party. It arose from requests for teachers for practical guidance on providing classroom-based provision for pupils with specific learning difficulties in mathematics. Thus, it deals explicitly with teaching such pupils in the classroom, where they are likely to be spending most of their time, whether or not they are also receiving some individualized interventions.

This book discusses:

(a) Assessment.

Assessment may take the form of testing; interviews with pupils or parents; and/or observation.

Standardized tests can be used to provide guidance about a pupil's mathematical achievement level relative to others. However, they do not describe individual strengths and weaknesses. "Thus, pupils with identical or near-identical scores may have entirely different abilities and weaknesses if an item analysis is made" (p. 14) (a point also made by Ginsburg, 1977 and others.) The problem is reduced, but not totally eliminated, by using tests which differentiate between different components of mathematics, such as the NFER's 'Profile of Mathematical Skills', and the National Curriculum Standardized Assessment Tests.

Pupil interviews can be useful in investigating pupils' self-ratings and attitudes, and serve to provide a more precise analysis of children's difficulties, and also of their interests. Such information could not be readily obtained from a standardized test. [However, it should be noted that though there is a correlation between children's self-ratings and actual performance it is far from perfect – Dowker, 2004 in press; Dowker and Green, 2003. Moreover, very young children may have difficulty in answering interview questions.)

Parent interviews can be useful in providing information about the child's previous history; about any family history of mathematical difficulties; about the child's mathematical experiences out of school; and about the parent's attitudes and priorities with regard to mathematics; and also in making plans for parental support. [Of course, their success depends very much on the parents' ability and willingness to co-operate; they cannot be forced to do so!]

Observation by the teacher provides the opportunity to discover individual strategies and working patterns during school mathematics lessons, and to ask the children questions about their written work which may lead to reflection and reconsideration. Thus, observation can involve both diagnosis (for example, differentiating between a mathematics learning difficulty and a specific misunderstanding) and intervention. [Limitations of this approach are that a teacher rarely has the opportunity for extensive work with an individual child, so that some of their working processes will inevitably go unobserved.]

(B) El-Naggar discusses frequently-occurring characteristics of children with specific learning difficulties in mathematics. Not every child will have all of these characteristics; and they can also be found in some children without mathematical difficulties. They include problems with short-term memory/working memory; problems with long-term memory; directional confusion; visual perceptual difficulties; sequencing problems; spatial awareness problems; difficulties with mathematical language; lack of problem solving strategies; and motor perception difficulties.

El-Naggar discusses both ways in which individualized programmes can be used in a classroom setting, and ways in which general classroom practice may assist those with special difficulties. She points out that individualized programmes do not necessarily require one-to-one teaching. They do involve assessment of the child's individual needs, and providing for these needs, for example by (i) small-group activities including several children with difficulties; (ii) including activities geared at the whole class revising and consolidating earlier learning in areas which the child with difficulties needs to master; (iii) providing

classroom activities which can be solved at several levels; e.g. with concrete materials by children with difficulties and in more abstract form by children without such difficulties.

Implications for general classroom practice (p.49) include such issues as “including something to see, something to listen to, and something to do, at each new stage of mathematical development; “capitalizing on classroom opportunities for group discussion and discussion”; “allowing plenty of classroom opportunities for discussion”; “rehearsing, as appropriate, earlier stages prior to the introduction of new stages and challenges”; etc.

Poustie (2001) discusses ways of helping pupils with mathematical difficulties, which can be applied within the classroom; in the context of individualized or small-group tuition; and/or by parents helping their children at home. Many of her recommendations are similar to El-Naggar’s, including the importance of revision and consolidation. She also emphasizes the need to take account of specific learning strengths and weaknesses, by presenting material in a variety of forms. For example, apparatus should be provided to help children who need concrete support; visual information (in the form of pictures and diagrams) to help children who have language difficulties and learn visually; and verbal explanations to help children who have visual processing difficulties.

Several teachers, such as Stewart (2003), have developed strategies for dealing with classes that include a significant number of children with mathematical difficulties. Stewart (2003) teaches a class of pupils of mixed low ability: many but not all of them have special needs in mathematics. Techniques that she has used include *multi-sensory* teaching of mathematics, involving motor activities: e.g. clapping and dancing as the children chant or sing multiplication tables, with the aim of assisting those children who, for example, have better motor than verbal memories. She has also demonstrated to children the *number patterns* that multiplication tables give: for example, the alternation of ‘5’ and ‘0’ as the final digits in the 5 times table, and the successive final digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 in the 11 times table. These ‘magic number’ patterns, which at first sight might appear trivial, have been highly motivating to many of her pupils, and assisted them in learning the tables.

Kay and Yeo (2003) focus on the mathematical difficulties that are often shown by dyslexic pupils (see also Miles and Miles, 1992; Chinn and Ashcroft, 1998; Yeo, 2003). They point out that most dyslexic pupils have difficulty with long-term memory for facts (e.g. multiplication tables); working memory difficulties; sequencing difficulties; and difficulties with language, including mathematical language.

Dyslexic pupils tend to think more slowly when solving mathematics problems, because they cannot retrieve facts as automatically. Therefore they need to be given more time for thinking, as well as practice in fact retrieval and reasoning. Some forms of practice, such as the rote learning of multiplication tables, are so difficult for most dyslexic pupils that they may be counter-productive.

Some suggested activities include:

*Rehearsal cards*, which include mathematical facts ( $5 \times 4 = 20$ ) or definitions (“Multiply means ‘groups of’ or ‘times’ or ‘x’”), Each individual child is given a small set of cards to practice each day under adult supervision.

*Counting activities.* These include general practice in counting objects. They also include practice in grouping objects, especially into tens. (Dyslexic children who over-rely on counting often have a units-based concept of number, and do not understand the decadal structure of number). Children may count objects into groups of ten; find individual numbers on a tens-structured track or a structured bead ring; build tracks of selected numbers using base ten materials; and represent groups of objects as numbers on a number line. They may then use base ten materials to count in larger groups; e.g. hundreds.

*Calculation activities.* These include learning addition facts up to ten. This can be aided by the use of patterns, such as domino and dice patterns. Games, e.g. involving flash cards, can be used to teach and reinforce the number pairs that add up to 10, and then to 20. Once children have learned some facts such as doubles ( $6 + 6 = 12$ ) they may be able to use derived fact strategies to figure out related facts (e.g.  $7 + 6$  is one more than  $6 + 6$ , so 13).

Multiplication may be assisted by practice in step counting in multiples of 2, 3, 4, 5 and 6; and by 'building' numbers through groups of concrete objects. Division should accompany multiplication and the inverse relationship between the two should be emphasized. Children should be encouraged to use whichever strategies are easiest for them; but some strategies tend to be easier for most dyslexic children. For example, subtraction is best understood through sequencing back (with practice on rulers and number lines) and through complementary addition; the latter method has the advantage that it involves forward rather than backward counting, and dyslexic children tend to find backward counting more difficult than forward counting.

### **6.1.3 The role of Teaching Assistants**

One possible method of assisting children with mathematical difficulties within a classroom is to employ Teaching Assistants to provide additional support for such pupils. Typically, their role has been to provide assistance to individuals or small groups in the context of whole-class activities, rather than to organize separate activities; though occasionally they also do the latter.

So far, Teaching Assistants have been used more frequently to support pupils with reading and other difficulties than those with mathematics difficulties.

A few projects have recently involved 'Numeracy Support Assistants' for the specific purpose of supporting low achievers in mathematics. Mujis and Reynolds (2003) describe the use of such assistants for 5-to 7-year-olds in the Gatsby Mathematics Enhancement Programme Primary: a project which principally emphasizes whole-class interactive learning. They compared 180 children who had received such support with 180 other children matched for gender, ethnicity, eligibility for free school meals, and initial level of mathematical achievement. After a year, there were no significant differences in mathematical achievement between those who had received such support and those who had not, indicating that the use of Numeracy Support Assistants was not effective in improving the performance of low achievers.

However, some other studies (OFSTED, 2002; Wilson, Schlapp and Davidson, 2002) have given more encouraging results regarding the role of Teaching Assistants.

In particular, a recent DfES pilot study with Year 6 pupils indicates that Teaching Assistants' *support of pupils with numeracy and other difficulties* has directly beneficial effects. It also – and crucially - indicates that *training has a positive impact on the quality of Teaching Assistants' interventions*.

Thus, it appears that the use of Teaching Assistants to support children with numeracy difficulties is not a panacea in itself, but may have beneficial effects if carried out appropriately. The important factors in determining the effectiveness of this strategy have yet to be fully established, but may include *the exact role of the Learning Support Assistants; the appropriateness of other aspects of the pupils' mathematics instruction; and certainly the training that they receive*.

## 7. Intervention programmes

The review will now discuss some specific intervention programmes that have been used. It will focus in particular on targeted interventions for individuals and small groups of children with diagnosed arithmetical difficulties. Some less specifically targeted interventions with disadvantaged groups will also be discussed, especially the relatively small number of such studies that deal with preschoolers.

This review is intended to describe and review important past and current programmes, but is not a “Which”-style guide to specific ‘recommended’ programmes.

### 7.1 *Preschool intervention programmes to prevent the later development of mathematical difficulties*

Some intervention programmes target children who are perceived to have an increased risk of academic difficulties: usually children in low socio-economic groups. Such programmes may or may not involve individualized assessment. Often they involve preschool children and are not always restricted to numeracy. For example, the Head Start programme in the United States includes significant emphasis on number concepts, among other aspects of early intervention (Arnold, Fisher, Doctoroff and Dobbs, 2002).

Arnold et al (2002) report a study within the context of the Head Start programme. Some Head Start classrooms instituted a 6-week classroom intervention, where math-related activities were introduced into the daily routine during circle time, transitions, mealtime and small-group activities. Other classrooms engaged in their typical activities. After the programme, the children who had undergone the intervention scored higher on a standardized mathematics test and both self-reports and teacher ratings indicated greater enjoyment of mathematics in this group. Boys improved more than girls.

Another project linked to Head Start is the *Berkeley Maths Readiness Project* (Starkey and Klein, 2000). This project is funded by the US Department of Education and is being carried out by Prentice Starkey and Alice Klein at the University of California at Berkeley. The researchers are developing and evaluating a pre-kindergarten mathematics curriculum including eight topical units: (1) enumeration and number sense; (2) arithmetical reasoning; (3) spatial sense; (4) geometric reasoning; (5) unit construction and pattern sense; (6) logical reasoning; (7)

measurement; and (8) computer mathematics. For example, the arithmetical reasoning unit includes a division activity (dividing a set of concrete objects into two equal subsets) and addition and subtraction activities such as using finger counting to solve problems such as "Three bears and one bear make how many bears altogether".

The math curriculum is delivered by both preschool teachers and parents. The teachers attend two workshops to learn about the curriculum and the materials. They then teach the curriculum in their classrooms, and also demonstrate related activities to parents and children in a series of home visits. The parents are given advice sheets and materials for use with their children.

The mathematical progress of children in demonstration classes using the curriculum is being compared with that of children in other classes, and children from the same classes a year before the project was implemented. Improvements in performance have already been demonstrated both in middle class children, and in disadvantaged children in Head Start and California State Preschool programmes. Evaluations are still in progress.

Other important American preschool mathematics programmes aimed at reducing mathematical difficulties are the Rightstart programme of Griffin, Case and Siegler (1994) and the Big Math for Little Kids programme of Ginsburg, Balfanz and Greenes (1999).

The *Rightstart* programme (Griffin et al, 1994) has been used in some inner-city kindergartens in the United States and Canada. It focuses on assisting children to acquire the 'central conceptual structure' of a mental number line. A child who has this conceptual structure will be able, at least for relatively small numbers, to tell which of two numbers is larger; to count backwards or forwards from a given number (e.g. to say which number comes two numbers after 7); and to use the addition strategy of counting on from the larger addend; e.g. if asked to add 2 and 5, will start with 5 and count on "6, 7"). Earlier research on the subject had indicated that children under 5 usually do not appear to use such a mental number line, while children over 5 generally do. However, some 5- and 6-year-old children, especially those from low-income backgrounds, demonstrated difficulty with the mental number line tasks.

The Rightstart programme includes thirty games to be played by small groups of four or five children, as well as some whole-class activities. The predominant mode of work with the children involves 20 minutes a day of small-group activities. The games and activities used in the programme involve counting; quantifying sets of objects; matching sets to written numerals; and predicting the result of adding 1 to or taking 1 away from a given set.

In an initial evaluation, 23 kindergarten children from the programme and 24 controls from similar backgrounds were followed up into first grade. Those who had been in the programme improved significantly more than the others on tests of oral arithmetic and word problem solving. They also did better on number line knowledge and on written arithmetic problems of the sort taught in school; but differences here were not significant, because children in both groups tended to do well at these tasks. The children who had been in the Rightstart group also received significantly better ratings from teachers on most aspects of number understanding and arithmetic.

Ginsburg, Balfanz and Greenes (1999) introduced the *Big Math for Little Kids* intervention programme. Like the Rightstart programme, it introduces mathematical games and activities into the general curriculum in preschool and kindergarten programmes in disadvantaged areas. Whereas the Rightstart programme emphasizes certain specific number skills that appear to be

important to the development of numeracy, the BigMath programme emphasizes introducing children early to a wide variety of important mathematical concepts: not necessarily relating to number.

The development of the programme reflects the view that mathematics education in the early years has often been limited to counting, shape identification, and some simple measurement comparisons. Where more sophisticated ideas are introduced, they have often been dealt with too briefly and not revisited. The BigMath programme is aimed at helping children to explore 'big' mathematical ideas over lengthy periods of time. It includes activities designed for individuals, small groups and the whole class. There are six major strands:

- (1) use of numbers, involving counting procedures and principles, the use of numbers as labels (e.g. house numbers), and the different ways in which numbers may be represented;
- (2) shape, involving not only recognition and naming of shapes, but exploration of their characteristics (e.g. number of sides and angles), symmetry, and ways of partitioning them into other shapes;
- (3) measurement, involving comparison, seriation, and iteration (repeated use of a measurement unit) with regard to a wide variety of quantities: length, weight, capacity, area, time, temperature and money;
- (4) working with numbers, including grouping of objects, adding and subtracting, and the relationships between sets and their subsets;
- (5) patterns, involving the systematic repetition of elements in the context of number, shape, colour, and sound (e.g. rhythm). Children copy patterns; extend them, e.g. adding 2 repeatedly to make 1, 3, 5, 7...; describe them; and create their own.
- (6) spatial relationships, involving describing and mapping positions and routes.

The programme includes a wide variety of games, stories and pictures. For example, one of the tasks involving line symmetry involves presenting children with pictures of halves of faces, and asking them how to complete the faces. Some of the spatial tasks include 'treasure hunts' where children attempt to locate an object from clues about its position relative to other objects. One of the number activities involves listening to a story, "So Many Fives", about children who represent the number five through number words in different languages, the written numeral, and various types of tally. The children are then encouraged to think of a wide variety of ways of representing other numbers,

The programme is still in its initial stages and has not yet undergone full evaluation. However, results so far are promising, and suggest that young children are far more ready to engage in mathematically challenging activities, and to go beyond very simple counting, than some traditional views have suggested.

There have been rather fewer preschool intervention studies in Britain. Partly this may be because British children start school earlier than in the United States, and standard preschool programmes have more mathematical content than is typical in the United States. Many children in Britain do not, however, experience preschool education though this is changing.

Roberts (2001) describes the Preschool Early Education Partnership (PEEP). This programme is based in a disadvantaged area of Oxford, England, and works with parents of children from birth to school age. It offers materials, group sessions and home visits to parents. The focus is on parents and children talking, singing and playing together, and on parents sharing books and similar materials with their children. The central focus of this programme is preparation for

literacy; but numeracy activities are also incorporated. These involve counting games; as well as encouraging parents to discuss number with children in the context of practical activities such as shopping and preparing meals, and also to discuss numbers in the environment, such as house and bus numbers.

Recently, the British Government has instituted "Family Literacy" and "Family Numeracy" programmes, where parents with limited education improve their own basic skills, while their preschool and early primary school age children are exposed to activities relating to language and number.

The programmes described so far target children who are at risk for mathematical difficulties for socio-economic reasons, rather than children who have been directly assessed as having actual or likely mathematical difficulties. Van Luit and Schopman (2000) carried out such a study in the Netherlands. They examined the effects of early mathematics intervention with young children attending kindergartens for children with special educational needs. The participants were 124 children between the ages of 5 and 7. They did not have sensory or motor impairments, or severe general learning disabilities. Most had language deficits and/or behavioural problems. All had scored in the lowest 25% for their age group on the Utrecht Test for Number Sense, a test of early counting skills and number concepts. 62 underwent intervention, and the other 62 served as a control group, who underwent the standard preschool curriculum. The intervention programme was the Early Numeracy Programme, which designed for children with special needs, and emphasizes learning to count. The programme involved the numbers 1 to 15, which were represented in various ways, progressing from the concrete (sets of objects) through the semi-concrete (tallies) to the abstract (numerals) sets of objects, and tally marks. Patterns of 5 were particularly emphasized, and were represented by 5 tally marks within and ellipse. The number activities were embedded in games involving families, celebrations and shopping. The children had two half-hour sessions per week in groups of three for six months. At the end, the intervention group performed much better than the control group on activities that had formed part of the intervention programme, but unfortunately did not transfer their superior knowledge to other similar but not identical numeracy tasks.

## *7.2 Interventions with school-age children with arithmetical difficulties*

### *7.2.1 Peer tuition and group collaboration*

One way in which schools can deal with mathematical weaknesses is by encouraging children to teach one another. This can involve older children teaching younger children; more able classmates teaching less able classmates; or collaborative learning between peers of similar ability. All of these techniques have been used extensively in general teaching. Collaborative group projects have been common in schools for many years, and have their adult counterparts in the classes, workshops and seminars that occur in higher education and professional development. The teaching of younger children by older children has a very venerable history, going back at least to the early 19th century 'monitorial' system, where older children became teachers as well as learners as a means of coping with the enormous class sizes and high teacher/pupil ratio of Britain's earliest free schools. The present discussion will be confined to the use of such techniques with regard to arithmetic.

The commonest forms of peer teaching involve collaborative group work, where several children co-operate in solving a mathematical problem. This can serve several purposes: increasing motivation; in encouraging children to put their mathematical ideas into words, and to reflect on the strategies that they use; and in enabling children to transmit mathematical knowledge and ideas to one another. The possible disadvantages of such approaches are that their success may depend on the dynamics of a particular group. There is the risk that a dominant child, or one who is perceived as 'clever', may do most of the work, while the others simply accept his or her decisions and are 'carried' along without really learning anything new. If we are specifically considering low achievers in mathematics, this is a particular danger. Such children may be ignored or dismissed in group discussions and decisions. However, some studies have indicated that collaborative learning can be beneficial to children with arithmetical difficulties.

Davenport and Howe (1999) looked at the effect of collaborative learning on children's ability to solve addition and subtraction word problems. Children worked in groups, using problem-solving guidelines that they had been given to solve the problems, and then 'taught' their problem to a fellow pupil. The children in this collaborative condition were compared with children who solved the same problems individually. The children in the collaborative condition performed better than those who worked individually, and in particular, children who were *below average* in arithmetic benefited from being the 'learners' who listened to peers.

More intensive forms of peer tutoring involve one-on-one tuition of one child by another: often following some form of explicit training of the child 'teacher' (FitzGibbon, 1981; Levin, Glass and Meister, 1984; Topping and Bamford, 1998; Vosse, 1999). Usually, older children teach younger children. Studies tend to show benefits both for the tutors - perhaps due to their need to reflect on the arithmetical concepts and procedures that they teach - and for the tutees.

Rohrbeck, Ginsburg-Block, Fantuzzo, and Miller (2003) carried out a meta-analysis, where they analyzed a large number of group comparison studies evaluating peer assisted learning interventions with elementary school pupils. Overall, improvements were highly significant ( $p < 0.0001$ ). The interventions were most effective with younger, urban, low income and ethnic-minority pupils. However, the researchers point out limitations due to the fact that descriptive information was often inadequate in the papers reviewed. The conclusion was that peer tuition is a promising approach, and should be developed further; and that papers should include more detailed information about pupils, schools and intervention components. From the point of view of the present review, the limited descriptive information makes it difficult to deduce exactly what proportion of the children in the studies were low achievers in arithmetic or had serious mathematical difficulties.

One recent meta-analysis has suggested that, while peer tutoring interventions are beneficial for children with arithmetical difficulties, they are less so than some other forms of intervention (Kroesbergen and Van Luit, 2003). This result needs to be interpreted cautiously, as the criteria for including children in peer tutoring programmes may be less strict than those involving other forms of intervention. Thus, children involved in peer tutoring may show less improvement just because they tend to have less serious problems in the first place. Nonetheless, this is an indication that peer tuition cannot be seen as a complete substitute for adult intervention.

### 7.2.2 Training children in Piagetian operations

At certain times in the past, training in Piagetian operations (see Piaget, 1952) was considered an important form of intervention to improve children's arithmetic. The basis for this approach was that understanding the underlying number concepts is a necessary prerequisite to arithmetic.

The acquisition of *formal* operations (involving the manipulation of symbols and abstractions) in adolescence appears to be less universal than the acquisition of earlier number concepts: those which, in Piaget's theory, are dependent on the attainment of *concrete* operations. There are considerable individual and cross-cultural differences as regards whether and when formal operations are attained, and they seem to depend highly on schooling. Research by Adey and Shayer (1994) indicates that training in formal operations does indeed appear to have a positive impact on the mathematical development of older children and adolescents. This research was predominantly aimed at improving performance in science; improvement in mathematics was a by-product. More recently, Adhami, Johnson and Shayer have used training in formal operations more specifically to enhance mathematical learning [Cognitive Acceleration in Mathematics Education, or CAME (Adhami, Johnson and Shayer, 1998)]. Current research by Shayer and his colleagues is aimed at training Year 1 and 2 children in concrete operations, and in particular at providing them with opportunities to use these in mathematical context: e.g. with regard to proportional reasoning. The CAME programme is aimed at all children rather than just those with mathematical difficulties.

Some earlier approaches that have emphasized training children with mathematical difficulties in Piagetian concrete operations have been relatively unsuccessful in improving arithmetical performance (Snorre Ostad; personal communication). On the other hand, some approaches which have combined Piagetian training with other forms of intervention have led to significant improvement (e.g. Van de Rijt and Van Luit, 1998).

### 7.2.3 Training in metacognition

Metacognition - awareness of one's own mental strategies and level of knowledge-is often regarded as particularly important to arithmetic. Current mathematics education in Britain and other countries has placed increasing emphasis on encouraging children to reflect on and discuss their mathematical ideas and strategies. Several intervention projects have involved training children, with and without arithmetical difficulties to reflect on their knowledge and plan and monitor their arithmetical strategies (Adey and Shayer, 1994; Verschaffel, DeCorte et al, 1999; Desoete, Roeyers and De Clercq, 2003) and have generally produced good results.

For example, Adey and Shayer (1994) found that pupils who had taken part in their Cognitive Acceleration programme (emphasizing both metacognition *and* Piagetian training as discussed above) were significantly more likely than controls to perform well in GCSE mathematics. The authors, who had previously worked out correlations between pupils' Piagetian Reasoning Test scores on entering secondary school and their subsequent performance in GCSE mathematics, used these to predict the GCSE levels of the pupils in their study, and compared them with their actual levels. 39% of the pupils who had undergone the Cognitive Acceleration programme obtained GCSE levels at least one standard deviation higher than the prediction, as compared with only 13% of those who had not. As is the case with most studies, it is not absolutely clear *which* aspect of the intervention had the most important effect, or which component(s) of GCSE maths were most facilitated.

An example of a small-scale project that emphasized metacognitive training was that of Naglieri and Johnson (1999). 19 children with moderate general learning difficulties were given an intervention that emphasized planning in arithmetic. All pupils completed mathematics worksheets during 7 baseline and 14 intervention sessions. During the intervention phase, the pupils were encouraged to reflect on and discuss the strategies that needed to be used. After the experiment, they were given the Cognitive Assessment System, and classified as having weaknesses in Planning, Attention, Simultaneous processing, Sequential processing, or no specific cognitive weakness. 3 had Planning weaknesses; 6 had other specific cognitive weaknesses; and 10 had no specific cognitive weaknesses. Those who had difficulties in Planning showed significant improvement; the other groups did not. This suggests both the possible usefulness of some forms of metacognitive training, the importance of targeting interventions to a pupil's specific weaknesses. However, it should be noted that the groups were very small.

Overall, it is not always absolutely clear which aspects of such programmes have been successful. 'Metacognitive training' can mean somewhat different things in different studies. Some studies may emphasize a form of reflection that is really an aspect of Piagetian formal operations, and thus comes within the previous section; some emphasize the ability to discuss and compare arithmetical strategies; others may emphasize a planning ability more akin to aspects of working memory; still others may emphasize awareness of one's own knowledge state, including 'knowing when one doesn't know'. More research is needed on exactly which aspects of metacognition are important here.

#### *7.2.4 Group interventions more specific to arithmetic*

There can be a rather fine line between the use of differentiated activities within a class, as described in section 5, and more specifically targeted small-group interventions concerning particular aspects of arithmetic (see also the discussion of Springboard in Section 9).

Askew, Bibby and Brown (2001) developed a small-group intervention technique which involved the use of *derived fact strategies*: the use of arithmetical principles such as commutativity and associativity to work out new arithmetical facts on the basis of known facts. The development and use of such strategies has been increasingly recognized in mathematics teaching as a whole. However, there are some children who do not develop such strategies. Often (though not always) these are among the children who know relatively few arithmetic facts and rely on counting strategies. Their failure to use derived fact strategies may further impede their developing a store of known facts, which in its turn may interfere with the development of derived fact strategies.

Rather than dealing with the problem by the traditional method of getting the pupils to memorize more facts, Askew et al (2001) developed a programme which emphasized working out strategies for deriving new facts on the basis of known facts. Teachers worked with small groups (four per group) of Year 3 children (7-to 8-year-olds) who had reached Level 2C or below in the National Curriculum Tests at age 7. The children underwent intervention once a week for 20 weeks. There were 48 children in the intervention group, who were compared with 48 matched controls. All children were given a diagnostic test of arithmetic problem solving, devised by the researchers, just before intervention, and again shortly after completing the intervention. The children in the programme improved significantly more than the controls,

both in accuracy, and in their use of known and derived facts rather than needing to resort to counting strategies.

## 8. *Individualized remediation in arithmetic*

We now turn from group-based techniques of helping children with arithmetical difficulties to more individualized component-based techniques, that take into account individual children's strengths and weaknesses in specific components of arithmetic. Some of these projects are totally individual; some include small-group work at least in part, but include individualized assessments.

### 8.1 *Assessments for targeted intervention*

Effective interventions imply some form of assessment, whether formal or informal, to (a) indicate the strengths, weaknesses and educational needs of an individual or group; and (b) to evaluate the effectiveness of the intervention in improving performance.

Assessments may or may not be 'tests' in the conventional sense. Ollerton and Watson (2001, p. 87) list 23 methods used to assess pupils. These range from traditional written tests and multiple choice tests through marking written work and extended projects to 'hearing a response which is unexpected and reveals other knowledge'.

There are a variety of standardized tests used for assessing children's arithmetic (Poustie, 2001). Many test batteries for measuring abilities (e.g. the British Abilities Scales and their American counterpart, the Differentiated Aptitude Tests) include tests both of calculation efficiency and of mathematical reasoning, the latter usually taking the form either of number pattern recognition or word problem solving. IQ scales, such as the Wechsler Intelligence Scale for Children and the Weschler Adult Intelligence Scale, include arithmetic subtests which tend to emphasize word problem solving. Some tests, used in school contexts (e.g. the NFER Mathematics tests, and the National Curriculum Tests), place greater emphasis on whether children have mastered particular aspects of the arithmetic curriculum. Others are devised by researchers for the specific purpose of assessing particular mathematical components, which are to be dealt with, or have been dealt with, in an intervention programme. Some researchers over the years have argued that the exclusive use of standardized tests may result in missing crucial aspects of an individual's strategies and difficulties, and have emphasized the importance of individual interviews and case study methods (Brownell and Watson, 1936; Ginsburg, 1977).

Butterworth (2002) has devised a computerized screening test of basic numerical skills: incorporating the recognition of small numerosities; estimation of somewhat larger numerosities; and comparisons of number size. These are intended to identify severe arithmetical difficulties (dyscalculia) rather than to assess individual differences in the general population.

There are advantages and disadvantages to the use of any type of assessment. Traditional tests may (but need not) lead to an emphasis on comparing pupils or schools rather than on identifying the educational needs of individual pupils. They may thus lead to anxiety in both teachers and pupils, and encourage or pressurize teachers to teach 'to the test' rather than to teach

for knowledge and understanding. They may be relatively unsuitable for people who have not had extensive experience of being tested. Observational and interview-based techniques involve fewer such problems, are potentially more flexible, and may provide richer and more detailed information about the strategies that a child is using at a given time. However, they usually require more time and resources to be carried out properly, and are more potentially vulnerable to the conscious or unconscious biases of the person who is carrying out the assessment.

## 8.2 *Some of the history of individualized intervention*

It is striking how many of the most modern practices have surprisingly early origins. Some forms of individualized, component-based techniques of assessing and remediating mathematical difficulties have been in existence at least since the 1920s (Buswell and John, 1927; Brownell, 1929; Greene and Buswell, 1930; Williams and Whitaker, 1937; Tilton, 1947). On the other hand, they have never been used very extensively; and there are many books, both old and new, about mathematical development and mathematics education, which do not even refer to such techniques, or to the theories behind them.

Weaver (1954) was a strong advocate of differentiated instruction and remediation in arithmetic. He put forward several important points that have since been strongly supported by the evidence, centrally that "arithmetic competence is not a unitary thing but a composite of several types of quantitative ability: e.g. computational ability, problem-solving ability, etc."; that "(t)hese abilities overlap to varying degrees, but most are sufficiently independent to warrant separate evaluations"; and that "children exhibit considerable variation in their profiles or patterns of ability in the various patterns of arithmetic instruction" (pp. 300-301). He argued (pp. 302-303) that any "effective programme of differentiated instruction in arithmetic must include provision for comprehensive evaluation, periodic diagnosis, and appropriate remedial work" and that "(e)xcept for extreme cases of disability, which demand the aid of clinicians and special services, remedial teaching is basically *good* teaching, differentiated to meet specific instructional needs".

For a long time, some researchers and educators have emphasized the importance of investigating the strategies that individual children use in arithmetic: especially those faulty arithmetical procedures that lead to errors (Buswell and John, 1926; Brownell, 1929; Van Lehn 1990). Thus, some children might add without carrying (e.g.  $23 + 17 = 310$ ); others might add all the digits without any reference to whether they are tens or units (e.g.  $23 + 17 = 13$ ); others, when adding a single-digit number to a two-digit number, might add it to both the tens and the units (e.g.  $34 + 5 = 89$ ). Much of the work over the years has looked at the faulty arithmetical procedures that children often demonstrate: e.g. when subtracting

52  
-28

a common faulty procedure is to always subtract the smaller number from the larger, in this case obtaining the answer 36.

Another faulty procedure is to omit borrowing and to write 0 when a larger digit seems to be subtracted from a smaller:

Many papers on mathematical difficulties have included lists of such faulty procedures. In early studies (Greene and Buswell, 1930; Tilton, 1947; Williams and Whitaker, 1937), such flaws tend to be described as 'bad habits'. In some more recent studies (Brown and Burton, 1978; Van Lehn, 1990), they are described as 'bugs', by analogy with malfunctioning computer programs.

Efforts are made to diagnose such incorrect strategies, so that they can be corrected. In the words of Tilton (1947, pp. 84-85), "...many errors are systematic. In other words, not as many errors are accidental and attributable to carelessness as teachers are inclined to think. ...If the youngsters who have such incorrect rules are to be helped, the teacher should know the child's rule, because the child's need is just as much to unlearn his incorrect rule as it is to learn the correct rule. To work in ignorance of his rules is to give him a feeling of confusion."

Tilton (1947) carried out his intervention study with a group of 38 fourth-grade (9-and 10-year-old pupils). They were selected from a sample of 138 children, because they obtained the lowest scores in the Compass Survey Tests in Arithmetic Elementary Examination. They were divided into an intervention group and a control group. The intervention group underwent 20 minutes of individualized intervention per week for four weeks. The intervention was based on the diagnosis and correction of faulty arithmetical strategies: e.g. always subtracting the smaller digit from the larger digit. Although the programme was short and non-intensive (80 minutes individualized intervention in total), the intervention group made 5 months more progress than the control group.

If componential theories of arithmetical ability, and their applications to differentiated instruction and remediation in arithmetic were already being advocated when our contemporary schoolchildren's great-grandparents were at school, why have they had comparatively little impact on theory and practice? Part of the reason is practical: in under-resourced classrooms, it is difficult to provide individualized instruction. Until the 1960s, primary class sizes of 40 or more were common in Britain. The average size of primary school classes in 2004 is 26.2 (DfES, 2004).

As described in the section on "How schools deal and have dealt with mathematical difficulties", if differentiated instruction is used at all under such conditions, it tends to take the form of grouping children according to their perceived overall ability, either in all academic areas, or, at best, in specific school subjects (here mathematics). Therefore, in some quarters, differentiated instruction may be viewed negatively, due to associations with educational methods which separate those labelled as 'less able' from those labelled as 'able'; and involve lower expectations and often worse teaching of the former group. In fact, the true aim of differentiated instruction is to identify strengths and weaknesses within the same domain in the same individual, and to use the strengths to overcome or compensate for the weaknesses. Such an aim is quite incompatible with labelling individuals as globally 'bad' at a subject, and with regarding their weaknesses as purely innate and not susceptible to intervention. Nevertheless, some people confuse the appropriate use of differentiated instruction with its misuse.

Another reason for the limited use of such intervention techniques is that there has been relatively little communication of findings: one of the problems that has bedevilled the whole area of mathematical development. Many an interesting study has remained in near-obscurity, or has only reached a particular category of individuals. Communications between teachers, researchers in education, researchers in psychology and policy-makers have been limited, as often have been communications between researchers within the same discipline in different countries and at different times.

### 8.3 *Recognizing and avoiding potential problems with individualized instruction and remediation*

What are the potential problems that can arise with individualized instruction and remediation? Apart from the practical problems arising from inadequate resources, there can be problems with the methods that are used. In particular, there may be gaps in the selection of arithmetical components for remediation. Indeed, in our present state of knowledge there must be, since we do not yet have a full understanding of the different components of arithmetic; the relationships between them; and the nature and causes of individual differences in them. Although such topics have been studied for many years, our understanding of them still has limitations, due to the number (considerably more than seems to be the case for reading) and complexity of the components; the difficulties and controversies involved in defining successful acquisition of numeracy (Brown, 1999; Baroody, 2003; Cowan, 2003); and the fact that some important methods of studying the issues, such as functional brain imaging, have only recently become available. Nevertheless, there has been sufficient knowledge in the area for quite some time to permit successful work in the area.

Moreover, appropriate individualized instruction depends on appropriate selection of the components of arithmetic to be used in assessment and intervention. This is still an issue for debate and one which requires considerable further research. One of the main potential problems, which was more common in the past than nowadays, is to assume that the components to be addressed must necessarily correspond to specific arithmetical operations: e.g. treating "addition", "subtraction", "multiplication", "division" etc. as separate components. It is, of course quite possible for children to have specific problems with a particular arithmetical operation. Indeed, as we have seen, it is possible for a particular arithmetical operation to be selectively impaired in adult patients following brain damage. Nevertheless, it is an over-simplification to assume that these operations are likely to be the primary components of arithmetical processing. Current classifications tend to place greater emphasis on the type of cognitive process; e.g. the broad distinctions between factual knowledge ("knowing that"), procedural knowledge ("knowing how"), conceptual knowledge ("knowing what it all means") and in some theories utilizational knowledge ("knowing when to apply it") (see, for example, Greeno, Riley and Gelman, 1984). A potential danger of over-emphasizing the different operations as separate components is that it may encourage children, and perhaps adults, to ignore the relationships between the different operations.

Another potential problem - again commoner in the past though still a danger nowadays - is looking at children's difficulties *only* in terms of *procedural* errors. It is, of course, important to investigate the strategies that individual children use in arithmetic, including those faulty arithmetical procedures that lead to errors. Nonetheless, diagnosing the incorrect strategies is not always the final step. There may be a conceptual reason why the incorrect strategy is

acquired and maintained or there may be unperceived conceptual strengths, which need to be noted and built on (Tilton, 1947; Ginsburg, 1977).

Such diagnostic work is vital. Children do indeed frequently acquire incorrect strategies, which can become entrenched, especially if the child is given too much of the wrong sort of arithmetical practice. Nonetheless, diagnosing the incorrect strategies is not always the final step. There may be a *conceptual reason* why the incorrect strategy is acquired and maintained. In the case of the faulty subtraction strategies described above, their acquisition could result from an assumption that larger numbers cannot be subtracted from smaller numbers, combined with an inadequate understanding of place value which makes it difficult for children to understand the nature and purpose of borrowing. As Tilton (1947, p. 85) goes on to remark, "Many of the errors made by these... children seemed to be due to an insufficient understanding of the meaning of numbers. It seems as if these children had been asked to learn the rules for the manipulation of numbers in addition, subtraction and multiplication without having learned the meaning of the symbols that they have been asked to manipulate".

More generally, interventions need to take into account the fact that arithmetical ability is made up of many components. Ginsburg (1972) pointed out that "children's knowledge of mathematics is extraordinarily complex and often much different from what we had supposed it to be... In the case of every child we have interviewed or observed, there have emerged startling contradictions, unsuspected strengths or weaknesses, and fascinating complexities". Of course, no intervention programme can take into account *all* possible components, but they are likely to be most effective if not restricted to one or two components, and if they allow for different children having different types of difficulty, which may not be restricted to procedural difficulties.

#### **8.4 *Individualized intervention programmes with primary school children***

Denvir and Brown (1986b) based an intervention project on the approximate hierarchies of skills that they had investigated (Denvir and Brown, 1986a). The pilot study was carried out individually over a three-month period with seven pupils, who took part in Denvir and Brown's (1986a) longitudinal study. They were taught skills which were regarded as 'next skills' up from their existing skills, in the approximate hierarchies constructed by Denvir and Brown (1986a): for example, a child who could add by 'counting all' might be taught to add by counting on from the first number. The teaching involved presenting children with problems; making concrete objects available; and encouraging them the children to discuss and reflect on the problems. All the children made progress; and they made more progress during the five months immediately following the teaching study than in subsequent periods of the longitudinal study.

The main study was also carried out over a three-month period with twelve pupils who had received low scores on the diagnostic assessment interview (Denvir and Brown, 1986a). They were taught in small groups (6 in each group) twice weekly for six weeks in sessions lasting five minutes. They were encouraged to use multiple methods to carry out problems (e.g. to solve arithmetic problems with a number line, a calculator, concrete objects such as Dienes blocks, in written form, etc.) and to discuss these methods and how and why they gave the same answers. Most conversation was between adults and children; children rarely discussed methods with each other.

The children improved in their performance. The children in this main study gained more skills than those in the pilot study. The children taught in groups seemed more relaxed and positive than those taught individually; but they were more often distracted; it was more difficult to ensure that each child was participating when they could 'hide behind' others; and target skills could not be so precisely matched to each child's existing level.

Interestingly, children in both studies 'did not always learn precisely what they were taught, so attempts to match exactly the task to the child did not always have the expected outcome'. In other words, the interventions resulted in the children acquiring new mathematical skills, but not always the specific skills that they were taught.

Recently, Trundley (1998) carried out a individualized intervention project focussing specifically on the development of derived fact strategies. It was based on the research by Askew, Bibby and Brown (2001) on the importance of derived fact strategies in boosting the performance of low mathematical attainers. In particular, it was based on the theory that use of known number facts and derived fact strategies reinforce one another. The more facts you know, the more you can derive; some facts you know become known facts. Some children rely persistently on counting-based strategies, and thus do not begin this mutual development of factual and conceptual knowledge. These tend to be the children with mathematical difficulties.

The project worked with two groups of 12 teachers in Devon. The first group met for 20 weekly full-day sessions during the autumn and spring terms of 1997/1998, and the second group for 20 weekly half-day sessions. Each teacher was asked to select six children in his/her class who were underachieving in mathematics specifically, All the Year 3 and 4 children in these classes were tested at the beginning and end of the project using an oral test. The six children in each teacher's focus group were selected on the basis of (a) the teacher assessment; (b) the oral test and (c) National Curriculum Tests at the end of Key Stage 1.

Each child underwent a weekly 20-minute individual session with his/her teacher in a 'mirror room', where they were observed by the other teachers in an adjacent room through a one-way mirror. Each session consisted of:

- 2-3 minutes practicing counting skills.
- 2 minutes revising individual known facts.
- 10-12 minutes practicing derived fact strategies building on known facts.
- 2 minutes playing with big numbers or working on a problem.

Teachers made notes on the sessions that they observed, and discussed them with the teacher who ran the sessions. All the teachers in the project ran their own sessions and also observed the other teachers' sessions. They also all read papers on numeracy teaching and assessment by M. Askew, S. Atkinson, A. Straker and others.

Those in the first, full-day group additionally engaged in afternoon group discussions of their readings, and related issues in mathematics teaching.

The teachers were also observed in their classrooms teaching mathematics twice during the course of the project, and observations were fed back.

The children, who were reassessed 5 months after the start of the project, showed considerable improvement. As regards *counting*, they were much more able to count in different steps, both forwards and backwards. Following intervention, they were correct and fluent at over 70% of the questions that they had previously been unable to do; and at 80% of the questions on which they had been mostly correct but made some errors. As regards *calculation*, they were now able to calculate 65% of the problems which they had previously been unable to do; used either derived or known facts on 40% of the problems which they had previously solved by counting objects; and used either derived or known facts on 68% of the problems which they had previously solved by counting in ones.

The teachers were enthusiastic about the project. Some comments related to feeling that it was acceptable to do small-group work with children with difficulties: “I am determinedly focusing on one group at a time”; “I spend the main part of the teaching activity with only one group – previously I felt too guilty to do this and perhaps spread myself too thinly”. The importance of derived fact strategies was emphasized in many comments: “The use of mental maths to build strategies has been the biggest development in my school stemming from the course”; “...this extra commitment to thinking about and explaining their mental strategies, looking for alternative strategies, making connections has paid dividends in terms of the children’s progress”; “The overwhelmingly powerful strand of the work done through the project has been the close observation of the different strategies children use to solve mathematical problems. It highlighted for me how much I knew of the way I taught children to do things and how little I knew of the way they actually did them”.

Kaufmann, Handl and Thony (2003) carried out a pilot study of an intervention project for Austrian children with arithmetical difficulties, which included factual, procedural and conceptual components. Six children between 6 and 7 with a diagnosis of developmental dyscalculia took part in the study. They underwent three individual 25-minute sessions weekly for a period of six months. The intervention involved training in components of increasing difficulty, beginning with counting principles; and proceeding through: writing and reading numbers; learning the number combinations that add up to 10; learning addition facts; learning subtraction facts; dealing with the inverse relationship between addition and subtraction; addition of numbers over 10; dealing with the base 10 system; learning multiplication facts; and learning how to carry out division procedures. They were compared with 18 children without mathematical difficulties on the Number Processing and Calculation Screening Battery. Overall, effects were greater in the intervention group.

Two larger-scale independently developed, individualized intervention programmes which address numeracy in young children, and take componential approaches based on cognitive theories of arithmetic, are the Mathematics Recovery programme (Wright, Martland and Stafford, 2000; Wright, Martland, Stafford and Stanger, 2002), and the Numeracy Recovery programme (Dowker, 2001). There are some important differences between the two programmes. Notably, the Mathematics Recovery programme is much more intensive than the Numeracy Recovery programme; and the Mathematics Recovery programme places more emphasis on methods of counting and number representation, and the Numeracy Recovery programme on estimation and derived fact strategy use. From a more theoretical point of view, the Mathematics Recovery programme places greater emphasis on broad developmental stages, while the Numeracy Recovery programme is treats mathematical development, to a greater extent, as involving potentially independent, separately-developing skills and processes. Despite these distinctive features, the two programmes have other important common features besides being individualized and componential. Both programmes are targeted at the often neglected

early primary school age group (6- to 7-year-olds); both deal mainly with number and arithmetic rather than other aspects of mathematics; and both place a greater emphasis than most programmes on collaboration between researchers and teachers.

#### *8.4.1 Mathematics Recovery*

The Mathematics Recovery programme was designed in Australia by Wright and his colleagues (Wright et al, 2000, 2002). In this programme, teachers provide intensive individualized intervention to low-attaining 6- and 7-year-olds. Children in the programme undergo 30 minutes of individualized instruction per day over a period of 12 to 14 weeks.

The choice of topics within the programme is based on the Learning Framework in Number, devised by the researchers. This divides the learning of arithmetic into five broad stages: emergent (some simple counting, but few numerical skills); perceptual (can count objects and sometimes add small sets of objects that are present); figurative (can count well and use 'counting-all' strategies to add); counting-on (can add by 'counting on from the larger number' and subtract by counting down; can read numerals up to 100 but have little understanding of place value); and facile (know some number facts; are able to use some derived fact strategies; can multiply and divide by strategies based on repeated addition; may have difficulty with carrying and borrowing).

Children are assessed, before and after intervention, in a number of key topics. They undergo interventions based on their initial performance in each of the key topics. The key topics that are selected vary with the child's overall stage. For example, the key topics at the Emergent stage are (i) number word sequences from 1 to 20; (ii) numerals from 1 to 10; (iii) counting visible items (objects); (iv) spatial patterns (e.g. counting and recognizing dots arranged in domino patterns and in random arrays); (v) finger patterns (recognizing and demonstrating quantities up to 5 shown by number of fingers); and (vi) temporal patterns (counting sounds or movements that take place in a sequence). The key topics at the next, Perceptual, stage are: (i) number word sequences from 1 to 30; (ii) numerals from 1 to 20; (iii) figurative counting (counting on and counting back, where some objects are visible but others are screened); (iv) spatial patterns (more sophisticated use of domino patterns; grouping sets of dots into "lots of 2"; "lots of 4", etc.); (v) finger patterns (recognizing, demonstrating and manipulating patterns up to 10 shown by numbers of fingers); and (vi) equal groups and sharing (identifying equal groups, and partitioning sets into equal groups). The key topics at later stages place greater emphasis on arithmetic and less on counting. Despite the overall division into stages, the programme acknowledges and adapts to the fact that some children can be at a later stage for some topics than for others.

There are many activities that are used for different topics and stages within the Mathematics Recovery programme. For example, activities dealing with temporal patterns at the Emergent stage include children counting the number of chopping movements made with the adult's hands; makes with his/her hands; producing a requested number of chopping movements with their own hands; counting the number of times they hear the adult clap; and clapping their own hands a requested number of times. Activities dealing with number word sequences in fives at the Counting-On stage include children being presented with sets of 5-dot cards; counting the dots as each new card is presented; counting to 30 in fives without counting the dots; counting to 30 in fives without the cards; counting to 50 in fives without the cards; and counting backward in fives from 30, first with and then without the cards.

Children in the programme improved very significantly on the topics that form the focus of the problem: often reaching age-appropriate levels in these topics. The teachers who worked on the programme found the experience very useful; felt that it helped them to gain a better understanding of children's mathematical development; and used ideas and techniques from the programme in their subsequent classroom teaching.

Children in the programme improved very significantly on the topics that form the focus of the project. In fact, during the period 1992-1997, over 75% of the pupils who had undergone the intervention reached age-appropriate or higher levels in the topics tested, despite the fact that all had been performing below age level on at least some of the topics at the start of the intervention. The teachers who worked on the programme found the experience very useful; felt that it helped them to gain a better understanding of children's mathematical development; and used ideas and techniques from the programme in their subsequent classroom teaching. So far, the evaluations of this project have not used standardized tests; this would be a desirable next step, if the results of the programme are to be easily compared with those of other programmes or none.

#### *8.4.2 Numeracy Recovery*

The Numeracy Recovery programme (Dowker, 2001, 2003), piloted with 6-and 7-year-olds (mostly Year 2) in some primary schools in Oxford, is funded by the Esmee Fairbairn Charitable Trust. The scheme involves working with children who have been identified by their teachers as having problems with arithmetic. 175 children (about 15% of the children in the relevant classes) have so far begun or undergone intervention.

These children are assessed on nine components of early numeracy, which are summarized and described below. The children then receive weekly individual intervention (half an hour a week) in the particular components with which they have been found to have difficulty. The interventions are carried out by the classroom teachers, using techniques proposed by Dowker (2001).

The teachers are released (each teacher for half a day weekly) for the intervention, by the employment of supply teachers for classroom teaching. Each child typically remains in the programme for 30 weeks, though the time is sometimes shorter or longer, depending on teachers' assessments of the child's continuing need for intervention. New children join the project periodically.

The interventions are based on an analysis of the particular subskills which children bring to arithmetical tasks, with remediation of the specific areas where children show problems. The components addressed here are not to be regarded as an all-inclusive list of components of arithmetic, either from a mathematical or educational point of view. Rather, the components were selected because earlier research studies and discussions with teachers have indicated them to be important in early arithmetical development, and because research has shown them to vary considerably between individual children in the early school years.

The components that are the focus of the project are

- (1) Counting procedures.

- (2) Counting principles: especially the order-irrelevance principle that counting the same set of items in different orders will result in the same number; and the ability to predict the result of adding or subtracting an item from a set.
- (3) Written symbolism for numbers.
- (4) Understanding the role of place value in number operations and arithmetic.
- (5) Word problem solving.
- (6) Translation between arithmetical problems presented in concrete, verbal and numerical formats (e.g. being able to represent the sum ' $3 + 2 = 5$ ' by adding 3 counters to 2 counters, or by a word problem such as 'Sam had 3 sweets and his friend gave him 2 more, so now he has 5' (see Hughes, 1986).
- (7) Derived fact strategies in addition and subtraction: i.e. the ability to derive and predict unknown arithmetical facts from known facts, for example by using arithmetical principles such as commutativity, associativity, the addition/subtraction inverse principle, etc.
- (8) Arithmetical estimation: the ability to estimate an approximate answer to an arithmetic problem, and to evaluate the reasonableness of an arithmetical estimate.
- (9) Number fact retrieval.

The intervention procedures make some use of existing published materials: for example materials by Hopkins (1997a,b) and Straker (1996) form part of the intervention for number fact retrieval. Most of the procedures, however, use materials and tasks specifically devised for the project.

For example, the assessments and interventions for *translation* (Component 6) includes tasks of translating in all possible directions between numerical (written sums); concrete (operations with counters); and verbal (word problem) formats for both addition and subtraction. In the intervention, children are shown the same problems in different forms, and shown that they give the same results. They are also encouraged to represent word problems and concrete problems by numerical sums, and to represent numerical problems and word problems by concrete objects.

The assessments and interventions for *estimation* (Component 8) involve presenting children with a series of problems of varying degrees of difficulty, and with estimates made for these problems by imaginary characters (Tom and Mary). The children are asked (a) to evaluate "Tom and Mary"'s estimates on a five-point 'smiley faces' scale from "Very good" to "Very silly"; and (b) to suggest "good guesses" for these problems themselves. They are encouraged to give reasons for their evaluations.

The children in the project, together with some of their classmates and children from other schools, are given three standardized arithmetic tests: the British Abilities Scales Basic Number Skills subtest (1995 revision), the WOND Numerical Operations test, and the WISC Arithmetic subtest. The first two place greatest emphasis on computation abilities and the latter on arithmetical reasoning. The children are retested at intervals of approximately six months.

The initial scores on standardized tests, and retest scores after 6 months, of the first 146 children to take part in the project have now been analyzed. Not all of the data from 'control' children are yet available, but the first 75 'control' children to be retested showed no significant improvement in standard (i.e. age- corrected) scores on any of the tests. In any case, the tests are standardized, so it is possible to estimate the extent to which children are or are not improving relative to others of their age in the general population.

The children in the intervention group have so far shown very significant improvements. (Average standard scores are 100 for the BAS Basic Number Skills subtest and the WOND Numerical Operations subtest, and 10 for the WISC Arithmetic subtest.) The median standard scores on the BAS Basic Number Skills subtest were 96 initially and 100 after approximately six months. The median standard scores on the WOND Numerical Operations test were 91 initially and 94 after six months. The median standard scores on the WISC Arithmetic subtest were 7 initially, and 8 after six months (the means were 6.8 initially and 8.45 after six months). Wilcoxon tests showed that all these improvements were significant at the 0.01 level.

One hundred and one of the 146 children have been retested over periods of at least a year, and have been maintaining their improvement.

### *8.5 Computer programs for individual instruction*

With the increasing development and availability of computer technology, a number of computer programs have been developed individualized instruction and intervention (Lepper and Gurtner, 1989; Poustie, 2001; Errera, 2002). Computerized individualized instruction systems have the same potential advantages (adaptability to individual patterns of learning; lack of social pressure) and disadvantages (lack of social interaction and communication; often exclusive emphasis on the response rather than on the cognitive process of reaching it) as other individualized self-teaching systems. In addition, they have the important advantage that computers are motivating to many children; and that, with increasing availability of home computers and computer games, they may be used outside of as well as within a school context.

Computer programs in the past tended to take a simplistic approach to children's errors and to reward correct answers, and reject incorrect answers, without scope for analyzing how the errors occurred (Hativa, 1988). The more sophisticated forms of programming that are available today make it much more possible to diagnose and interpret misconceptions; though, as with any test, they may not pick up a particular individual's interpretations and misinterpretations, especially if these are somewhat untypical of the population as a whole.

Most studies of computer-based intervention with children with mathematical difficulties are as yet relatively small-scale. For example, Earl (2003) investigated the use of RM (Research Machines) mathematics software, an integrated learning system designed for use in primary schools, with eight Year 7 and 8 pupils with communication disorders in a specialized unit within a mainstream secondary school. They were evenly divided between Years 7 and 8. Seven of the eight pupils were boys. Four were diagnosed with speech and language disorders and four with autistic spectrum disorder.

RM Maths presents maths questions to pupils both orally through headphones and visually, often with illustrations involving cartoon animals. Pupils are given three chances to answer a question, with increasing clues each time, until the answer is finally given. The difficulty level of subsequent questions is adjusted upwards or downwards, in view of the pupil's success or failure on a given question. Pupils typically undergo three 15-minute sessions per week.

The children in this project used the RM program. They and their teachers were encouraged to keep a Computer Diary, describing their work and impressions. They also used self-assessments of their own understanding using 'red', 'amber' and 'green' to indicate limited,

partial or complete understanding. They were assessed at the beginning and end of the project through the NFER Mathematics tests, and the computer-based assessment programme 'Snapshot'.

Interviews revealed that most of the pupils preferred 'easy' work. When asked what they found easy or hard, they tended to find addition easy and multiplication and/or division hard.

Most staff and pupils liked the software; though two of the Year 8 pupils complained of the childishness of the graphics; and another pupil said that he would have preferred 4-D graphics! The interpretability of interview and self-assessment measures were limited by the fact that these *were* pupils with communication difficulties.

The three who made greatest use of the software made considerable progress on the NFER tests. Four made considerable progress on 'Snapshot'; and four made little progress; the four who did make considerable progress had made more use of the software than the others.

Pennant (2001) carried out action research on computer-based intervention as a primary teacher and maths co-ordinator working with low-achieving Year 5 pupils. The study was based on the observation that pupils with learning difficulties were often better at concentrating when using a computer than in other situations. This led the researcher to investigate whether they would acquire mathematical skills better when using a computer than in traditional classroom situations. The pupils were 13 Year 5 children, aged 9 and 10, with maths NFER scores between 76 and 110. The lesson objectives were multiplying by 4, multiplying by 20, multiplying by 5, and adding 4. They alternated between practising on the computer and working with the teacher in a small group on interactive dice and card games.

The CDROM was produced by Sherston Software, and is called Mental Maths Olympics Year 4. It targets mental maths strategies in the National Numeracy Strategy. It is set up so that each mental strategy is introduced by a computer 'coach' who explains how to carry out the method. Mathematics problems are then put on the screen one at a time; and the computer provides feedback as to whether the child's responses are right or wrong. At the end of each set of questions, scores are provided in terms of an Olympic sport (e.g. time or distance obtained). Children attempt to improve on their scores in subsequent sets of problems, and are awarded 'certificates' at the end. They can refer to the coach as often as they wish.

All pupils improved. They progressed from getting 0 to 20% correct to getting 85 to 100%. Scores a couple of months later ranged from 65 to 100%, indicating generally good retention of learning. Notably, children concentrated on the computer activities for at least 20 minutes, whereas their concentration span was more typically 10 minutes or less.

It should be noted, however, that as with peer tuition, Kroesbergen and Van Luit's (2003) meta-analysis indicated that computer-based interventions tend to result in less progress than interventions carried out by teachers. These results may be based in part on sampling differences, and certainly do not mean that computer-based interventions are worthless; however, they should not be seen as a replacement for interventions by human beings. Computer-based interventions, in any case, take many forms and some will be more effective than others. On the whole, it may be appropriate to view the computer-based aspects of such interventions predominantly as 'tools of access', increasing motivation, and reducing the impact of emotional, communication, or motor difficulties.

## 9. *Interventions currently used within the National Numeracy Strategy*

The National Numeracy Strategy (DfEE 1999, and subsequently) has introduced some intervention techniques for children who are struggling with arithmetic. The Numeracy Strategy, like the Literacy Strategy, incorporates three 'waves' or levels of intervention: Wave 1, or whole-class teaching for all children (e.g. the Daily Mathematics Lesson); Wave 2, or interventions in small groups with children who are experiencing mild or moderate difficulties in the subject; and Wave 3, or targeted interventions for children with special educational needs.

The main Wave 2 intervention programme is the Springboard programme, used with children in Years 3 to 7 (7 to 12 years). The target group is children with relatively mild arithmetical difficulties; e.g. those who perform at Level 2C in standardized school tests at age 7. The Springboard programme provides additional tuition for small groups of six to eight children as a supplement to the daily mathematics lesson that they undergo with the whole class. Typically, it provides two 30-minute sessions which consolidate the work currently being taught in the daily mathematics lessons.

Wave 3 materials for mathematics have been piloted since June 2003 in 25 local education authorities across the country (Primary Strategy). They are still in draft form, and awaiting full evaluation. They emphasize individualized diagnosis of the errors and misconceptions shown by children with significant difficulties, specific or non-specific, with mathematical learning (usually, children who are performing at least one National Curriculum level below age-related expectations)

The children in the pilot study are given diagnostic interviews to determine conceptual and procedural difficulties in coping with the objectives of the major areas of the mathematics curriculum. For example, one of the important skills at Year 2 in the area of Addition and Subtraction is to understand the inverse relationship between addition and subtraction (e.g. if  $4 + 13 = 17$ , then  $17 - 4 = 13$ ). Difficulties could involve a complete failure to apply the inverse principle, or its application only to small numbers, for example those below 10. This difficulty can be assessed by giving the child such problems as "What is the answer to 30 add 20? If 30 add 20 is 50, what is 50 subtract 20?"

Wave 3 materials are being developed for use with children to correct the errors and misconceptions that have been spotted. For example, children who have difficulty in understanding the values of digits may be shown 3-digit numbers made out of arrow cards. The teacher then replaces one of the digits (e.g. changing 233 to 203) and asks the child how it has changed. Another related activity involves using a number fan to make a 3-digit number (e.g. 523) and asking the child to change one digit to make a larger number; such activities can also be carried out with 2-digit, 4-digit or higher numbers.

Many of the materials and activities involve presenting children with a variety of words, symbols, models and images to represent the same concept or process. For example, activities for children who have difficulty with remembering and using multiplication facts include representing such facts by groups of cubes, beads and domino patterns; verbal counting by 2s or larger sets; and representing the facts on a multiplication grid.

The materials can be used within or outside the daily mathematics lesson, and are used individually by the child with the class teacher, a teaching assistant, or a special needs teacher. The child typically undergoes one 20-minute individual session per week, and 5-minute 'spotlight' sessions on each of the following days, in addition to following the mathematics curriculum with or without specialist support.

#### 10. *Children with general learning difficulties: the Spiral Mathematics Programme*

In general, programmes for children outside mainstream education are not the focus of this review. However, the Spiral Mathematics Programme (Banes, 1999) will be mentioned here, as it is directly influenced by the concepts of the National Numeracy Strategy. It is designed for children with general learning difficulties working within Level 1 in mathematics, and has been predominantly used in special schools and units, though it may also be used for children with learning difficulties in mainstream schools. The programme includes three phases. The first or introductory phase includes three strands of mathematical learning: (1) prediction/anticipation (e.g. predicting the movements and positions of one's own body or of an object); (2) classification; and (3) comparison. The second phase includes appreciation of pattern, and incorporates the three above strands at a more advanced level, and also the additional strands of pattern and estimation. The third phase is the language level, and emphasizes different ways of describing and communicating mathematical findings.

The programme is based on the QCA's "P" levels prior to Level 1, ranging from P1 (where the child is aware of, and shows reflex responses to, sensory stimuli) to "P8" (where the child can compare lengths, masses and quantities of items; recognize shapes and make simple pictures and patterns from them; uses mathematical language such as "circle" and "larger"; counts ten objects reliably and beyond ten by rote; estimates small quantities; recognizes a few written numerals; and finds one more and one less than a given number of objects).

Tasks in the programme include *mazes* (finding one's way through mazes, and designing mazes). They also include LOGO and Turtles: the programme originally designed by Seymour Papert, where children plan a route and give instructions to a computer turtle as to how to follow the route. More advanced activities involve recognizing, using and constructing patterns from a variety of cultures: Islamic, African, ancient Roman, etc.

#### 11. *Conclusions*

Research strongly supports the view that children's arithmetical difficulties are highly susceptible to intervention. It is not the case that a large number of children are simply 'bad at maths', and that nothing can be done about it. For example, it is notable that Dowker's (2001, 2003) study indicates strong improvements occurred in the WISC Arithmetic subtest: part of a test used predominantly to test 'IQ' rather than school achievement.

Moreover, individualized work with children who are falling behind in arithmetic has a significant impact on their performance. The amount of time given to such individualized work does not, in many cases, need to be very large to be effective. Quite small amounts of

individualized work may bring a child to the point where (s)he can profit much better from the general ('Wave 1') teaching that (s)he receives.

### 11.1 *Need for further development and investigation of intervention programmes*

Further investigations are of course necessary show whether and to what extent the individualized interventions described here are more effective in improving children's arithmetic than *other* interventions which provide children with individual attention: e.g. interventions in literacy, or interventions in arithmetic which are conducted on a one-to-one basis but not targeted toward individual strengths and weaknesses. It is also desirable to investigate whether different approaches to such intervention (e.g. age when intervention starts; degree of intensiveness; the particular components emphasized) may be differentially appropriate to different groups of children.

It is important to compare the different programmes using *similar forms of assessment*. At present, as pointed out by Kroesbergen and Van Luit (2003) and by Rohrbeck et al (2003), it is difficult to compare programmes, because most researchers and project managers have worked in relative isolation, unaware of each other's programmes. Most programmes have involved different methods of sampling and different forms of assessment, rendering it difficult or impossible to make valid comparisons.

It would also be desirable to investigate the potential for similar types of intervention in areas of mathematics other than numeracy: e.g. geometry and measurement.

It would also be desirable to investigate who are the best people to carry out individual and small-group interventions: classroom teachers (perhaps being relieved of classroom duties for short periods for the purpose); learning support assistants; special needs teachers; or in certain cases parents?

Future goals should include further development and investigation of individualized and small-group interventions with children *outside* the age range (schoolchildren over 7) which has so far received most attention from this point of view in British practice. Action research by teachers could play a significant role in such studies.

In particular, given the already-demonstrated importance of preschool interventions with at-risk children, it would be desirable to have more investigations of methods of *assessing* preschool children's early mathematical abilities; of *predicting* different forms of mathematical difficulty; and of targeting early interventions to have maximum impact in *preventing* such difficulties.

Among school age children, it would be desirable to focus more on interventions for young children: those under 7. This younger age group seems particularly important from this point of view, as the first two years of school are when much of the foundation is being laid for later mathematical learning; and identifying and intervening with difficulties at this stage has the potential prevent children from developing inappropriate arithmetical strategies which may handicap them in later work, and from developing negative attitudes toward arithmetic.

At the other end of the scale, another goal of research should be to investigate the role of targeted interventions for *adults* with mathematical difficulties. Most intervention programmes have been with children or adolescents. Since numeracy difficulties have lifelong implications,

it is important that more work be carried out on diagnosis and intervention for such difficulties in adults. Numeracy is increasingly included in 'basic skills' programmes for adults; though most such programmes do not differentiate between adults who have not learned such skills due to lack of educational opportunity, and those who have specific difficulties in arithmetic.

Greater communication and collaboration between teachers, researchers and policy-makers is vital. This was indeed pointed out by Piaget (1971), but has only rarely been put into practice.

## 11.2 *Implications for educational practice*

It is clear that many children have difficulties with some or most aspects of arithmetic. Arithmetical thinking involves such a wide variety of components; so that there are many forms and causes of arithmetical difficulty, and many degrees of severity. Only a minority of children have dyscalculia as defined by the DfES (severe specific difficulties with most or all aspects of arithmetic). However, a very significant proportion of the population have difficulties with certain aspects of arithmetic, which are sufficient to cause them at least some practical and educational problems, especially without intervention.

It is important to recognize those individuals who do have major difficulties with very basic number concepts: e.g. perceiving and distinguishing small quantities (Butterworth, 1999, 2002), which are likely to result in severe general mathematical difficulties.

Most children with mathematical difficulties do not have such extreme problems, and should be seen more as part of a continuum of mathematical performance. This implies that there is also a continuum of intervention needs. This may range from:

*No active intervention needs, but pupil may benefit from teachers and others being aware of his or her specific strengths and weaknesses;*

through:

*Need for flexible adaptations of programme of activities within whole-class setting ('Wave One' intervention').*

*Need for small group provision ('Wave Two' intervention)*

*Need for individualized ('Wave Three') provision of an infrequent and/or non-intensive nature.*

*Need for intensive individualized ('Wave Three') provision*

to

*Need for totally individualized programme and/or special educational setting.*

Children may require different degrees and types of intervention at different times in their school career, or for different aspects of the mathematics curriculum. Relatively small amounts of individual intervention may make it possible for a child to benefit far more fully from whole-class teaching.

There are some children who have a greater than average chance of developing arithmetical difficulties, and should be targeted for assessments. These include children with any degree of general learning difficulties, and those with dyslexia and/or oral language difficulties. Also, arithmetical difficulties sometimes run in families, so that children whose parents or siblings have trouble with arithmetic are at increased risk of having difficulties with arithmetic themselves. However, not all children with these risk factors will have difficulty with arithmetic; and some children with none of these risk factors have specific difficulties in arithmetic.

Children who have difficulty with mathematics usually (but not always) have particular difficulty in remembering number facts. This is probably especially true of children who are also dyslexic or have other language difficulties. Often, such children can learn to compensate for their difficulties by using strategies based on reasoning; and eventually by using a calculator effectively. However, some children have more serious difficulties and are restricted to using cumbersome strategies that are based on counting. Interventions can also help these children, but may need to be longer and/or more intensive.

No two children with arithmetical difficulties are the same. It is important to find out what specific strengths and weaknesses an individual child has; and to investigate particular misconceptions and incorrect strategies that they may have. Interventions should ideally be targeted toward an individual child's particular difficulties. If they are so targeted, then most children may not need very intensive interventions.

Mathematics assessment is useful when it enables teachers to identify the particular problems that children are experiencing, and to profile their strengths and weaknesses. Forms of assessment that compare children in a linear fashion and label some children simply as 'bad at maths' are likely to be counter-productive.

Peer tuition and computerized teaching play a useful role in mathematics interventions, but cannot substitute for interaction with a teacher.

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