

Investigation and Exploration

Dynamic Geometry Software

What is Mathematics Investigation?

A complete mathematical investigation requires at least three steps:

- finding a pattern or other conjecture;
- seeking the logical interconnections that constitute proof; and
- organizing the results in a way that can be presented coherently.

(Facilitator's guide to "Way to Think about Mathematics" by Steven Benson)

A performance view of understanding

Understanding is a matter of being able to do a variety of thought provoking thing with a topic such as:

- explaining,
- justifying,
- finding evidence and examples,
- generalizing,
- applying,
- analogizing, and
- presenting the topic in new ways.

By definition, understanding performances are activities that go beyond the rote and the routine.

Dynamic Geometry Software

- Interactive geometry software (also called Dynamic Geometry Software) is computer programmes which allow one to create and then manipulate geometric constructions, primarily in plane geometry.
- One starts construction by putting a few points and using them to define new objects such as lines, circles or other points.
- After some construction is done, one can move the points one started with and see how the construction changes.

The Geometer's Sketchpad (GSP)

The GSP is one of the dynamic mathematics software that provides opportunities for teachers and students to use their ability to access, drag, visualize, and create graphic. The GSP enable students to construct graphical and numerical representation together. These enhance students' understanding in interpreting both visually and numerically mathematics.

The Geometer's Sketchpad (GSP)

When the GSP start up, you will see a box with the words “The Geometer’s Sketchpad” in the centre of the screen. Click once anywhere to clear the box. Now you shall see The Sketchpad document window as shown in Figure 1. The document window displays one page at a time. Each document window consists of the Title Bar, the Menu Bar; the Minimize and Maximize Boxes; the Close Box, GSP layout, called a sketch; Page Tabs; Scroll Bars; and Resize area.

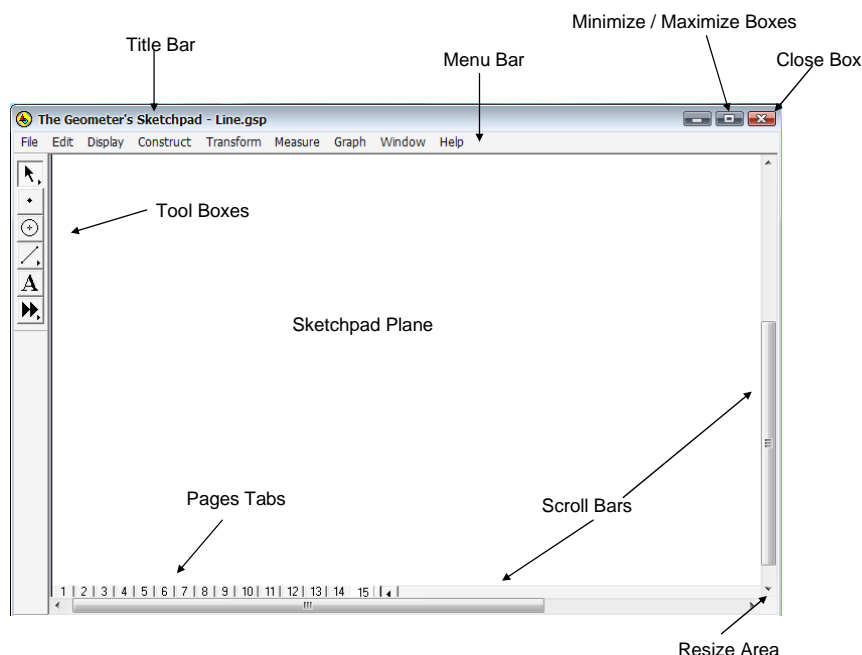


Figure 1 Sketchpad Document Window

On the left side of your sketch plane, you will see the Sketchpad’s toolbox as shown in Figure 2. The Sketchpad’s toolbox consists of 6 tools, namely:

- The Selection Arrow Tool
- The Point Tool
- The Compass Tool
- The Straightedge Tool
- The Text Tool
- The Custom Tool

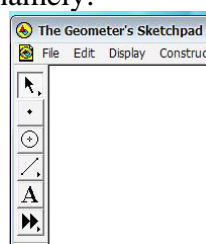
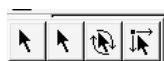







Figure 2 Sketchpad’s Toolbox

The Selection Arrow Tool




The Selection Arrow Tool is an important tool of the Sketchpad’s Dynamic Geometry capacities. It is the tool you use to select or deselect objects and to move or drag objects in your sketch. The Selection Arrow Tools include of three variation tools as follows:


- Translate Tool 
- Rotate Tool 
- Dilate Tool 

When Sketchpad starts, the active Selection Arrow Tool is the Translate Arrow Tool . Choose a different Arrow Tool by pressing and holding on the arrow tool icon in the Tool box. When you press and hold, a menu  pops out and you can choose one of the three arrow tools.

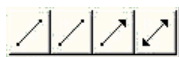
The Point Tool

-  The Point Tool is used to construct points in your sketch. When the point tool is active, you can:
 - Click in any empty area of your sketch to create an independent point;
 - Click on a path object such as a segment, a circle, or the edge of a polygon interior to construct a point on the path; and
 - Click at the intersection of two path objects.




The Compass Tool

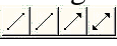
-  The Compass Tool is used to construct circle in your sketch. Construct a circle by using the Compass Tool:
 - Choose the Compass tool,
 - Press the mouse button at the center point, drag and release the button at the radius point.

The Straightedge Tool




The Straightedge Tool is used to construct straight objects: segments, rays and lines in your sketch. The Straightedge Tools include the segment tool, the ray tool and the line tool.

-  The Segment Tool is used to construct a segment between its two endpoints.
-  The Ray Tool is used to construct a ray from one endpoint through another point.
-  The Line Tool is used to construct a line through two points.

When Sketchpad starts, the active Straightedge Tool is the segment tool. Choose a different Straightedge tool by pressing and holding on the Straightedge tool icon in the Toolbox. When you press and hold, a menu  pops out and you can choose any of the three tools.

The Text Tool

-  The Text Tool is used to create, show, hide and edit labels, to add text to your sketch, and to change the text displayed with measurements, calculations and parameters.

Text Palette

You can use Text Palette to format the font, size, style and colour of the label, captions, measurements and other text. Normally the Text Palette appears at the bottom of the sketchpad application window automatically when you edit a caption.



Figure 3 Text Palette

You can also use the text palette to insert mathematical symbols and formatting into captions. Press the Symbolic Notation button in the Text Palette to displays additional notation tools as show in Figure 4

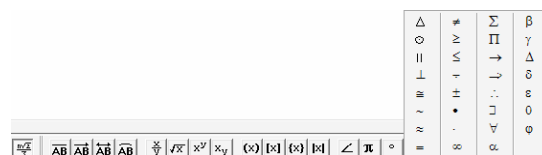
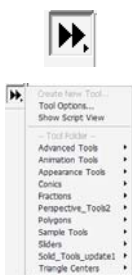


Figure 4 Mathematical Symbols

The Custom Tool



The Custom Tool is used to create new construction tool that extended Sketchpad’s built-in construction capabilities.

When you press the Custom Tool icon in the Toolbox, Sketchpad displays the Custom Tools menu. This menu has several parts, such as Create New Tool, Tool Options, Show Script View and other Document.

Sketchpad Activities for young learners

Activity 1 Balance

Balance	Interactive Sketchpad Exploration
Description	This sketch contains a collection of five different shapes: a star, square, circle, triangle, and diamond. The weight of these shapes can be compared by using the provided balance scale; simply drag one or more shapes onto the balance and it will tilt in the direction of the heavier shape(s).
Grade	3, 4, or 5
Total Time	45-60 minutes
What You Need	<ul style="list-style-type: none"> • The sketch Balance.gsp • Paper and pencil to help students keep track of their work
Standards	<p>Number & Operations “Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.”</p> <p>Algebra “Describe, extend, and make generalizations about geometric and numeric patterns; represent and analyze patterns and functions, using words, tables, and graphs.”</p> <p>Problem Solving “Build new mathematical knowledge through problem solving; apply and adapt a variety of appropriate strategies to solve problems.”</p> <p>Reasoning & Proof “Recognize reasoning and proof as fundamental aspects of mathematics.”</p> <p>Communication “Analyze and evaluate the mathematical thinking and strategies of others.”</p>
Sketchpad Skills	<p>This activity is suitable for first-time Sketchpad users. Students will be required to:</p> <ul style="list-style-type: none"> • Use a button • Change the value of a parameter <p>To “press” a button, click it once using the Arrow tool.</p> <p>To change the value of a parameter, select it with the Arrow tool and press either + or – on the keyboard to increase or decrease. Another way is to double-click the parameter with the Arrow tool, then change the value directly in the dialog box that appears.</p>

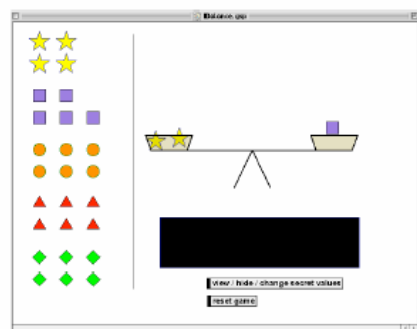
Balance

Teacher Notes

This sketch contains a collection of five different shapes—a star, square, circle, triangle, and diamond. The weight of these shapes can be compared by using the provided balance scale—simply drag one or more shapes onto the balance and it will tilt in the direction of the heavier shape(s).

We suggest that you begin the activity with a whole-group discussion, as described in “Introducing the Activity,” then invite students to play the Balance Game in pairs, as described in “How to Play the Balance Game.”

Introducing the Activity



A balance scale is a useful metaphor for comparing the size of numbers. Note, however, that this model has a few quirks. When students play with the scale, they will discover the following:

Regardless of how much weight is placed on a pan, the balance will always “tip” by the same amount.

The weights do not remain sitting on the pans when the balance tips up or down.

Discuss these unexpected behaviors with your students. Once the differences are noted, students will probably not find them to be an issue. Some students assume that the star, the largest of the objects, weighs the most. You might point out here that a piece of paper, while larger than a small rock, weighs less than the rock.

Invite students to interact with the balance scale. Once they are comfortable with the way it works (for instance, all stars “weigh” the same), challenge them to rank the five shapes from lightest to heaviest. Once the students have accomplished this, ask them to generate different combinations of shapes that achieve balance (for example, with the default values, **triangle + star** equals **square + circle**). This can lead into a discussion of how much each shape weighs. As a hint to students, announce that the star weighs one unit. Using this information, challenge the students to find the values of the remaining shapes using the star as unit of weight. (**Star = 1, square = 2, circle = 3, triangle = 4, diamond = 5.**) If you do not provide the students with the hint, they may come up with a different set of correct values such as **star = 2, square = 4, circle = 6, triangle = 8, diamond = 10.** (In some classrooms, you may wish to not provide the hint, then discuss the equivalence of different possible answers.)

After this initial exploration with the balance, invite students to play the Balance Game, as described here, or (if you feel your students need a bit more introductory time, or perhaps if you are short on computers) to explore these additional questions:

- 1) Rather than rank all the shapes, can students find—with as few weighings as possible—the shape with value of 3?
- 2) Using just a single star, square, circle, triangle, and diamond, is it possible to balance the scale? (All five shapes must be used.)
- 3) Is it possible to balance the scale using just four shapes, all different? What’s the same in all possible solutions? What’s different?
- 4) How many different ways is it possible to balance the shape with value 5?

- 5) Reveal the values of the five shapes. Then put all six copies of any shape on one side of the balance. Can the scale now be balanced using some combination of the remaining shapes?

How to Play the Balance Game

1. Player One presses the *view/hide/change secret values* button to reveal the numerical value of the five shapes. While Player Two looks away, Player One double-clicks these numbers and enters new values. Each shape should be assigned a value from 1 to 5, and no two shapes should share a value. You may wish to ease these restrictions for subsequent rounds. When Player One is done, he/she should press the button again to hide the secret information.
2. Player Two now drags one or more shapes onto both sides of the balance. The balance scale will tip to indicate which shape(s) are heavier.
3. Player Two continues to experiment, dragging shapes to and from the balance scale to test different weight comparisons. Player One keeps track of Player Two's moves.
4. When Player Two thinks he/she knows the values of the five shapes, he/she should predict the outcome of several new weight comparisons and then check them to see if these predictions hold. As a final check, she should click the button to reveal the secret values.

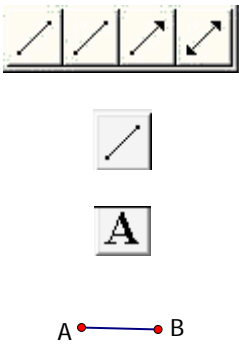

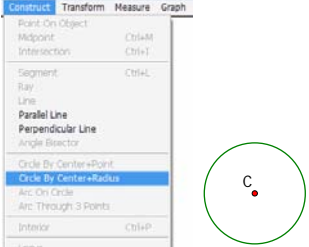
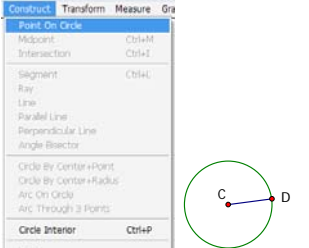
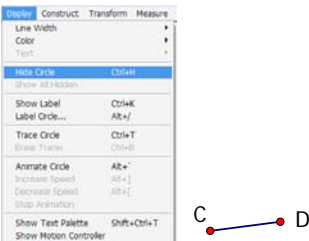
Players One and Two now switch roles and play the game again.

(Nathalie Sinclair, Department of Mathematics, Michigan State University)

Activity 2 Construction a Line Segment

Construction: Duplicating a Line Segment

This method for duplicating a line segment with Sketchpad is equivalent to the standard compass and straightedge construction. However, Sketchpad's compass (the Circle tool) is collapsible, like Euclid's compass. Duplicating a segment using only Sketchpad's freehand tools (i.e. Euclid's tools) is more complicated than you'd expect. We'll get around this by using the Construct menu. This construction is a building block for many more complex constructions. For this reason, you may want to record and save a script for duplicating a line segment.

<p>Sketch</p> 	<p>In the File Menu, Choose New Sketch and follows the instruction step-by-step.</p> <p>Step 1 Construct segment AB. This is your given segment.</p> <ul style="list-style-type: none"> Choose the Segment Tool and move onto your sketch plane Move the pointer on one of the two points and press the mouse left button and hold it down and drag the mouse until the pointer is on the other point; Release the mouse button. The two points are connected with a segment. Click on the Text Tool and move a point's label over each point and click.
	<p>Step 2 Construct point C. This is one endpoint of your new segment</p> <ul style="list-style-type: none"> Choose Point Tool from the Toolbox and move pointer to the Sketch plane. Click any where in the sketch plane Label point C
	<p>Step 3 Select segment AB and point C and choose Circle By Centre + Radius in the Construct menu.</p> <ul style="list-style-type: none"> Click on the Construct menu, there is a pop down menu and choose Circle By Centre + Radius.
	<p>Step 4 Construct segment CD, where point D is on the circle.</p> <ul style="list-style-type: none"> Click on the Construct menu, choose Point On Circle. Choose point C and D, click on the Construct menu and choose Segment.
	<p>Step 5 Hide the circle.</p> <ul style="list-style-type: none"> Choose the circle Click on the Display menu, choose Hide Circle.

Construction: Duplicating a Line Segment

Investigate	<ul style="list-style-type: none"> • Move points C and D. Do they behave as you would expect them too? • Move point A or B. What effect does changing the length of AB have on CD? • In the space below write a paragraph describing why this construction works. • Compare it to how you would duplicate a segment using a compass and straightedge. • Why is the command Circle By Centre + Radius necessary? (What can you do with an actual compass that you can't do with Sketchpad's compass?) <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>
Present your finding	Discuss your construction with your partner or group. To present your findings you could create a script that duplicates a line segment, commented to explain why it works. Save this as a utility script for duplicating a segment.
Explore More	Construct three unconnected segments. Now try to construct a triangle whose sides have the same lengths as the three segments. Are all the triangles you can construct with these given lengths congruent? Change the lengths of the segments. Can any three segments make a triangle?
Student Audience	Primary level
Prerequisites	None
Sketchpad Proficiency	Beginner
Construction Tips	<ul style="list-style-type: none"> • This is a very simple construction. • The purpose of this activity is to familiarize students with a useful Sketchpad construction technique and connect to experience they may have with compass and straightedge. • Students can also create a utility script for duplicating a segment. • Even though it only takes four steps to duplicate a segment using the Construct menu, a script could come in handy for this very useful construction.
Investigate/Conjecture	Segment CD is equal to segment AB because it's a radius of a circle with radius AB, and all radii of a circle are equal. When you duplicate

LESSON OVERVIEW

What You Will Learn

1. Introduction to Dynamic Geometry Software
2. Enhancing Primary Students' Mathematical Learning through Exploration and Investigation using the Geometer's Sketchpad

1. INTRODUCTION TO DYNAMIC GEOMETRY SOFTWARE

Dynamic geometry software (DGS, also called "dynamic geometry environments", DGEs, or "Interactive geometry software", IGS) are computer programs which allow one to create and then manipulate geometric constructions, primarily in plane geometry. One starts construction by putting **points, lines or circles** and using them to define new objects. After some construction is done, one can move the points, lines or circles one started with and see how the construction changes.

Some dynamic geometry software presently available are:

- | | | |
|------|-----------------------------|--------------|
| i) | <i>Cabri</i> | - Commercial |
| ii) | <i>Cinderella</i> | - Commercial |
| iii) | <i>GeoGebra</i> | - Free |
| iv) | <i>Geometer's Sketchpad</i> | - Commercial |
| v) | <i>Winplot</i> | - Free |

The Geometer's Sketchpad (GSP)

The Geometer's Sketchpad is an interactive geometry software program for exploring Euclidean geometry, algebra, calculus, and other areas of mathematics. It was created by Nicholas Jackiw. Geometer's Sketchpad includes the traditional Euclidean tools of classical Geometric constructions; that is, if a figure can be constructed with compass and straight-edge, it can also be constructed using this program.

2. Enhancing Primary Students' Mathematical Learning through Exploration and Investigation using the Geometer's Sketchpad

The Geometer Sketchpad, like the other mathematical software, is only a tool aiding in the understanding of mathematical concepts. The software by itself does not guarantee understanding. Teacher instruction and facilitation are indispensable not only to enable learning but also to enhance it.

When mathematical exploration and investigation is concerned, it is even more relevant that strategies and approaches are so designed with the human touch. Timely conversations by the teacher to guide, encourage and challenge are keys to effective learning incorporating investigation and exploration strategies. This is in harmony with social constructivism which promotes interactions through communication between teacher and students, and among students themselves while investigating and exploring mathematical concepts.

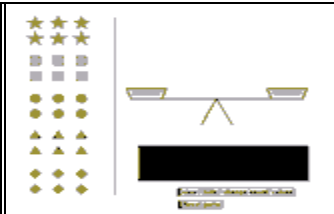
In recent times, the *process* in mathematical learning is taken to be as important as the *content* or *product*, yet it is often sacrificed in the classroom. This lesson is thus planned in such a way so as to place as equal importance in the process as the content or product as is possible.

Today's Investigation and Exploration Activities:

- ACTIVITY 1 BALANCE**
- ACTIVITY 2 CIRCLES AND SQUARES**
- ACTIVITY 3 GROUPING**
- ACTIVITY 4 GHOST PATHS**
- ACTIVITY 5 ROOBOOGOO**
- ACTIVITY 6 LULU**
- ACTIVITY 7 COLOR BY NUMBERS**

ACTIVITY 1 BALANCE

- to introduce number sense, relative values, enhance logical thinking and problem solving skills

	<p>Balance: This sketch contains a collection of five different shapes: a star, square, circle, triangle, and diamond. The weight of these shapes can be compared by using the provided balance scale; simply drag one or more shapes onto the balance and it will tilt in the direction of the heavier shape(s).</p> <p>Sketch and activity: Daniel Scher</p>
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ACTIVITY 2 CIRCLES AND SQUARES

- to introduce number & operations, pre-algebra, enhance logical thinking and problem solving skills

	<p>Circles and Squares: Using arithmetic reasoning and logic, players try to determine the secret values of the square and circle in as few turns as possible.</p> <p>Sketch and activity: Daniel Scher</p>
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ACTIVITY 3 GROUPING

- to introduce number and operations, pre-algebra, problem solving skills

	<p>Grouping I: The goal of this activity is twofold. First, it introduces students to Sketchpad's most important tools and menu commands. Second, it helps students appreciate how grouping objects can make them easier to count, and how the same quantity of objects can be grouped in different ways. In addition, it will help you developed a classroom vocabulary that can be used in Grouping II.</p> <p>Grouping II: The idea of this activity is for students to appreciate how grouping objects can make them easier to count, and how the same quantity of objects can be grouped in different ways. Younger students might be encouraged to think about how to group objects "evenly," that is, into group sizes that are factors of the total number of objects. The idea that n groups of m is the same as m groups of n (the <i>commutative property of multiplication</i>) can also be explored. Older students might be encouraged to explicitly consider the remainder objects--that is, the number of objects left over after they have been divided into certain group sizes.</p> <p>Sketch: Nick Jackiw (based on an idea by Spario Soon); Activity: Nathalie Sinclair</p>
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ACTIVITY 4 GHOST PATHS

- to introduce geometry and logical thinking

	<p>Ghost Paths: Students move two ghost characters (Doug and Linda) around the screen to explore geometric concepts. Students discover connections between the ghosts' constructed paths by comparing them and looking for similarities and differences. Vocabulary introduced: line segment, line, ray, circle, radius, hexagon, and octagon.</p> <p>Sketch and activity: Judy Dussiaume</p>
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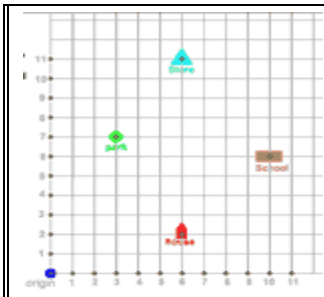
ACTIVITY 5 ROOBOOGOO

- to introduce geometry, enhance logical thinking and visualization

	<p>RooBooGoo: The idea of this activity is to introduce students to some of the basic transformational behaviors. The emphasis is less on properties than on relationships between objects that have been reflected, rotated, or translated.</p> <p>Sketch and activity: Nathalie Sinclair</p>
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ACTIVITY 6 LULU

- to enhance geometry

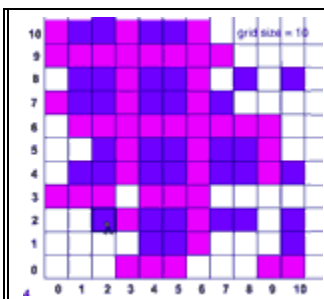


Lulu: Students interact with a first-quadrant coordinate system, learning how to specify points through their co-ordinates and to describe horizontal and vertical motion on the grid.

Sketch and Activity: Nathalie Sinclair

ACTIVITY 7 COLOR BY NUMBERS

- to enhance numbers & operations



Color by Numbers: Students investigate patterns produced on a grid of numbers generated through addition and multiplication of the whole numbers. The focus is on elementary number theory concepts such as even/odd, factors, multiples, divisibility, and the commutativity of addition and multiplication.

Sketch and Activity: Nathalie Sinclair

REFERENCES

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<http://www.cabri.com/v2/>

Cinderella

<http://cinderella.de/tiki-index.php>

GeoGebra

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Geometer's Sketchpad

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Interactive Geometry Software

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Winplot

<http://www.math.hawaii.edu/lab/241/winplot.html>

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