PROBLEMS PROBLEMS AND PROBLEMS

Problem Set 2, *The* 15^{th} *December 2004*[1]

Problem 1 (Inequality) *We assume that* a, b, c *are nonegative real numbers such that* $\max\{ab, bc, ca\} \leq 1$. *Find the greatest value and the least value of the following expression*

$$2(a+b+c) - \left(a^2+b^2+c^2\right)$$

Problem 2 (Functional Equation) *Find all function* $f : \mathbb{Z}_+ \longrightarrow \mathbb{Z}_+$ *such that for any* $m, n \in \mathbb{Z}_+$ *there exists an* $k \in \mathbb{Z}_+$ *such that*

$$3f^4(k) = f^4(m) + f^2(m)f^2(n) + f^4(n)$$

where $f^4(x) = \left(f(x)\right)^4$ *for* $x \in \mathbb{Z}_+$.

Problem 3 (Number Theory) *Find all positive intereger* x, y, z, t *such that*

$$(x+y+z)(xy+yz+zx)(xyz+1) = 3^t$$

Problem 4 (Number Theory) *Find all integer numbers* a *such that for all* $n > 1$ *positive integer the equation*

$$x^3 + a \equiv 0 \pmod{3^n}$$

has exactly three roots in modulo 3^n.

Problem 5 (Geometry) *Let be given an equivalent triangle* $\triangle ABC$. *A moving point* M *on the line* BC. *Draw two lines* MP, MQ *are respectively perpendicular to* AB *and* AC *(*P, Q *lie on* AB, AC *respectively). Find the locus of the centriod of the triangle* $\triangle MPQ$.

Problem 6 (Geometry) *Prove that for all triangle* $\triangle ABC$ *and for all point* M *in the plane we have the following inequality*

$$\frac{MA}{a} + \frac{MB}{b} + \frac{MC}{c} \geq \sqrt{3}$$

where a, b, c *are respectively the lengths of the segments* BC, CA, AB.

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[1] All problems of this month is proposed by me