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NEW PROBLEMS IN THIS ISSUE

Problems for Lower Secondery School

Problem T1/327.[For 6^{th} grade] Can my friend write 7 distince 7-digit numbers so that :

i) for writting each number , he uses 7 distinct digits $1, 2, 3, 4, 5, 6, 7$;

ii) the sum of the 7^{th} powers of some (distinct) numbers among them is equal to sum of the 7^{th} powers of the others .

Problem T2/327.[For 7^{th} grade] Prove that

$$\frac{1}{65} < \frac{1}{5^3} + \frac{1}{6^3} + \cdots + \frac{1}{2004^3} < \frac{1}{40}$$

Problem T3/327. Find all integers x such that $x^3 - 2x^2 + 7x - 7$ is divisible by $x^2 + 3$.

Problem T4/327. Solve the equation

$$4x^2 - 4x - 10 = \sqrt{8x^2 - 6x - 10}$$

Problem T5/327. Prove that

$$(1 + \frac{1}{a^3})(1 + \frac{1}{b^3})(1 + \frac{1}{c^3}) \geq 1 + \frac{729}{512}$$

where a, b, c are positive real numbers satisfying $a + b + c = 6$.

Problem T6/327. The circle (O_1) with center O_1 , radius R_1 cuts the circle (O_2) with center O_2 ,radius R_2 at the points A and B . The tangent to (O_1) at A cuts (O_2) at C and the tangent to (O_2) at A cuts (O_1) at D . Let M be the point of intersection of AB and CD , let N be the midpoint of CD. Prove that $\widehat{CAM} = \widehat{DAN}$ and $\frac{MC}{MD} = \frac{R_2^2}{R_1^2}$.

Problem T7/327. The quadrilateral $ABCD$ is incribed in a circle with radius R and circumscribes about in a circle with radius R and circumscribes about a circle with radius r . Prove that $R \geq r\sqrt{2}$.

Problems for Lower Secondery School

Problem T8/327. The sequence $\{x_n\}_{n=1}^{+\infty}$ and $\{y_n\}_{n=1}^{+\infty}$ are defined by : $x_1 = -1, y_1 = 1, x_{n+1} = -3x_n^2 - 2x_n y_n + 8y_n^2, y_{n+1} = 2x_n^2 + 3x_n y_n - 2y_n^2$ for all $n = 1, 2, \ldots,$. Find all prime number p such that $x_p + y_p$ is not divisible by p .

Problem T9/327. The positive real numbers a, b, c, d satisfying the conditions $a \leq b \leq c \leq d$ and $bc \leq ad$. Prove that $a^b \cdot b^c \cdot c^d \cdot d^a \geq a^d \cdot b^a \cdot c^b \cdot d^c$.

Problem T10/327. For each positive integer n , consider the function $f_n(x) = e^x \cdot \sum_{m=0}^{n} \frac{x^m}{m!}$, defined on the set of positive real numbers .

i) Prove that for every positive real numbers k with $0 < k < 1$ and for every positive integer n , the equation $f_n(x) = k$ has a unique root

ii) Let α_n be the above mentioned root . Find $\lim\limits_{n \to +\infty} \frac{1}{\alpha_n}$

Problem T11/327. Let be given a triangle ABC with $BC = a, CA = b, AB = c$ and with circumradius R .Let l_a, l_b, l_c be respectively the measure of the angled bisector of the angle A, B, C and let r_a, r_b, r_c be respectively the radius of es-cribed circle in the angle A, B, C . Prove that

$$\frac{l_a^2 \cdot l_b^2 \cdot l_c^2}{a^2 \cdot b^2 \cdot c^2} \leq (\frac{r_a + r_b + r_c}{6R})^3$$

Problem T12/327. Let $A_1A_2A_3A_4$ be a tetrahedron , circumscribing about a sphere with center O . Let B_i be the touching point of the sphere with the face opposite to the vertex A_i $(i = 1, 2, 3, 4)$. Prove that omong the angles formed by a pair of ditinct rays OB_1, OB_2, OB_3, OB_4 there exists an angle α with : $\sin\alpha \leq \frac{2\sqrt{2}}{3}$.

END.

Hà Duy Hưng
Hanoi University of Education , Vietnam
Email : pluricomplex_amath@yahoo.com
Url : www.geocities.com\pluricomplex_amath .