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Problems for Lower Secondery School
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Problem T1/326. [For $6^{th}$ grade] Factorize $2003^{2004}$ in the product of two natural numbers $a$ and $b$. Is the sum divisible by $2004$?

Problem T2/326. [For $7^{th}$ grade] The positive real numbers $a,b,c,d$ satisfying the conditions

$$\frac{a^4}{b}+\frac{b^4}{d}=\frac{1}{b+d}$$

$$a^2+c^2=1$$

Prove that

$$\frac{a^{2004}}{b^{1002}}+\frac{b^{2004}}{d^{1002}}=\frac{2}{(b+d)^{1002}}$$

Problem T3/326. Find the least prime number $p$ such that $p$ can be written in ten sums of the forms

$$p=x_1^2+y_1^2=x_2^2+2y_2^2=x_3^2+3y_3^2=\cdots=x_{10}^2+10y_{10}^2$$

where $x_i,y_i$ $(i=1,2,...,10)$ are positive integers.

Problem T4/326. Solve the equation

$$\sqrt[3]{3x+1}+\sqrt[3]{5-x}+\sqrt[3]{2x-9}-\sqrt[3]{4x-3}=0$$

Problem T5/326. Prove that

$$\frac{1}{a(b+1)}+\frac{1}{b(c+1)}+\frac{1}{c(a+1)}\geq\frac{3}{abc+1}$$

Problem T6/326. Prove that for every polygon, there exist at least two sides such that the measures $a,b$ of which satisfy the conditions $a\leq b\leq 2a$.

Problem T7/326. From a point $P$ at outside of a circle with center $O$ , draw two tangents $PA, PB$ to the circle . Let $M, N$ be respectively the midpoints of $AP$ and $OP$ . The line $BM$ cuts again the circle at $K$ . Prove that $KN \perp AK$

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Problem T8/326. Find integer-solution of the equation of two unknown

$$x^{y^x} = y^{x^y}$$

Problem T9/326. Prove that

$$xy + max\{x, y\} \leq \frac{3\sqrt{3}}{4}$$

for arbitrary real nonegative numbers $x, y$ satisfying the condition $x^2 + y^2 = 1$ .

Problem T10/326. Find all functions $f : R^+ \to R^+$ satisfying the condition $xf(xf(y)) = f(f(y))$ for all $x, y$ in $R^+$ .

Problem T11/326. Let $R, r$ be respectively the circum radius and inradius of a triange ABC and I be its incenter . Prove that

$$\sum_{cyclic} \frac{1}{IA \cdot IB} \leq \frac{5R + 2r}{8Rr^2}$$

Problem T12/326. Let $ABCD$ be a regular tetrahedron with side $a$ . Let $H$ and $K$ be the midpoints of $AB$ and $CD$ respectively . An arbitrary plane containing the line $HK$ cuts the sides $BC$ and $AD$ at $E$ and $F$ respectively . Prove that $EF \perp HK$ . Find the least value of the area of the quadrilateral $HEKF$

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