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New Problems

For lower secondery schools

Problem 1/325 . (for 6th grade) Prove that the sum

$$A = \frac{2004}{2003^2+1} + \frac{2004}{2003^2+2} + \cdots + \frac{2004}{2003^2+2003} \quad \text{(2003 terms)}$$

is not an integer number .

Problem 2/325 . (for 7th grade) Let ABCD be a rectangle with AB = 2AD and M be the midpoint of the segment AB . Let H be the point on side AB such that $\angle ADH = 15^0$. The lines CH and DM intesect at K . Compare the lengths of the segments DH and DK .

Problem 3/325 . Solve the system of equations

$$\begin{cases} x^3(2+3y) = 1 \\ x(y^3-2) = 3 \end{cases}$$

Problem 4/325 . Find the least value of the expression

$$A = \frac{1}{x^3+y^3} + \frac{1}{xy}$$

where x, y are positive real numbers satisfying $x + y = 1$.

Problem 5/325 . Let be given a convex quadrilateral ABCD . O is the midpoint of side BC ,E is symetry to D with repect to O . A point M moves on the side AD . The line passing through I , parallel to BC , cuts AB anf AC respectively at K and H . Prove that the expression

$$\frac{AB}{AK} + \frac{AC}{AH} - \frac{AD}{AM}$$

takes constant value .

For upper secondery Schools

Problem 6/325 . Let be given $a > 1$. Find all triples (x, y, z) such that $|y| \geq 1$ and

$$\log_a^2(xy) + \log_a\left(x^3y^3 + xyz\right)^2 + \frac{8+\sqrt{4z-y^2}}{2} = 0$$

Problem 7/325 . Find the greatest value of the expression $ac + bd + cd$ where a, b, c, d are real numbers satisfying the conditions $a^2 + b^2 = 4$ and $c + d = 4$.

Problem 8/325 . The circles C_1, C_2, C_3 internally touch the circle C respectively at A_1, A_2, A_3 and they externally touch each other . Let B_1, B_2, B_3 be respectively the touching point of C_2 and C_3 , of C_3 and C_1 , of C_1 and C_2 . Prove that the lines A_1B_1, A_2B_2, and A_3B_3 are concurrent .

CONTEST ON THE 40th ANNIVERSARY OF THE JOURNAL

For lower secondery school

Problem 9/325 . Find all positive integers x, y such that $A = x^2y^4 - y^3 + 1$ is a perfect square .

Problem 10/325 . The sequence of numbers $\{x_n\}_{n=1}^{+\infty}$ is defined by the fomulas

$$x_n = \begin{cases} 1 \ \ when \ \left[(n+1)\sqrt{2004}\right] - \left[n\sqrt{2004}\right] \ is \ odd \\ 0 \ \ when \ \left[(n+1)\sqrt{2004}\right] - \left[n\sqrt{2004}\right] \ is \ even \end{cases}$$

Find the sum

$$S = x_{1964} + x_{1965} + \cdots + x_{2004}$$

For upper secondery school

Problem 11/325 . Let be given a natural number n and a prime number p . Determine the number of sets of p distinct natural numbers $\{a_0, a_1, ..., a_{p-1}\}$. satisfying the conditions

a) $1 \le a_i \le n$ for all $i = 0, 1, ..., p-1$.

b) $lcm[a_0, a_1, ..., a_{p-1}] = p \min\{a_0, a_1, ..., a_{p-1}\}$

Problem 12/325 . Consider a convex hexagon inscribed in a circle such that the opposite sides are parallel . Prove that the sums of the lengths of opposite sides are the same if and only if the distances of the opposite sides are the same .