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Problems for Lower Secondery School

Problem T1/324. How many digits does contain the decimal reprentation of the number 2^{100}? What is the first digit on the left in this reprentation?

Problem T2/324. Let ABC be a triangle with $\widehat{ABC} = 70^0, \widehat{ACB} = 50^0$. On the side AC take M so that $\widehat{ABM} = 20^0$, on the side AB, take N so that $\widehat{ACN} = 10^0$. Let P be the point of intersection of BM and CN Prove that $MN = 2PM$.

Problem T3/324. Solve the system of equations

$$\begin{cases} x^3 + y = 2 \\ y^3 + x = 2 \end{cases} \tag{1}$$

Problem T4/324. Prove the inequality

$$\sqrt{a^4 + b^4 + c^4} + \sqrt{a^2b^2 + b^2c^2 + c^2a^2} \geq \sqrt{a^3b + b^3c + c^3a} + \sqrt{ab^3 + bc^3 + ca^3}$$

where a, b, c are non-negative real numbers.

Problem T5/324. Let ABC be a triangle right at A. For every point K on the side AC, construct the circle (K) with center at K, touching the line BC at E. Draw the line BD touching the circle (K) at D (distince from E). Let M, N, P, Q be the midpoints of $AB, AD, BD,$ and, MP respectively. Let S be the point of intersection of QN and BD. Find the line on which the point S moves when K moves on the side AC.

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Problem T6/324. Let $f(x)$ be a polynomial of degree 2003 with $f(k)=\frac{k^2}{k+1}$ for $k=1,2,3,\ldots,2004$. Calculate $f(2005)$.

Problem T7/324. Prove that $4x^2+4y^2\leq xy+yz+zx+5z^2$ where x,y,z are positive real numbers satifying conditions $x\leq y\leq z$. When does the equality occur?

Problem T8/324. Let r_a,r_b,r_c be the radii of the escribed circles in angles A,B,C of a triangle ABC . Prove that

$$\sum_{sym} r_a sin\frac{A}{2}\leq\frac{\sum_{sym} r_a^3}{6}(\sum_{sym}\frac{1}{r_a^2})$$

Contest on the 40^{th} Anniversary of the Journal

For lower Secondery Schools

Problem T7/LSS. Let be given positive real numbers a,b,c satifying the condition

$$6(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2})\leq 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

Prove that

$$\frac{1}{10a+b+c}+\frac{1}{a+10b+c}+\frac{1}{a+b+10c}\leq\frac{1}{12}$$

Problem T8/LSS. Let ABC be a triangle , right at A and $\widehat{ABC}=60^0$. A line passing through B cuts the line AC at D and cuts the circle with center A and radius AC at E and F . Prove that

$$|\frac{1}{AC}-\frac{1}{BF}|=\frac{1}{BD}$$

For upper Secondery Schools

Problem T7/USS. Find the greatest real number c satisfying the condition : for arbitrary given positive integers m,n there exits a real number x such that $sin(mx)+cos(mx)\geq c$

Problem T8/USS. Let be given a cube $ABCD.A'B'C'D'$. A plane touches the sphere inscribed in the cube at Q and cuts the sides $AB,AD,A'B',A'D'$ of the cube at M,N,M',N' respectively . Prove that $\widehat{MQN}+\widehat{M'QN'}=90^0$