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Problems for Lower Secondery School

Problem T1/322.[For 6^{th} grade] Find all integers x satisfying

$$|x-3|+|x-10|+|x+101|+|x+990|+|x+1000|=2004$$

Problem T2/322.[For 7^{th} grade] Let ABC be a triangle with its median AM and let O_1, O_2 be the incenters of triangles ABM, ACM respectively . Prove that $MO_1 = MO_2$ when and only when $AB = AC$.

Problem T3/322. Consider the six pairs of marbles and consider the sum of masses of two marbles of each pair . Let a, b, c, d, e, f be these sums . Determine the mass of each marble , known that

$$a+b+c+d+e+f=a^3+b^3+c^3+d^3+e^3+f^3=6$$

Problem T4/322. Find the least value of the expressions

$$\frac{a^6}{b^3+c^3}+\frac{b^6}{c^3+a^3}+\frac{c^6}{a^3+b^3}$$

where a, b, c are real numbers satifying the condition $a+b+c=1$

Problem T5/322. The circumcircle of triangle ΔABC has center O and diameter AD . Let I be the incenter of triangle ΔABC . The lines AI, DI cut again the circumcircle at H, K respectively . Draw the line IJ perpendicular to BC at J . Prove that H, K, J are colinear .

Problems for Lower Secondery School

Problem T6/322. Prove the inequality

$$\frac{1}{2}(\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \frac{1}{x_i}) \geq n - 1 + \frac{n}{\sum_{i=1}^{n} x_i}$$

where $x_i (i = 1, 2, \dots, n)$ are positive real numbers satifying $\sum_{i=1}^{n} x_i^2 = n$ and n is an integer greater than 1 .

Problem T7/322. The sequence of numbers $\{u_n\}_{n=1}^{\infty}$ is defined by

$$u_n = \sum_{i=1}^{n} \frac{1}{(k!)^2}$$

Prove that this sequence has a limit and this limit is an irrational .

Problem T8/322. Let $SABC$ be a tetrahedron . The points M, N, P lie respectively on the sides SA, SB, SC so that $AM = BN = CP$ (M, N, P are distinct from the vectices S, A, B, C) . Let G be the centroid of triangle MNP . Prove that G lies on a fixed line when M, N, P move on SA, SB, SC respectively .

Contest on the 40^{th} Anniversary of the Journal

For lower Secondery Schools

Problem T3/LSS. Let x, y be two integer numbers distinct from -1 such that $\frac{x^3+1}{y+1} + \frac{y^3+1}{x+1}$ is an integer . Prove that $x^{2004} - 1$ is divisible by $y + 1$

Problem T4/LSS. Let H be the orthocenter of a nonright triangle ΔABC . Let D, E be the midpoints of BC and AH respectively . Let F be the orthogonal projection of H on the angle bisector $\widehat{BAC}$. Prove that D, E, F are collinear .

For upper Secondery Schools

Problem T3/USS. Find all positive integers a, m, n satifying the condition $(a-1)^m = a^n - 1$

Problem T4/USS. Let A, B, C, D be four points lying on a circle . Prove that the three radical axes of three pairs of cirlces respectively with diameters AB and CD , BC and DA , AC and BD are concurrent .