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Problems for Lower Secondery School

Problem T1/321. [For 6^{th} grade] Write the number 2003^{2004} as the sum of two positive integers. What is the remainder of the division by 3 of the sum of the cubes of these integers.

Problem T2/321. [For 7^{th} grade] Simplify the expression

$$\frac{(a-2)(a-1002)}{a(a-b)(a-c)}+\frac{(b-2)(b-1002)}{b(b-c)(b-a)}+\frac{(c-2)(c-1002)}{c(c-a)(c-b)}$$

Problem T3/321. Prove that

1) $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq\frac{a+b+c}{\sqrt[3]{abc}}$

2) $\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{d^2}+\frac{d^2}{a^2}\geq\frac{a+b+c+d}{\sqrt[4]{abcd}}$

where a, b, c, d are positive numbers.

Problem T4/321. Find a necessary and sufficient condition on the number m so that the following system of equations has a unique solution

$$\begin{cases} x^2=(2+m)y^3-3y^2+my \\ y^2=(2+m)z^3-3z^2+mz \\ z^2=(2+m)x^3-3x^2+mx \end{cases} \qquad (1)$$

Problem T5/321. Let $ABCD$ be a trapezoid insribed in a circle with radius $R=3cm$ such that $BC=2cm$, $AD=4cm$. Let M be the point on the side AB such that $MB=3MA$. Let N be the midpoint of CD. The line MN cuts AC at P. Calculate the area of the quadrilateral $APND$

Problems for Lower Secondery School

Problem T6/321. Let be given three positive integers m, n, p such that $n+1$ is divisible by m . Find a fomula to calculate the number of $p-$uples of positive integers $(x_1, x_2, \dots, x_p)$ satisfying the conditions : the sum $x_1 + x_2 + \cdots + x_p$ is divisible by m and $x_1, x_2, \dots, x_p$ are not greater than n .

Problem T7/321. a, b are arbitrary positive numbers such that the equation $x^3 - ax^2 + bx - a = 0$ has three roots greater than 1 . Determine a, b so that the expression $\frac{b^n - 3^n}{a^n}$ attains its least value and find this value .

Problem T8/321. The incircle of triangle ABC touches the sides BC, CA, AB respectively at D, E, F . Prove that

$$\frac{DE}{\sqrt{BC \cdot CA}} + \frac{EF}{\sqrt{CA \cdot AB}} \frac{FD}{\sqrt{AB \cdot BC}} \leq \frac{3}{2}$$

Contest on the 40^{th} Anniversary of the Journal

For lower Secondery Schools

Problem T1/LSS. Write 2004 natural numbers from 1 to 2004 in an arbitrary order to get a sequence $a_1, a_2, \dots, a_{2004}$ and calculate the sum

$$P = \sqrt{a_1 + a_2} + \sqrt{a_3 + a_4} + \cdots + \sqrt{a_{2003} + a_{2004}}$$

Find the greatest value of P for all these sequences.

Problem T2/LSS. Let $ABCD$ be a convex quadrilateral such that $AC = BD$ and AC is perpendicular to BD . Construct at the ouside of the quadrilateral triangles triangles ABX, BCY, CDZ, DAT . Prove that $XZ = YT$ and XZ is perpendicular to YT

For upper Secondery Schools

Problem T1/USS. The sequence of numbers $\{a_n\}_{n=1}^{\infty}$ is defined by

$$a_1 = \frac{1}{2^{1965}}, a_n = -\frac{1}{2^{1964+n}}$$

for $n = 2, 3, \dots, 40$. Prove that $\sum_{i,j=1}^{40} a_i a_j |b_i - b_j| \leq 0$ for arbitrary given real numbers $b_1, b_2, \dots, b_{40}$.

Problem T2/USS. Find all function $f : R^+ \to R^+$ satisfying the condition

$$f(\frac{f(x)}{y}) = yf(y)f(f(x)) \text{ for all positive numbers } x, y$$