Vietnam journal of Mathematics and Youth
(A math magazine for Secondery students)

Volume320-41st-February , 2004

God made our number
We makes their properties

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New problems in this issues

For lower secondery schools

T1/320: Write the sum of 18 fractions $1,\frac{1}{2},\frac{1}{3},...,\frac{1}{17},\frac{1}{18}$ in the form of an irreducible fraction $\frac{a}{b}$ .Prove that b is divisible by 2431

T2/320: Given a triangle ABC with AB > AC the foot of its atirtude AH lies inside BC The angled-bisectors of $\hat{ABC}$ and of $\hat{ACB}$ cut AH respectively at E and F .Prove that

$$BE > EF + FC$$

T3/320: Find positive integers $a \geq b \geq c$ and $x \geq y \geq z$ so that

$$\begin{cases} a+b+c = xyz \\ x+y+z = abc \end{cases}$$

T4/320: Solve the equation $(x-2)\sqrt{x-1} - \sqrt{2}x + 2 = 0$

T5/320: Find the greatest value of the expression $\sqrt{4x - x^3} + \sqrt{x + x^3}$ where $0 \leq x \leq 2$

T6/320: The circle (O) with center O cuts the circle (O') with center O' at P and Q.Their common tagent(nearer to P) touches (O) at A,(O') at B.Let C be the point of intersection of the tagents to (O) at P with the cirlce (O') .Let D be the point of intesection of the tagents to (O') at P with the circle (O).Let M be the point such that AB and PM has common midpoint.the line AP cuts BC at E and the line BP cuts AD at F.Prove that AMBEQF is a hexagone inscribed in a circle.

T7/320: Construct a triangle ABsC with given P,Q,R so that B is the midpoint of AP,C is the midpoint of BQ ,A is the midpoint of CR.

***For upper secondery schools*:**

T8/320:Prove the following equalities for positive integer n

a) $$\sum_{k=1}^{n}\frac{(-1)^{k-1}}{2k-1}C_n^k.C_{n+k-1}^{k-1}=1$$

b) $$\sum_{k=1}^{n}\frac{(-1)^{k-1}k^n}{2k-1}C_n^k=\frac{(n!)^2.2^n}{(2n)!}$$

T9/320:Solve the follwing system of equations of n unknowns

$$\begin{cases}\sqrt{x_1+8}+\sqrt{x_2+8}+...+\sqrt{x_3+8}=3n\\ \sqrt{x_1}+\sqrt{x_2}+...+\sqrt{x_n}=n\end{cases}$$

(n is a given positive integer)

Generalize the problem

T10/320: Find the greatest value of the fuction

$$f(x)=\sqrt{2}\sin x+\sqrt{15-10\sqrt{2}\cos x}$$

T11/320: Let r and R be respectively the inradius and the circumradius of triangle ABC Let p and p' be respectively the perimeter of ΔABC and $\Delta A'B'C'$ where A',B',C' are touching points of BC,CA,AB with the incricle .Prove that

$$\frac{r}{R}\le\frac{p'}{p}\le\frac{1}{2}$$

T12/320: Let be given a cube $ABCD.A_1B_1C_1D_1$ with the side AB=a From a point E on the side CD (E distince from C,D) draw a line cutting the lines and B_1C_1 respectively at M and N .From M draw a line cutting the lines BC and C_1D_1 respectively at F and P.Determine the position of E so that the perimeter of triangle MNP attains its least value and calculate this value.