Module 4

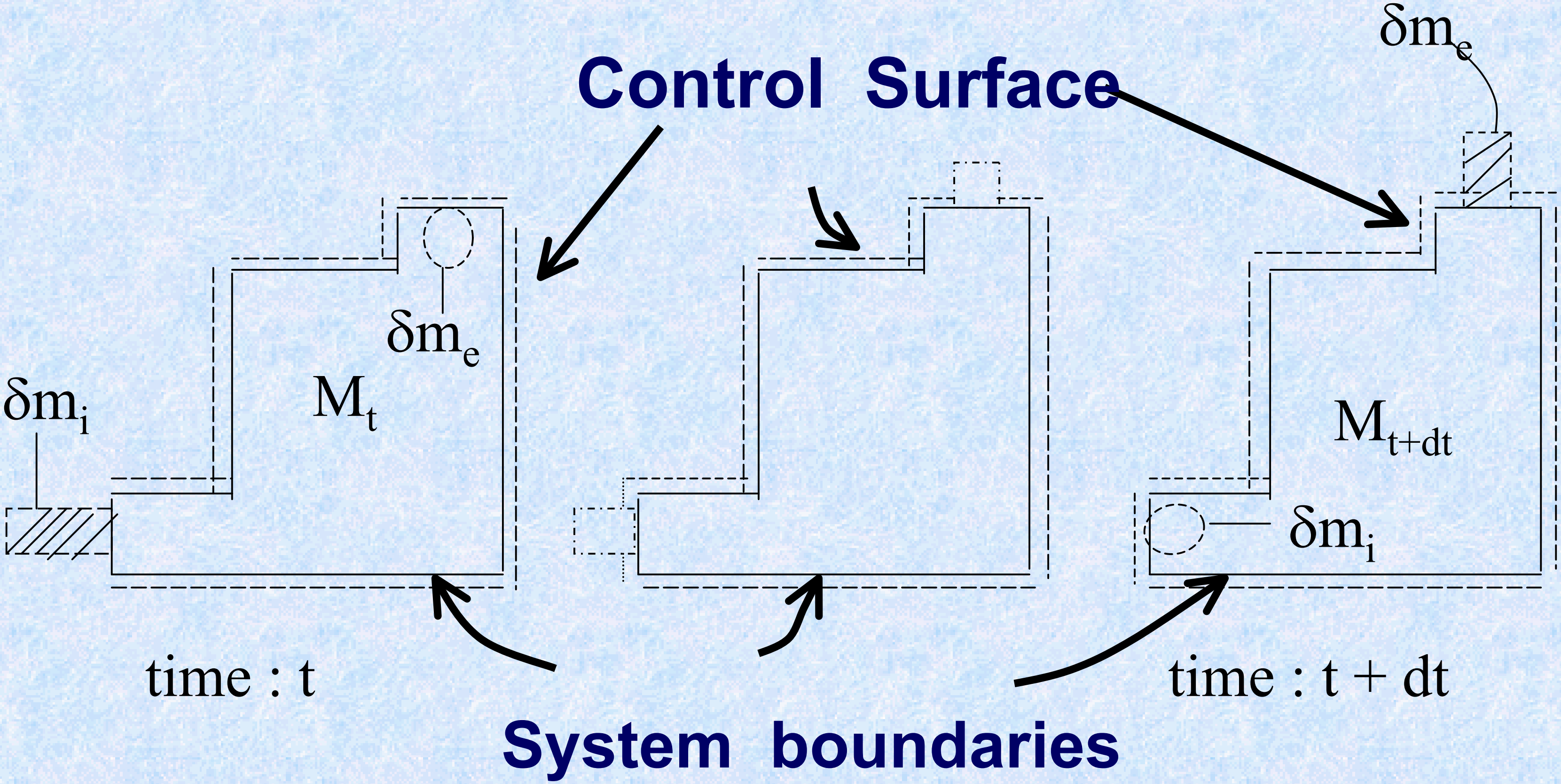
Open Systems

Lecture 4.1

Basic Equations

ANALYSIS OF OPEN SYSTEMS

FLOW PROCESS VISUALISED AS A SERIES OF NON-FLOW PROCESSES


Control Surface

System boundaries

ANALYSIS OF OPEN SYSTEMS

Conservation of MASS

$$M_t + \delta m_i = M_{t+\delta t} + \delta m_e$$

$$\delta m_i - \delta m_e = M_{t+dt} - M_t$$

{Net flow into C.V.} = {Increase of mass within CV}

ANALYSIS OF OPEN SYSTEMS

$$\frac{M_{t+\delta t} - M_{t}}{\delta t} + \frac{\delta m_{e}}{\delta t} - \frac{\delta m_{i}}{\delta t} = 0$$

or

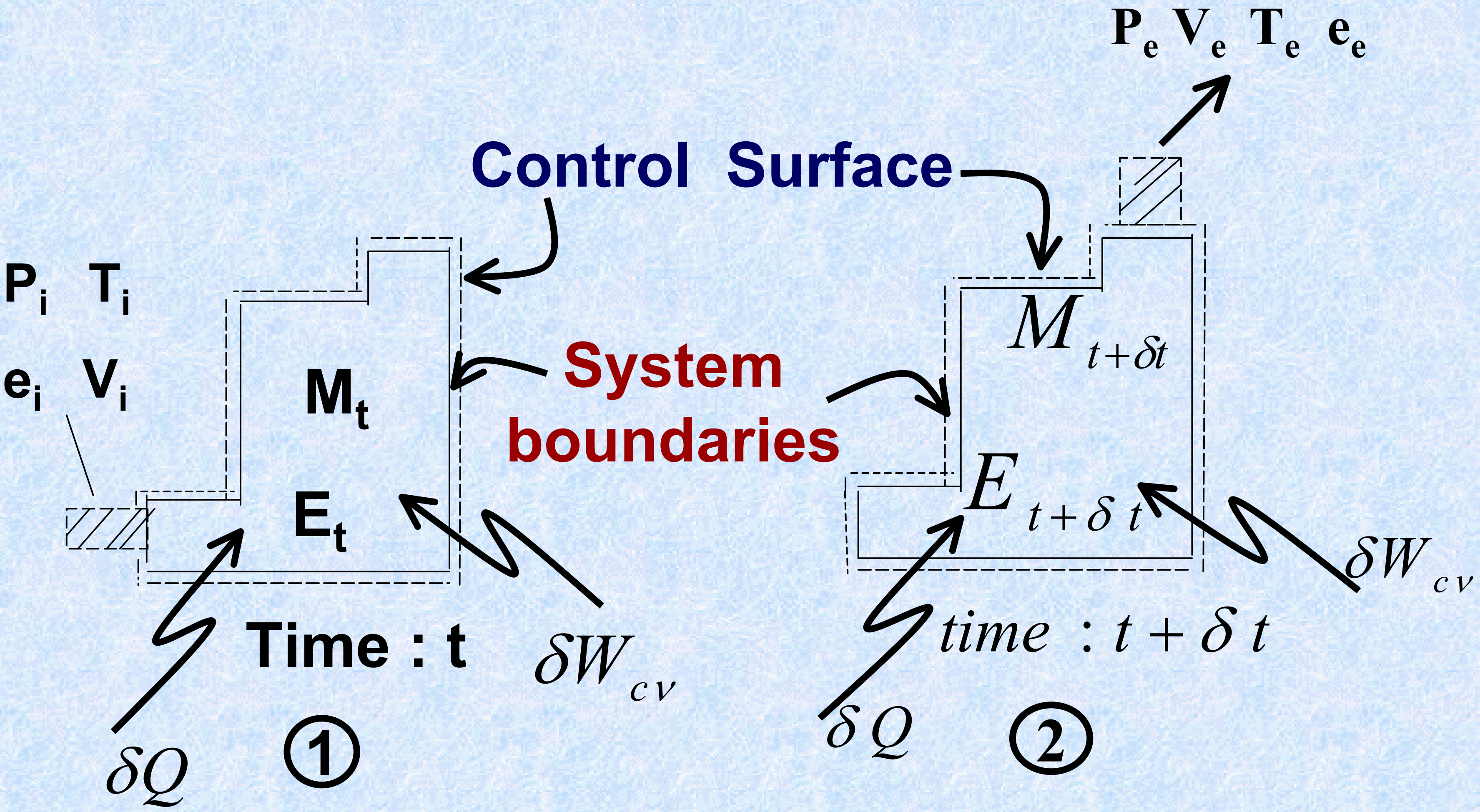
$$\frac{dM_{cv}}{dt} = \sum \dot{m}_{i} - \sum \dot{m}_{e}$$

Rate of increase of mass within CV

Inst. rate of flow of mass entering the CV

Inst. Flow rate of mass leaving the CV

FIRST – LAW FOR C.V.



P_e V_e T_e e_e

Control Surface

P_i T_i

e_i V_i

M_t

E_t

System boundaries

$M_{t+\delta t}$

$E_{t+\delta t}$

δW_{cv}

δW_{cv}

Time : t

$time \; : t + \delta t$

δQ

①

δQ

②

FIRST – LAW FOR C.V.

Total Work done on the System $= \delta W_{cv} +$

Work transfer associated with boundary deformation

FIRST – LAW FOR C.V.

$$W_{12} = \delta W_{cv} + \mathrm{P}_i \underbrace{\upsilon_i}_{sp.vol.} \delta m_i - \mathrm{P}_e \underbrace{\upsilon_e}_{sp.vol.} \delta m_e$$

Work transfer across deforming boundaries

$$\left\{ -P_i \times \left[-\left(\upsilon_i \delta m_i \right) \right] \right\} \qquad \left\{ -P_e \times \upsilon_e \delta m_e \right\}$$

FIRST – LAW FOR C.V.

For system undergoing the process 1-2

first law gives

$$ {}_1Q_2 = E_2 - E_1 - W_{12} $$

Where $E_1 = E_t + e_i\delta m_i \; ; \; E_2 = E_{t+\delta t} + e_e\delta m_e$

Sp. energy of incoming stream

Sp. energy of outing stream

FIRST – LAW FOR C.V.

or

$$\frac{Q_{12}}{\delta\, t} = \frac{E_{t+\delta t} - E_t}{\delta\, t} + \frac{e_e \delta m_e - e_i \delta m_i}{\delta\, t} -$$

$$\frac{\delta W_{cv}}{\delta\, t} + \frac{P_e v_e \delta m_e}{\delta\, t} - \frac{P_i \upsilon_i \delta m_i}{\delta\, t}$$

FIRST – LAW FOR C.V.

or

$$\frac{Q_{12}}{\delta\ t}+\frac{\delta m_i}{\delta\ t}\left(e_i+P_i\upsilon_i\right)=\frac{E_{t+\delta t}-E_t}{\delta\ t}+$$

$$\frac{\delta m_e}{\delta\ t}\left(P_e\upsilon_e+e_e\right)-\frac{\delta W_{CV}}{\delta\ t}$$

$$Writing \quad e=u+gz+\frac{V^2}{2}$$

FIRST – LAW FOR C.V.

$$e_i + P_i\upsilon_i = \underbrace{u_i + P_i\upsilon_i}_{} + gz_i + \frac{V_i^2}{2} = h_i + gz_i + \frac{V_i^2}{2}$$

$$e_e + P_e\upsilon_e = \underbrace{u_e + P_e\upsilon_e}_{} + gz_e + \frac{V_e^2}{2} = h_e + gz_e + \frac{V_e^2}{2}$$

$$and \quad taking \quad limit \quad as \quad \delta t \rightarrow 0$$

FIRST – LAW FOR C.V.

$$\dot{Q}_{C_V} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i\right) = \frac{dE_{C_V}}{dt}$$

$$+ \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e\right) - \dot{W}_{C_V}$$

SPECIAL CASES

→ **SS SF** $\quad \dfrac{dE_{C_V}}{dt} = 0$

$$\dot{m}_L,\ \dot{m}_e \rightarrow Constant\{not\ fn\ of\ time\}$$

FIRST – LAW FOR C.V.

∴ **For 1 stream entering & 1 stream leaving**

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

& S.F.E.E. becomes

$$`Q_{Cv} + \dot{W}_{Cv} = \dot{m}\{(h_e - h_i) + \frac{V_e^2 - V_i^2}{2} + g(z_e - z_i)\}$$

FIRST – LAW FOR C.V.

Defining energy transfers as per unit mass of flowing stream

$$q = \frac{\dot{Q}_{cv}}{\dot{m}} \qquad w = \frac{\dot{W}_{cv}}{\dot{m}}$$

SFEE becomes :

$$q + w = (h_o - h_i) + \frac{V_e^2 - V_i^2}{2} + g(Z_e - Z_i)$$

End of Lecture

Lecture 4.2

Basic Equations ….contd

FIRST – LAW FOR C.V.

Transient Analysis

Uniform State Uniform Flow assumption

→ The state of mass within c.v. may change with time, but is uniform throughout the cv at every instant

→ The state of streams entering / leaving the cv is constant with time although their $\dot{m}$ may be time varying.

FIRST – LAW FOR C.V.

Integrating the basic eq. over t, the process duration

$$\int \dot{Q}_{CV}\,dt + \int \dot{m}_i \underbrace{\left(h_i + \frac{V_i^2}{2} + gz_i \right)}_{\text{Const.}} dt = \int \frac{dE_{CV}}{dt}\,dt$$

$$+ \int \sum \dot{m}_e \underbrace{\left(h_e + \frac{V_e^2}{2} + gz_e \right)}_{\text{Const.}} dt - \int \dot{W}_{CV}\,dt$$

FIRST – LAW FOR C.V.

We get

$$Q_{CV}+\sum m_i\left(h_i+\frac{V_i^2}{2}+gz_i\right)=\left(M_2e_2-M_1e_1\right)+$$

$$\sum m_e\left(h_e+\frac{V_e^2}{2}+gz_e\right)$$

$$-W_{CV}$$

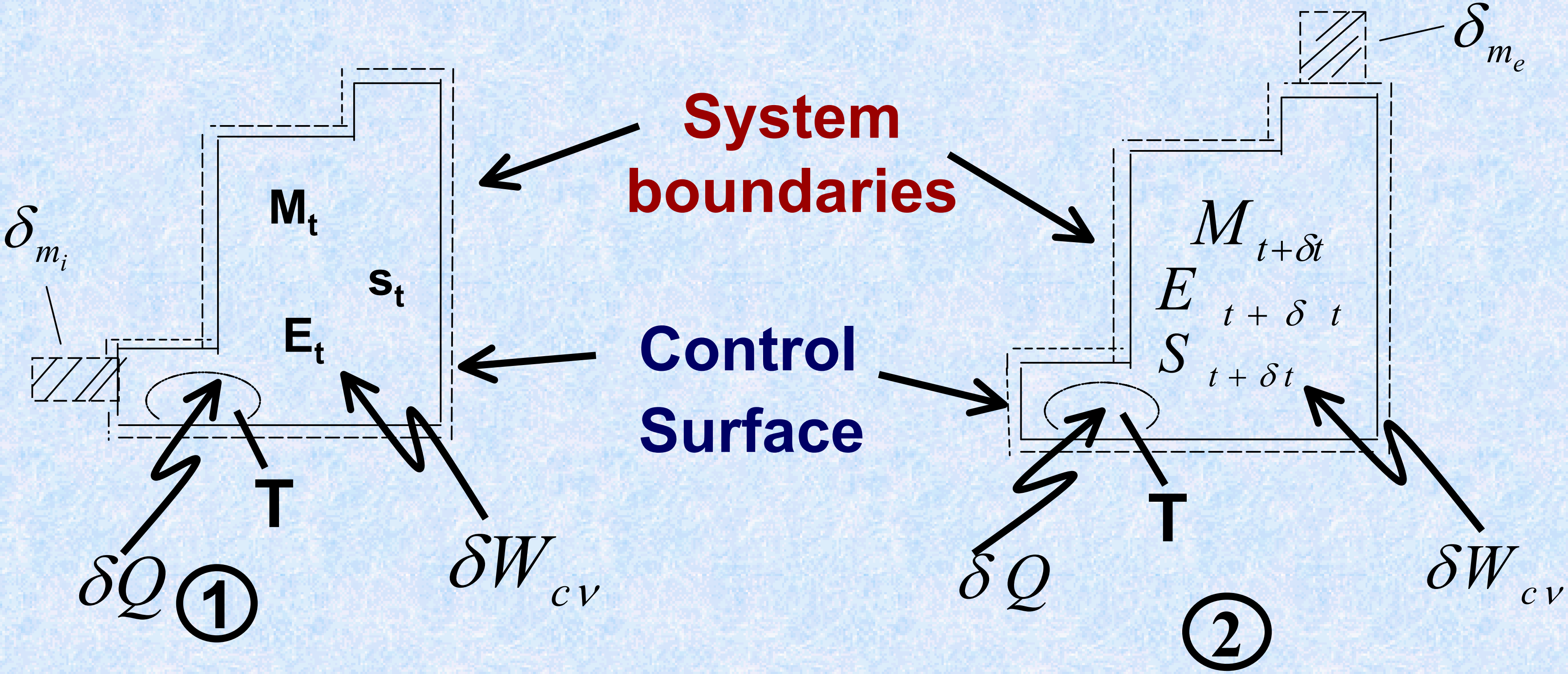
Solving few problems

End of Lecture

Lecture 4.3

Second Law Analysis of Open systems

SECOND LAW ANALYSIS OF C.V.



System boundaries

Control Surface

$$S_1 = S_t + \delta m_i \; s_i$$

$$S_2 = S_{t+\delta t} + \delta m_e s_e$$

SECOND LAW ANALYSIS OF C.V.

Second Law e.q. $S_2 - S_1 \geq \frac{\delta Q}{T}$

or $\frac{S_2 - S_1}{\delta t} \geq \frac{\delta Q / \delta t}{T}$

$$\Rightarrow \quad \frac{S_{t+\delta t} - S_t}{\delta t} + \frac{s_e \delta m_e - s_i \delta m_i}{\delta t} \geq \frac{\delta Q / \delta t}{T}$$

Taking limit as $\delta t \to 0$

SECOND LAW ANALYSIS OF C.V.

$$\frac{dS_{cv}}{dt} + s_e \dot{m}_e - s_i \dot{m}_i \geq \frac{\dot{Q}}{T}$$

In order to permit the possibility of additional streams entering / leaving the c.v. & h.t. across different portions of c.v. with different temps.

$$\frac{dS_{cv}}{dt} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \sum \dot{Q} / T$$

SECOND LAW ANALYSIS OF C.V.

$$\frac{dS_{cv}}{dt} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \sum \dot{Q}/T$$

Equality sign for reversible process

$\therefore$ for an irreversible process

$$\frac{dS_{cv}}{dt} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \dot{Q}/T + \dot{\sigma}$$

Where σ is the entropy generation
due to irreversibility

SECOND LAW ANALYSIS OF C.V.

Thus $$\dot{\sigma} = \frac{dS_{cv}}{dt} - \sum \frac{\dot{Q}_{cv}}{T} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i$$

Second law statement becomes :

$$\dot{\sigma} \geq 0$$

SPECIAL CASES

SSSF Processes $\left\{ \right.$ with one stream entering & one stream leaving

SECOND LAW ANALYSIS OF C.V.

$$\dot{m}_e = \dot{m}_i = \dot{m} \quad ; \quad \frac{dS_{cv}}{dt} = 0$$

$\therefore$ **2nd law eq. becomes** $\dot{m}\left(s_e - s_i\right) \geq \sum \frac{\dot{Q}_{cv}}{T}$

for an adiabatic process, $\dot{Q}_{cv} = 0$

$$\therefore s_e - s_i \geq 0$$

SECOND LAW ANALYSIS OF C.V.

Reversible adiabatic process :

$$s_e - s_i = 0$$

$\therefore$ **from basic eq. relating properties**

$$Tds = dh - \upsilon dp$$

it follows $ds = 0, \quad \therefore h_e - h_i = \int_i^e \upsilon dp$

SECOND LAW ANALYSIS OF C.V.

Using first law

$$q + h_i + \frac{v_i^2}{2} + gZ_i = h_e + \frac{v_e^2}{2} + gZ_e - w$$

Since $q = 0$

$$w = \int_i^e \upsilon dp + \frac{V_e^2 - V_i^2}{2} + g\left(z_e - z_i\right)$$

SECOND LAW ANALYSIS OF C.V.

Reversible isothermal process

Second law eq. $\dot{m}\left(s_e - s_i\right) = \dfrac{\dot{Q}_{cv}}{T}$

$$\therefore \quad T\left(s_e - s_i\right) = \frac{\dot{Q}_{cv}}{\dot{m}} = q$$

Again from basic eq. $Tds = dh - \upsilon dp$

SECOND LAW ANALYSIS OF C.V.

$$T\left(s_e - s_i\right) = h_e - h_i - \int_i^e \upsilon dp$$

= q (from above)

First law eq.

$$q + h_i + \frac{V_i^2}{2} + gZ_i = h_e + \frac{V_e^2}{2} + gZ_e - w$$

$$\boxed{w = \int \upsilon dp + \frac{V_e^2 - V_i^2}{2} + g\left(Z_e - Z_i\right)}$$

SECOND LAW ANALYSIS OF C.V.

Since above eq. is valid both for rev. isothermal process & rev. adiabatic process, it is valid for any reversible SSSF process since

Any rev. process

$\equiv$ series of alternate adiabatic + isothermal processes

Specific cases : Incompressible flow, processes involving negligible Δ KE Δ PE

SECOND LAW ANALYSIS OF C.V.

Specific cases : Incompressible flow, processes involving negligible Δ KE Δ PE

$$w = \int_i^e v\,dP$$

Specific cases : Incompressible flow, no work transfer

$$v(P_e - P_i) + g(z_e - z_i) + \frac{V_e^2 - V_i^2}{2} = 0$$

Bernoulli's equation

End of Lecture

Lecture 4.4

Second Law Analysis of Open systems

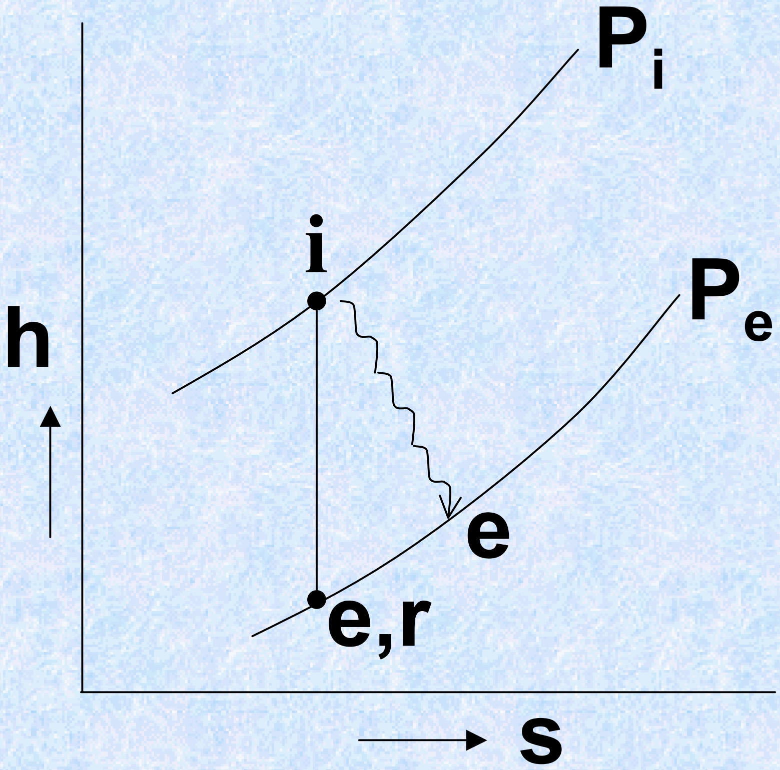
SECOND – LAW ANALYSIS OF SOME SIMPLE PROCESSES

(i) Steady flow through a turbine

$$Cont.: \dot{m}_i = \dot{m}_e = \dot{m}$$

Ist Law : $\Delta KE,\ \Delta PE \approx 0$

$$\dot{Q}_{CV} = 0$$

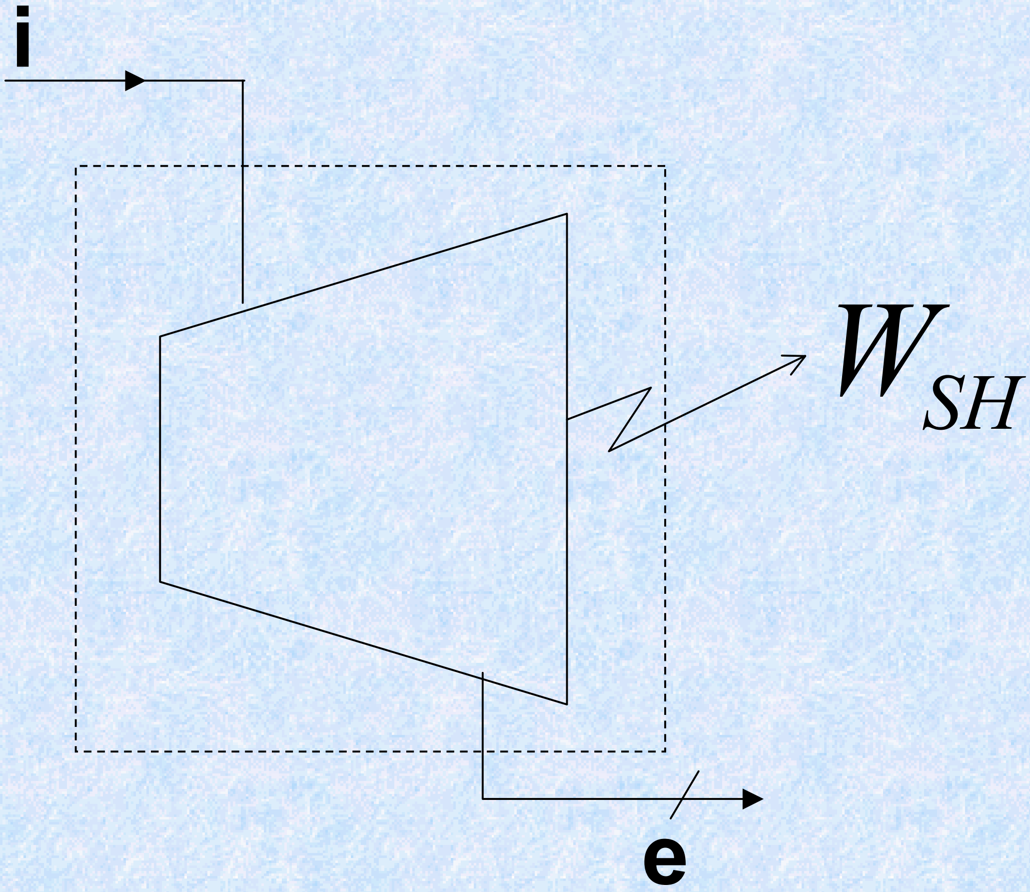


SECOND – LAW ANALYSIS OF SOME SIMPLE PROCESSES

$$-W = \dot{m}(h_i - h_e)$$

$$= W_{SH}$$

{shaft work output}



SECOND – LAW ANALYSIS OF SOME SIMPLE PROCESSES

IInd Law : $\dot{\sigma} = \dot{m}(s_e - s_i)$

For rev process $W_{SH,r} = \dot{m}(h_i - h_{e,r})$

$$\dot{\sigma} = 0 = s_{e,r} - s_i$$

$$\& \; W_{SH,r} - W_{SH} = \dot{m}(h_e - h_{e,r}) \left\{ \begin{array}{l} \text{isentropic} \quad \eta \\ \\ \eta = \dfrac{W_{Sh}}{W_{Sh,r}} = \dfrac{h_i - h_r}{h_i - h_{e,r}} \end{array} \right.$$

SECOND – LAW ANALYSIS OF SOME SIMPLE PROCESSES

Now, from basic eq. Tds = dh + vdP it follows that, at constant pressure

$$dh = Tds, \qquad \therefore h_e - h_{er} = \int_{e,r}^{e} TdS \approx T_{av.}\left(s_e - s_{e,r}\right)$$

where $$T_{e,av} = \frac{1}{s_e - s_{er}} \int_{e,r}^{e} Tds$$

SECOND – LAW ANALYSIS OF SOME SIMPLE PROCESSES

$$\therefore \quad W_{Sh,r} - \dot{W}_{Sh} = \dot{m}\ T_{e,av}\left(s_e - s_{er}\right)$$

$$= T_{e,av}\left\{\dot{m}\left(s_e - s_i\right)\right\} = T_{e,av}\ \dot{\sigma}$$

$$= W_{s,r}(1-\eta)$$

Lost Work = Av. Temp. * Entropy Generation

$$\dot{\sigma} = \frac{W_{sr}\,(1-\eta)}{T_{e,av}}$$

Solving a few problems

Show that for an ideal gas passing through a diffuser the actual exit pressure P_e is related to the ideal exit pressure P_{er} by the relation :

$$P_e = P_{er} \exp(-\frac{\dot{\sigma}}{\dot{m}R})$$

Where symbols have their usual meanings

SECOND – LAW ANALYSIS OF USUF PROCESS

$$\frac{dS_{cv}}{dt} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \sum \dot{Q}_{cv} / T$$

Integrating over a time interval t, since s_e, s_i are constant over t

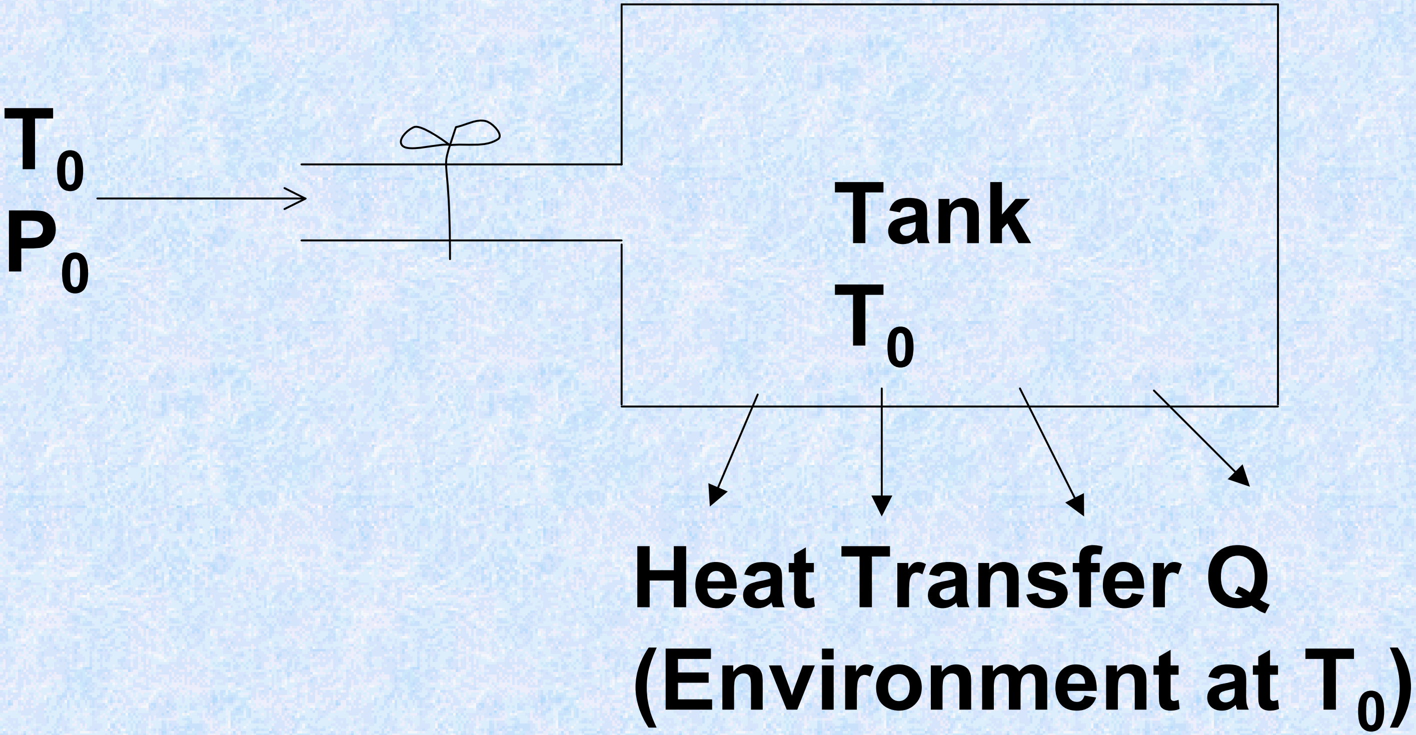
SECOND – LAW ANALYSIS OF SOME SIMPLE PROCESSES

$$\left[M_2 s_2 - M_1 s_1\right]_{c.v.} + \sum m_e s_e - \sum m_i s_i \geq \int_0^t \frac{\left(\sum \dot{Q}_{cv}\right)}{T} dt$$

$$\int \left(\sum \dot{Q}_{cv} / T \right) dt = \sum \int (\dot{Q}_{cv} / T) dt$$

Solving a few problems

Filling of an Evacuated Tank



T_0
P_0

Tank
T_0

Heat Transfer Q
(Environment at T_0)

Filling of an Evacuated Tank

Continuity $\dot{m}_i - 0 = \dot{m}_i = \dfrac{dM_{cv}}{dt}$

$$\Rightarrow \quad M_{cv} = \dot{m}_i t = m_i$$

=The total mass of air admitted

Filling of an Evacuated Tank

Ist Law

$$Q_{cv} + m_i h_i = 0 + m_i \left(u_2\right) - 0$$

$$Q_{cv} = m_i \left(u_2 - h_i\right)$$

{ Final state same as entering air

$$u_2 = u_i = h_i - \underset{\displaystyle p_o}{\underset{\downarrow}{p_i}} \upsilon_i$$

$$= -m_i p_0 \upsilon_i$$

$$= -p_0 V \qquad \left\{V : vol \quad of \quad container\right\}$$

Filling of an Evacuated Tank

IInd Law

$$\dot{\sigma} = m_2 s_2 - m_i s_i - \frac{Q_{cv}}{T_0}$$

$$= m_i s_2 - m_i s_i - \frac{Q_{cv}}{T_0}$$

$$= -\frac{Q_{cv}}{T_0} = \frac{p_0 V}{T_o}$$

final state of
C.V.= T_0, P_0
i.e. environ state

∴ $s_2 = s_i$

Filling of an Evacuated Tank

Remarks

$T_0 \; \dot{\sigma} = p_0 V =$ Work done by atmosphere during the filing process

How could we do reversible filing ?

Replace valve by reversible expander

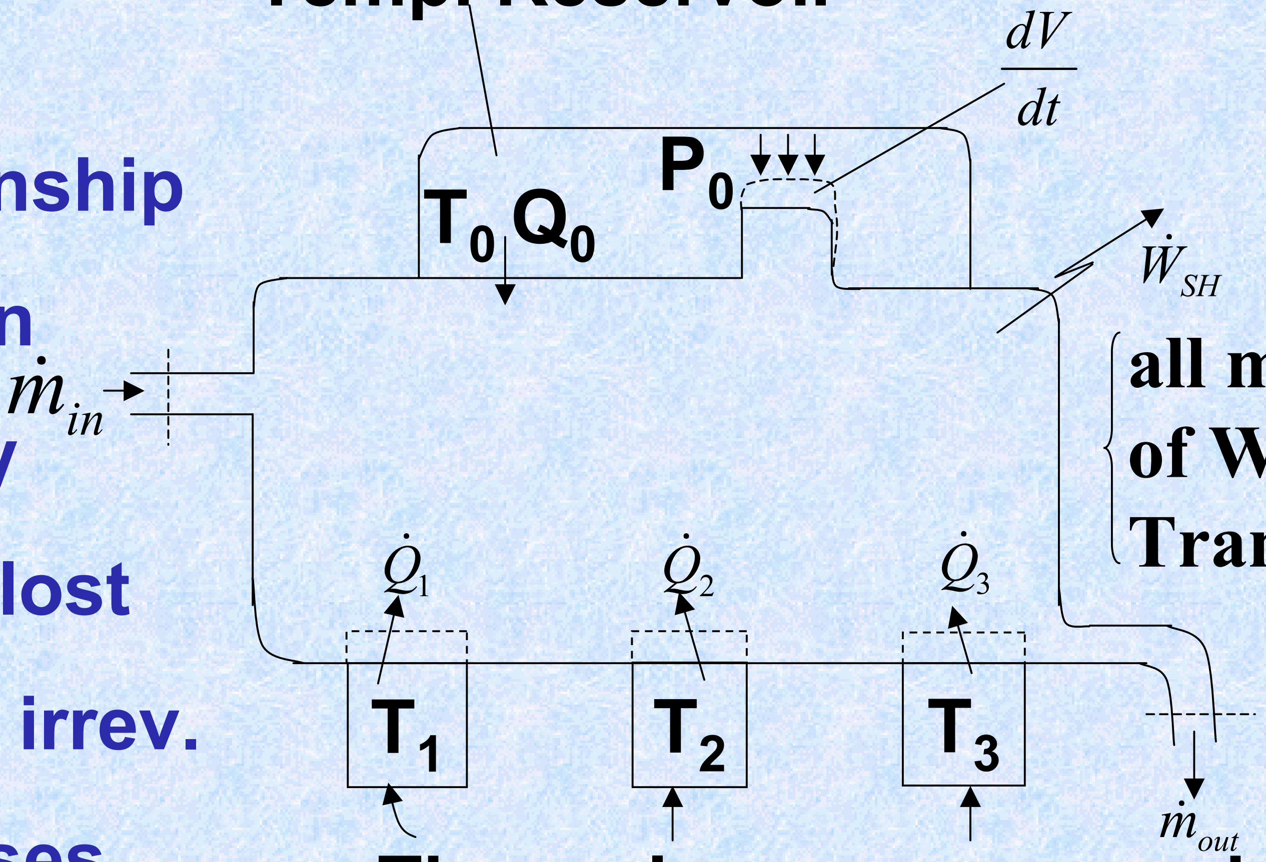
End of Lecture

Lecture 4.5

Availability

GOUY-STODOLA THEOREM

Gen. Relationship between Entropy Gen. & lost work in irrev. processes



Atmospheric Press & Temp. Reservoir

T_0 Q_0

P_0

$\frac{dV}{dt}$

$\dot{W}_{SH}$

all modes of Work Transfer

$\dot{m}_{in}$

$\dot{Q}_1$ $\dot{Q}_2$ $\dot{Q}_3$

T_1 T_2 T_3

$\dot{m}_{out}$

<u>Thermal energy reservoirs</u>

GOUY-STODOLA THEOREM

Ist Law

$$\sum_{0}^{n}\dot{Q}_i+\sum\dot{m}_i\left(h_i+\frac{V_i^2}{2}+gz_i\right)=\frac{dE_{CV}}{dt}+\sum\dot{m}_e\left(h_e+\frac{V_e^2}{2}+gz_e\right)$$

$$+\dot{W}_{SH}$$

define : $h_i+\dfrac{V_i^{\,2}}{2}+gz_i=h_i^{\,0}$ **: generalised enthalpy or <u>methalpy</u>**

$$\therefore\dot{W}_{SH}=Q_0+\sum_{1}^{n}Q_i+\sum\dot{m}_ih_i^0-\sum\dot{m}_eh_e^0-\frac{dE_{CV}}{dt}$$

GOUY-STODOLA THEOREM

IInd Law

$$\dot{\sigma} = \frac{dS_{CV}}{dt} - \sum_{i=0}^{n} \frac{Q_i}{T_i} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq 0$$

or

$$Q_0 = T_0 \left\{ \frac{dS_{CV}}{dt} - \sum_{i=1}^{n} \frac{Q_i}{T_i} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \right\} - T_0 \dot{\sigma}$$

GOUY-STODOLA THEOREM

Eliminating Q_0 between these two eqs. gives

$$\dot{W}_{SH} = \sum \dot{m}_i h_i^0 - \sum \dot{m}_e h_e^0 - \frac{dE_{CV}}{dt} + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_1}\right) Q_i +$$

$$\sum \dot{m}_e s_e T_0 - \sum \dot{m}_i s_i T_0 + \frac{d}{dt}\left(T_0 S_{CV}\right) - T_0 \dot{\sigma}$$

GOUY-STODOLA THEOREM

or

$$\dot{W}_{SH} = \sum \dot{m}_i \left(h_i^0 - T_0 s_i \right) - \sum \dot{m}_e \left(h_e^0 - T_0 s_e \right)$$

$$- \frac{d}{dt} \left(E_{CV} - T_0 S_{CV} \right) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_1} \right) Q_i$$

$$- T_0 \dot{\sigma}$$

GOUY-STODOLA THEOREM

$$\left(\dot{W}_{SH}\right)_{\max} = \left(\dot{W}_{SH}\right)_{rev} = \dot{W}_{SH} \quad for \quad \dot{\sigma} = 0$$

and Loss of work due to irreversibility

i.e. $\boxed{\textbf{LAW = I =} \quad T_0 \dot{\sigma}}$

GOUY-STODOLA THEOREM

Since $\dot{\sigma} \geq 0 \quad ; \quad I \geq 0$

=> If W_{SH} is +VE **{work output}**

$$W_{SH} \leq W_{SH,\max} = W_{SH,rev}$$

If W_{SH} is -VE **{work input}**

$$W_{input} \geq W_{input,rev}$$

GOUY-STODOLA THEOREM

For SSSF process this gives

$$\left(\dot{W}_{SH}\right)_{\max} = \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i - T_0 S_i \right)$$

$$- \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_i - T_0 S_e \right)$$

$$+ \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_1} \right) Q_i$$

define h - T_0S = b, availability

Is it a thermodynamic property?

AVAILABILITY BALANCE

$$\left(\dot{W}_{SH}\right)_{\max} = \left(\sum \dot{m}_i b_i - \sum \dot{m}_e b_e\right) + \left(\sum \dot{m}_i \frac{V_i^2}{2} - \sum \dot{m}_e \frac{V_e^2}{2}\right)$$

← **flow availability** →

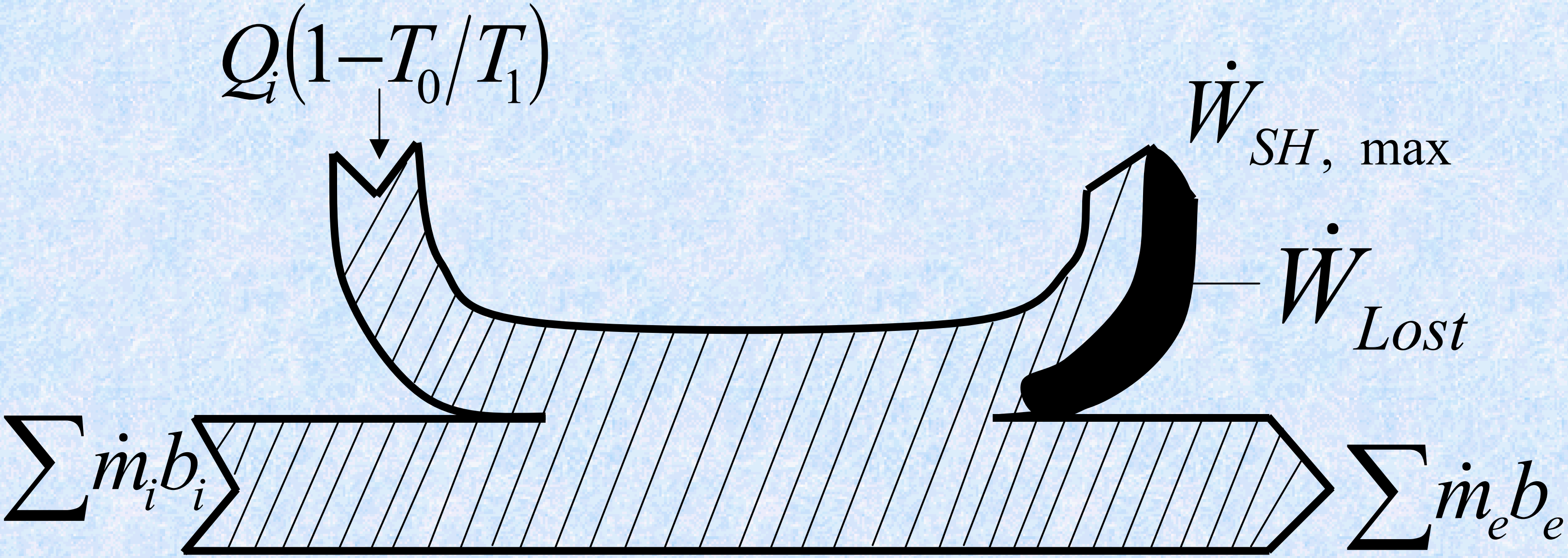
Δ K.E.

$$+ \left(\sum \dot{m}_i g z_i - \sum \dot{m}_e g z_e\right) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_1}\right) Q_i$$

Δ P.E.

"availability" of heat interaction

AVAILABILITY BALANCE



NB **While changes in KE & PE are fully converted to work, changes in enthalpy h & h.t. Q_i are not fully converted**

GOUY-STODOLA THEOREM

K.E. & P.E. are “ordered” forms of energy

Q, U => “disordered” energy stored in the form of random molecular motion

What are others ordered forms of energy?

MAX WORK IN A CLOSED SYSTEM

$$\dot{m}_i = 0, \quad \dot{m}_e = 0, \quad \dot{\sigma} = 0$$

$$\dot{W}_{SH} = \sum \dot{Q}_i \left(1 - \frac{T_0}{T_1} \right) - \frac{d}{dt} \left(E_{CV} - T_0 S_{CV} \right)$$

Integrating over process duration

$$W_{SH} = \sum Q_i \left(1 - \frac{T_0}{T_1} \right) + \left[\left(E - T_0 S \right)_1 - \left(E - T_0 S \right)_2 \right]$$

MAX WORK IN A CLOSED SYSTEM

Useful Work = Shaft work –
Work done against atmospheric pressure

$$W_{US} = W_{Sh} - P_0\left(V_2 - V_1\right)$$

$$= \sum Q_i\left(1 - \frac{T_0}{T_1}\right) + \left[\left(E + P_0V - T_0S\right)_1 - \left(E + P_0V - T_0S\right)_2\right]$$

$$E + P_0V - T_0S \equiv A$$

Closed system availability

MAX WORK IN A CLOSED SYSTEM

$$a = u + P_0 \upsilon - T_0 s$$

(per unit mass basis)

Max work obtainable from a closed system exchanging heat only with environment

$W_{max} = a_1 - a_2$

End of Lecture

Lecture 4.6

EXERGY

EXERGY

Maximum useful work output possible from a given stream in any s.f. process(neglecting ΔKE & ΔPE)

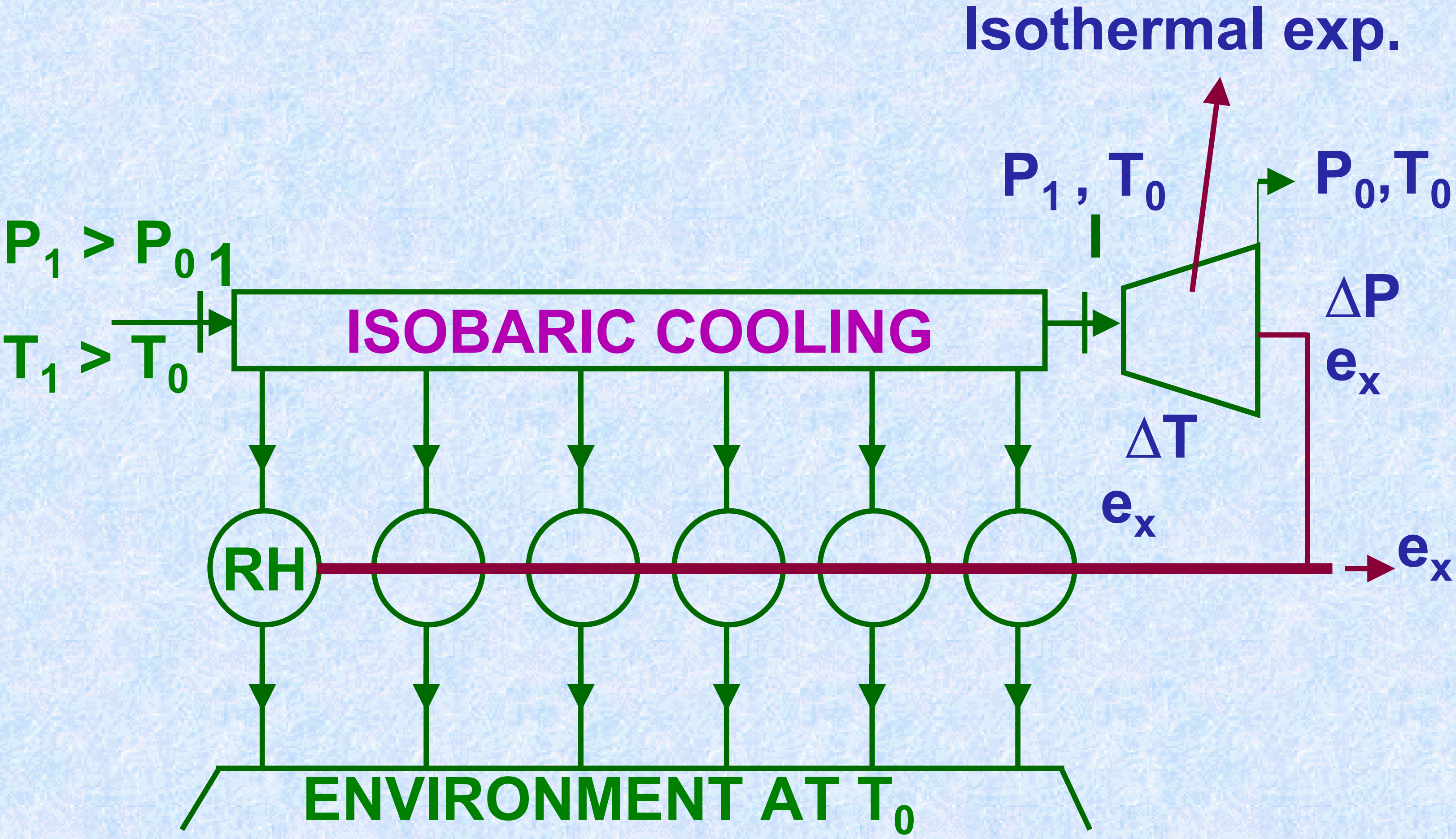
$$e_x = b - b_0 = (h - h_0) - T_0\,(s - s_0)$$

Components of exergy (thermo-mechanical)

Useful work arising from press. difference w.r.t. P_0 & that from temp. difference w.r.t. T_0

EXERGY

Conceptual Visualisation



EXERGY

$$e_x^{\Delta T} = \left\{ \int_{T_1}^{T_0} \left(1 - \frac{T_0}{T}\right)(-dh) \right\}_{P_1} = -\int_{T_1}^{T_0} (T - T_0)\,ds$$

$$e_x^{\Delta P} = (h_i - T_0 s_i) - (h_0 - T_0 s_0)$$

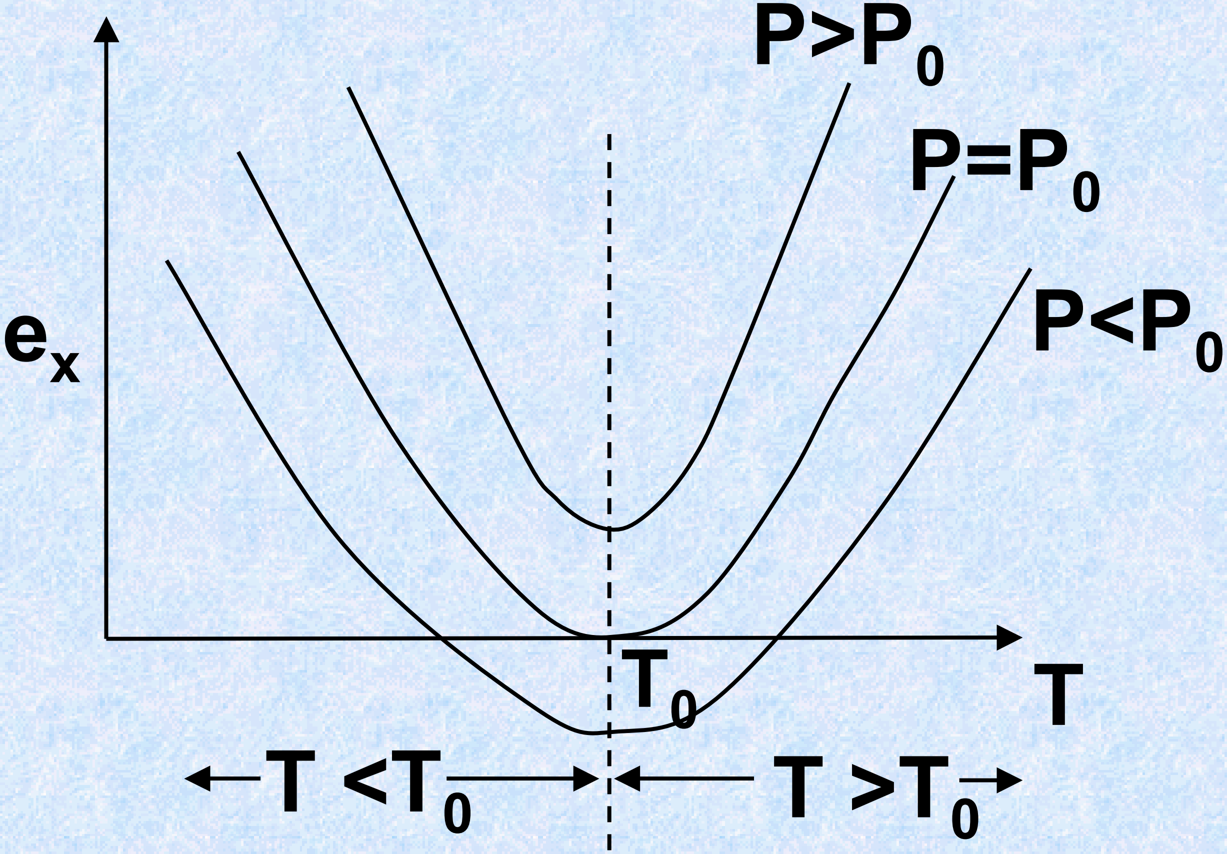
$$= T_0(s_0 - s_i) - (h_0 - h_i)$$

$$\boxed{e_x = e_x^{\Delta P} + e_x^{\Delta T}}$$

{verify}

FLOW EXERGY OF AN IDEAL GAS

$$e_x = C_p(T - T_0) - T_0\left(C_p \ ln \ \frac{T}{T_0} - R \ ln \ \frac{P}{P_0}\right)$$

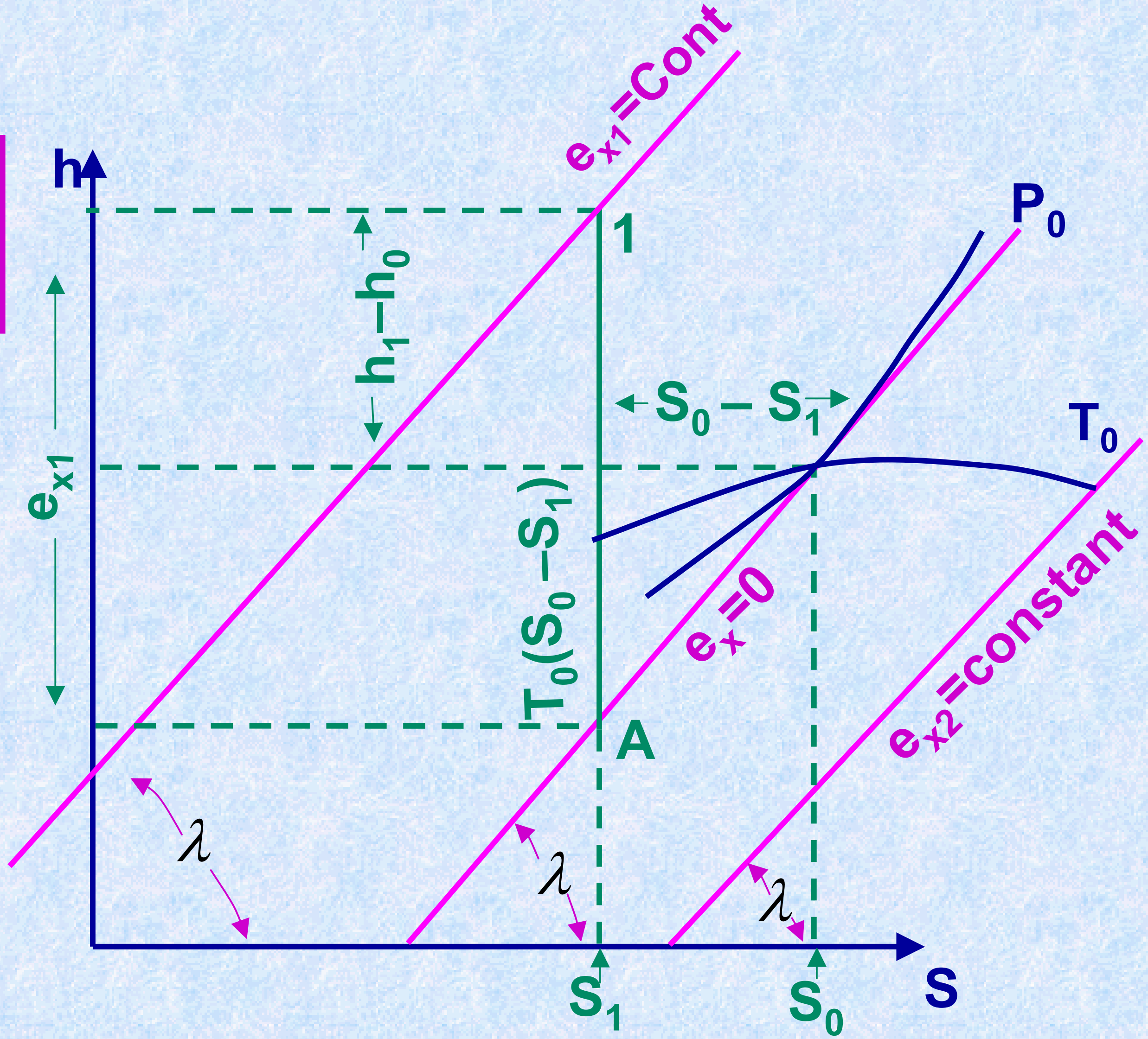


FLOW EXERGY FROM h-S CHARTS

$$\boxed{\left(\frac{\partial h}{\partial s}\right)_P = T}$$

Implies →

$$\boxed{\tan\lambda = T_0}$$



FLOW EXERGY FROM h-S CHARTS

Hence $1 - A \equiv e_{x1}$

Further since

$e_x = (h - T_0S) - (h_0 - T_0S_0)$

$$\frac{\partial e_x}{\partial S} = \left(\frac{\partial h}{\partial S}\right) - T_0$$

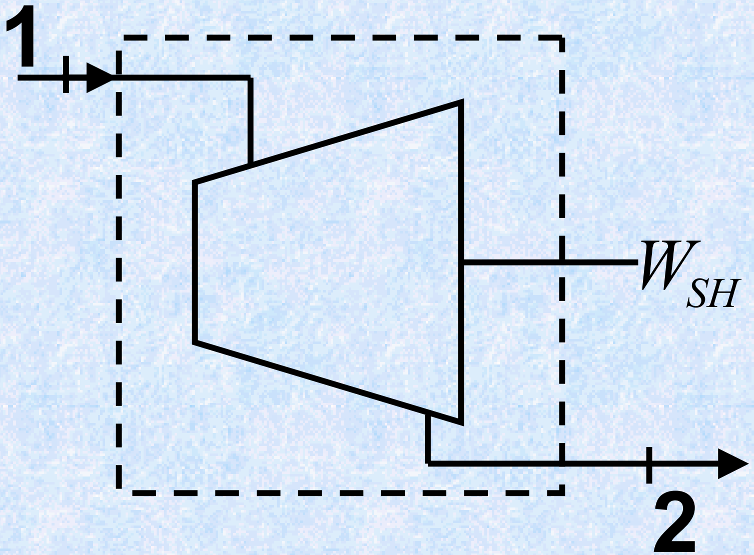
$\therefore$ For $\dfrac{\partial h}{\partial S} = T_0$; $\dfrac{\partial e_x}{\partial S} = 0$

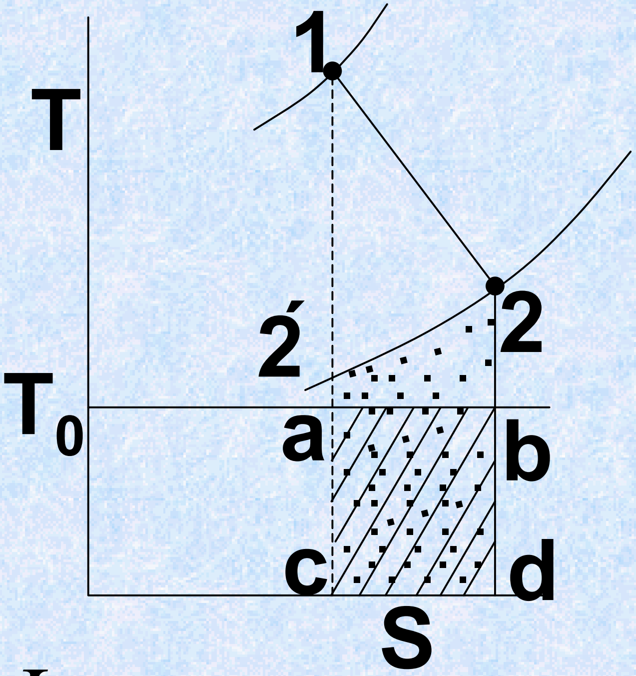
=> e_x = Const for lines II to tangent to const P_0 curve.

EXERGY ANALYSIS – SIMPLE PROCESSES

EXPANSION IN TURBINE

(adiabatic) for simplicity





$$e_{x_1} = e_{x_2} + W_{Sh} + I$$

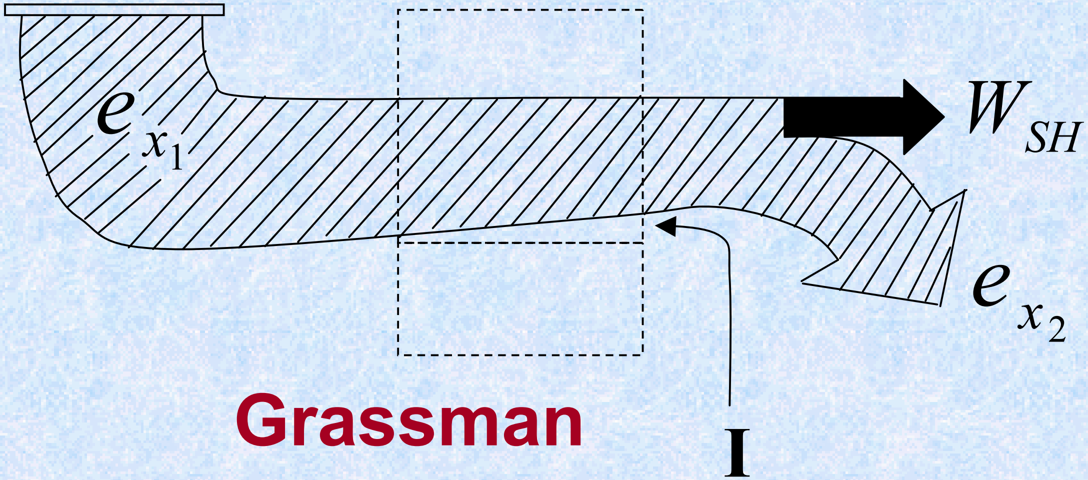
$$I = T_0 (s_2 - s_1)$$

EXERGY ANALYSIS – SIMPLE PROCESSES

Second Law Efficiency $\psi = \dfrac{W_{SH}}{e_{x_1} - e_{x_2}}$

$$= 1 - \frac{I}{e_{x_1} - e_{x_2}}$$

$$= \frac{h_1 - h_2}{e_{x_1} - e_{x_2}}$$



Grassman Diagr.

{assuming mech. losses to be small

EXERGY ANALYSIS – SIMPLE PROCESSES

Compare with isentropic efficiency

$$\eta_S = \frac{h_1 - h_2}{h_1 - h_2'}$$

$$\psi = \frac{h_1 - h_2}{(h_1 - h_2) + \underbrace{T_0(s_2 - s_1)}_{\downarrow\, = i}}$$

Sp. Irreversibility

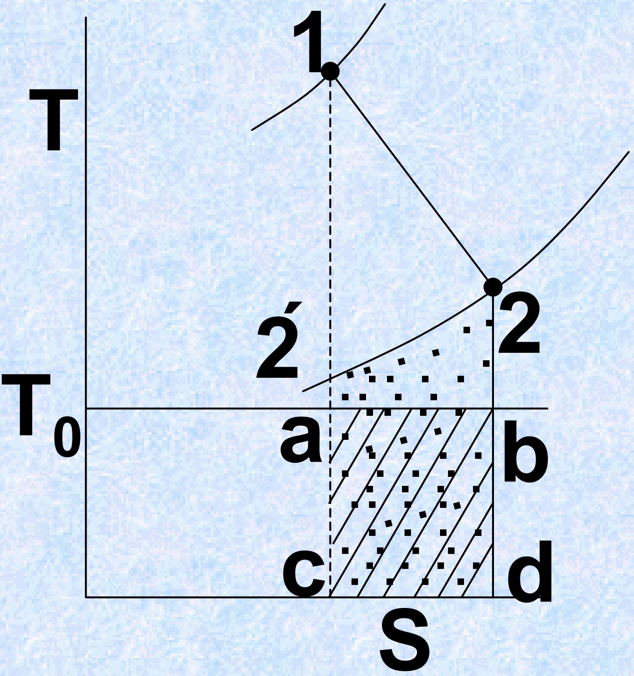
{area a b d c a}

EXERGY ANALYSIS – SIMPLE PROCESSES

$$\eta_S = \frac{h_1 - h_2}{\left(h_1 - h_2\right) + \underbrace{\left(h_2 - h_2'\right)}_{\downarrow\ = r}}$$

Frictional reheat

{area 2 2′ c d 2}



End of Lecture

Lecture 4.7

Exergy Analysis-Flow processes

Exergetic (2nd Law) Efficiency

- 2nd Law implies :

$$\sum (exergy)_{output} = \sum (exergy)_{input} - I$$

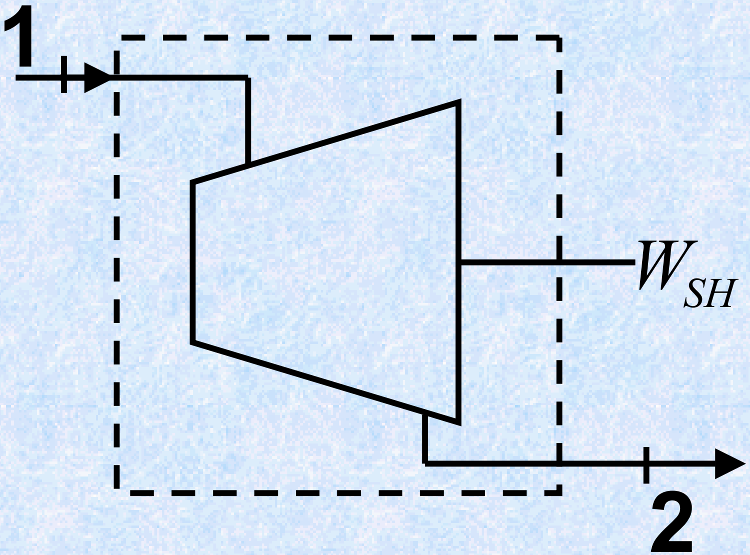
Second Law Efficiency

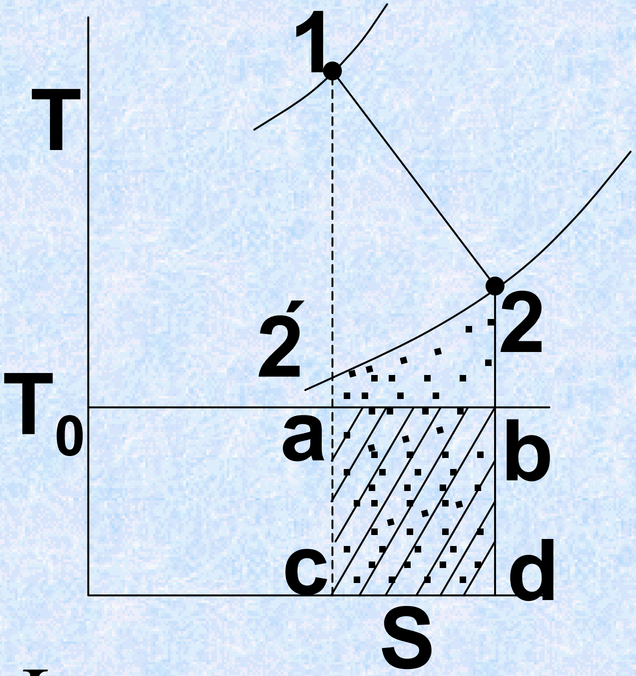
$$\psi = \frac{\sum (exergy)_{output}}{\sum (exergy)_{input}}$$

EXERGY ANALYSIS – SIMPLE PROCESSES

EXPANSION IN TURBINE

(adiabatic) for simplicity





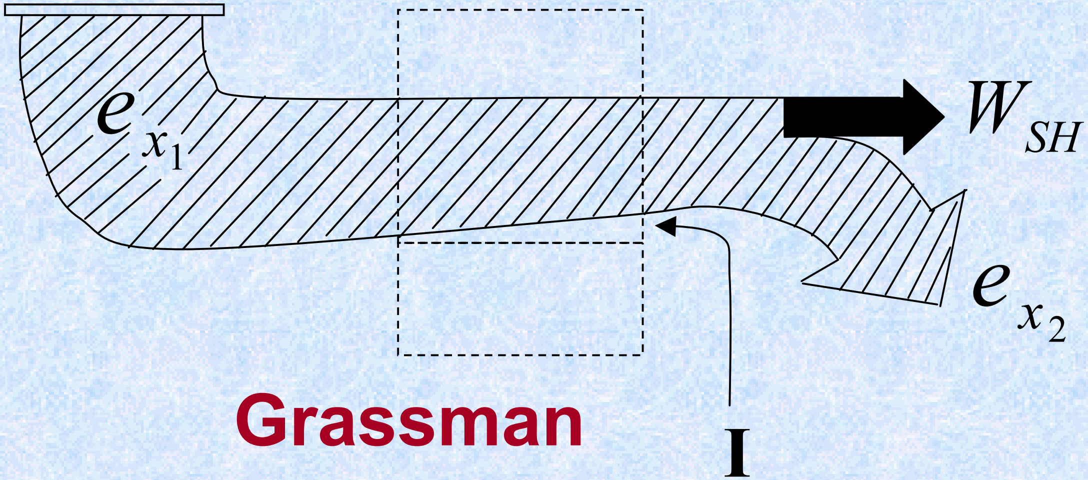
$$e_{x_1} = e_{x_2} + W_{Sh} + I$$

$$I = T_0(s_2 - s_1)$$

EXERGY ANALYSIS – SIMPLE PROCESSES

Second Law Efficiency $\psi = \dfrac{W_{SH}}{e_{x_1} - e_{x_2}}$

$$= 1 - \frac{I}{e_{x_1} - e_{x_2}}$$



$$= \frac{h_1 - h_2}{e_{x_1} - e_{x_2}}$$

Grassman Diagr.

{assuming mech. losses to be small

EXERGY ANALYSIS – SIMPLE PROCESSES

Compare with isentropic efficiency

$$\eta_S = \frac{h_1 - h_2}{h_1 - h_2'}$$

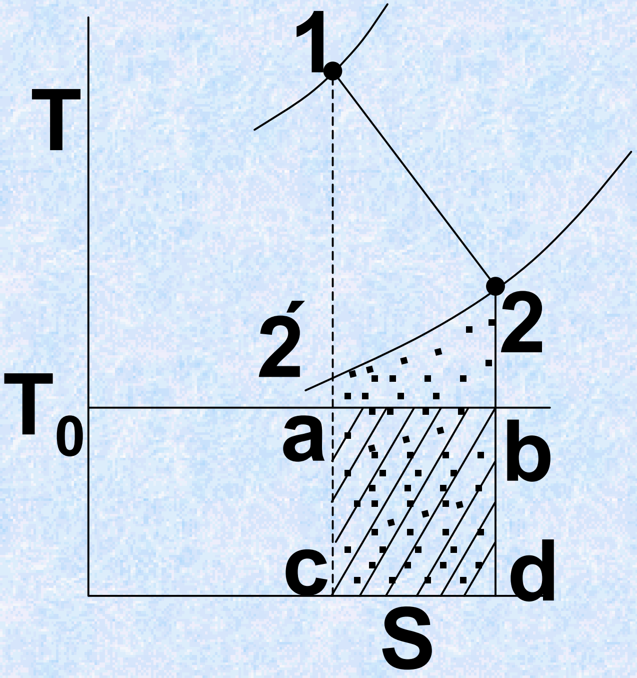
$$\psi = \frac{h_1 - h_2}{\left(h_1 - h_2\right) + \underbrace{T_0\left(s_2 - s_1\right)}_{= i}}$$

Sp. Irreversibility

{area a b d c a}

EXERGY ANALYSIS – SIMPLE PROCESSES

$$\eta_S = \frac{h_1 - h_2}{(h_1 - h_2) + \underbrace{(h_2 - h_2')}_{\downarrow\ = r}}$$



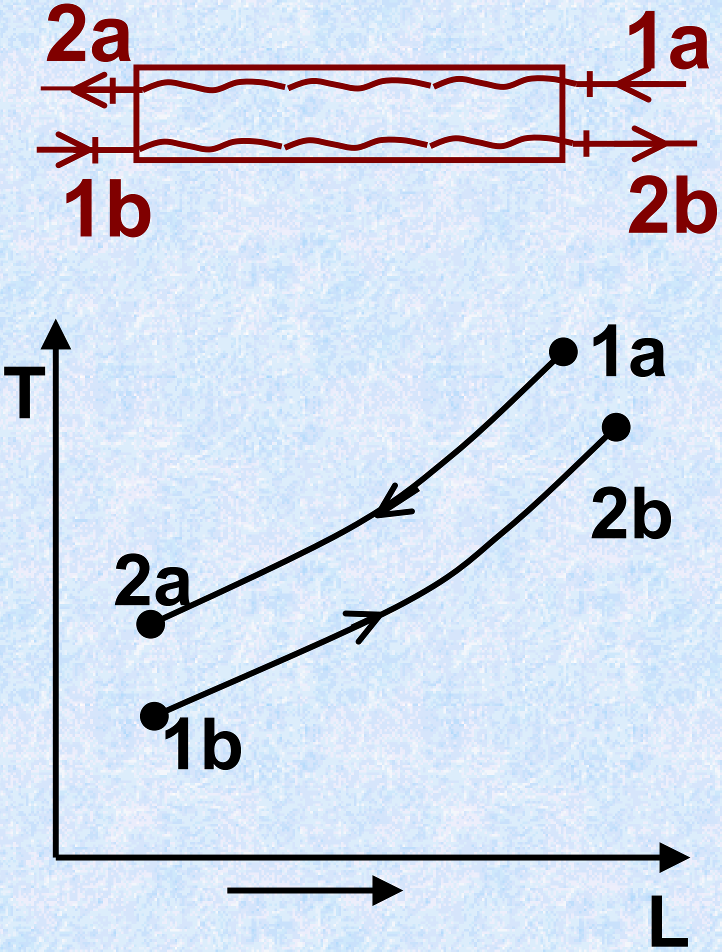
Frictional reheat

{area 2 2′ c d 2}

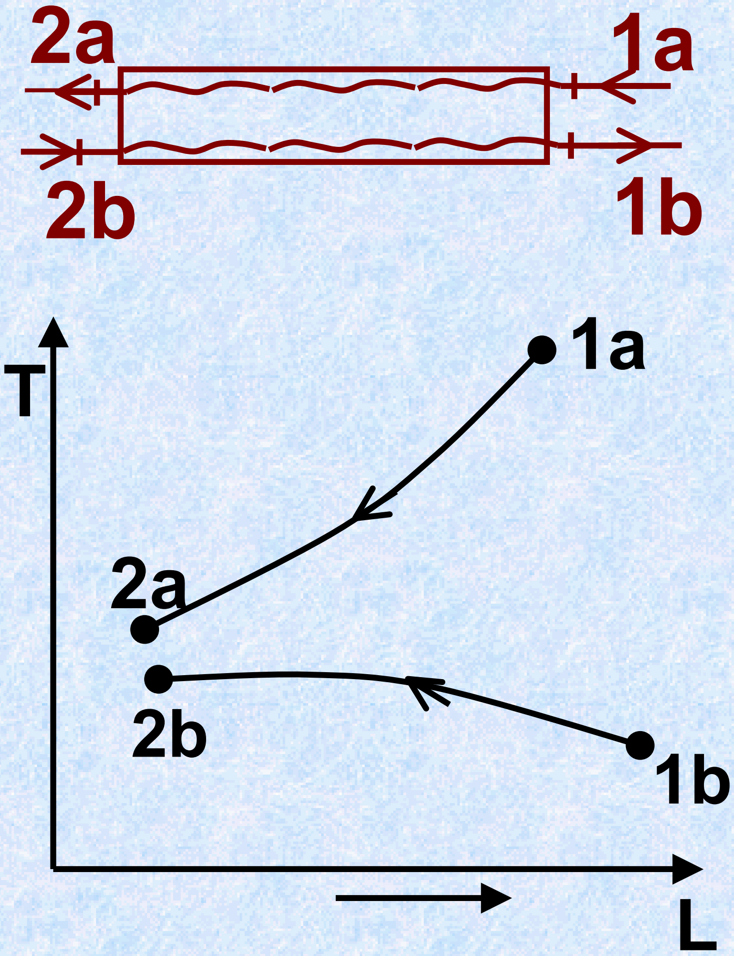
$e_{x2} - e_{x2'} = r - i$ **(Prove it)**

HEAT TRANSFER PROCESSES

Isobaric Heat Transfer



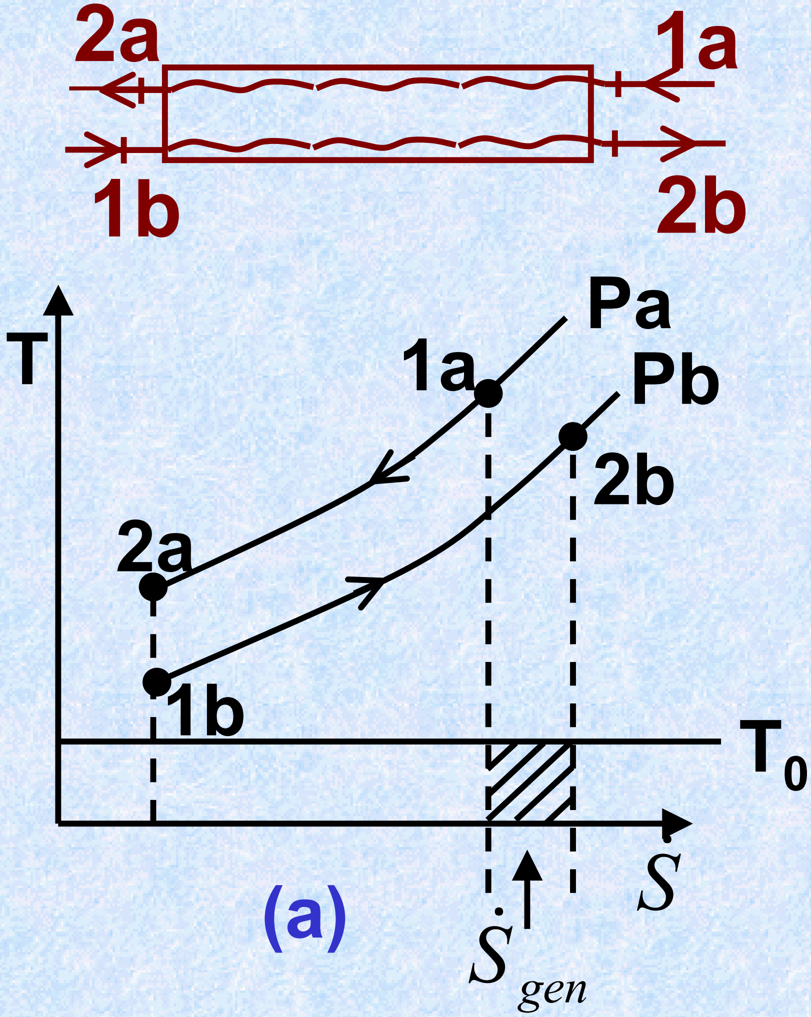
COUNTER FLOW H.E.



PARALLEL FLOW H.E.

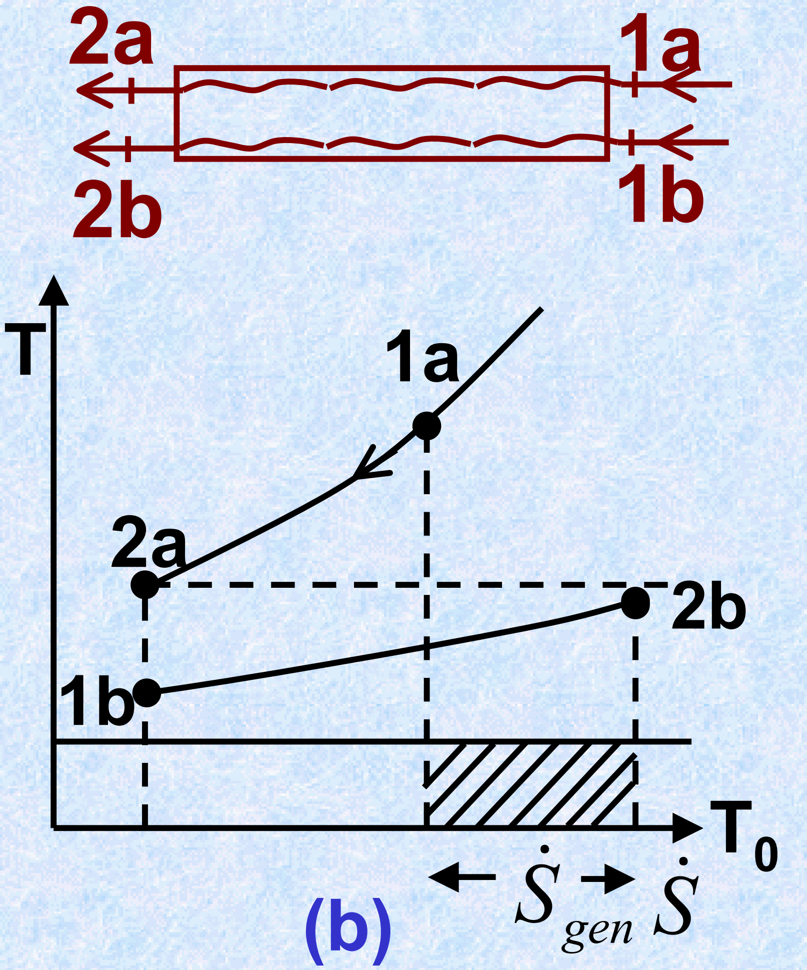
HEAT TRANSFER PROCESSES

Isobaric Heat Transfer



(a)

COUNTER FLOW H.E.



(b)

PARALLEL FLOW H.E.

HEAT TRANSFER PROCESSES

$$I = T_0 \dot{\sigma} = T_0[(s_{2a} - s_{1a})\dot{m}_a + (s_{2b} - s_{1b})\dot{m}_b]$$

$$= T_0[(\dot{S}_{2a} - \dot{S}_{1a}) + (\dot{S}_{2b} - \dot{S}_{1b})]$$

Also from exergy balance

$$I = \dot{m}_a\left(e_{x1a} - e_{x2a}\right) - \dot{m}_b\left(e_{x2b} - e_{x1b}\right)$$

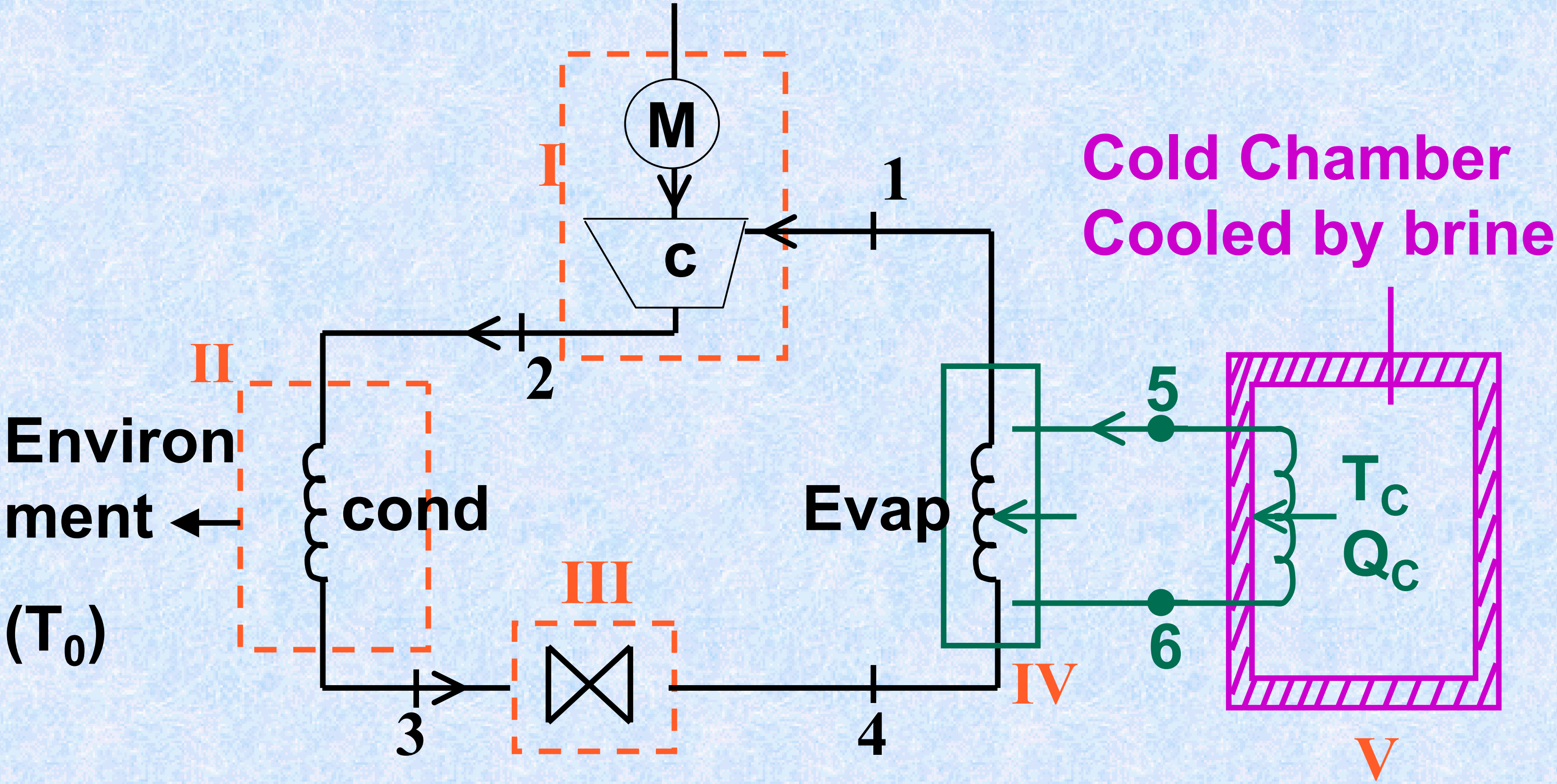
{are these expressions equal to each other}

HEAT TRANSFER PROCESSES

$$\psi_{HE} = \frac{Exergy\ increase\ of\ cold\ stream}{Exergy\ decrease\ of\ hot\ stream}$$

$$\psi_{HE} = \frac{\dot{m}_b (e_{x2b} - e_{x1b})}{\dot{m}_a (e_{x2a} - e_{x1a})}$$

EXERGY ANALYSIS OF A REFRIG PLANT



Working fluid : NH_3 **Coolant** **Brine**

EXERGY ANALYSIS OF A REFRIG PLANT (ref: Kotas)

Design Parameters

Q_C = 93.03kW T_0 = 20^0C T_c = -1^0C

Comp inlet, T_1 = -10^0C T_e = -12^0C

Comp outlet, T_2 = 119^0C

Condenser temp = 28^0C

Condensate outlet temp = 25^0C

EXERGY ANALYSIS OF A REFRIG PLANT

$$Compressor,\ \eta_{mech} = .83 \qquad \underset{(elect.)}{\eta_{motor}} = 0.9$$

Brine temps.

$$\begin{cases} T_5 = -5^0C \\ T_6 = -7^0C \\ C_p = 2.85\ kj/kg\ K \end{cases}$$

Assumptions

Negligible heat leaks; adiabatic compression

Negligible Press Drop, ΔKE, ΔPE

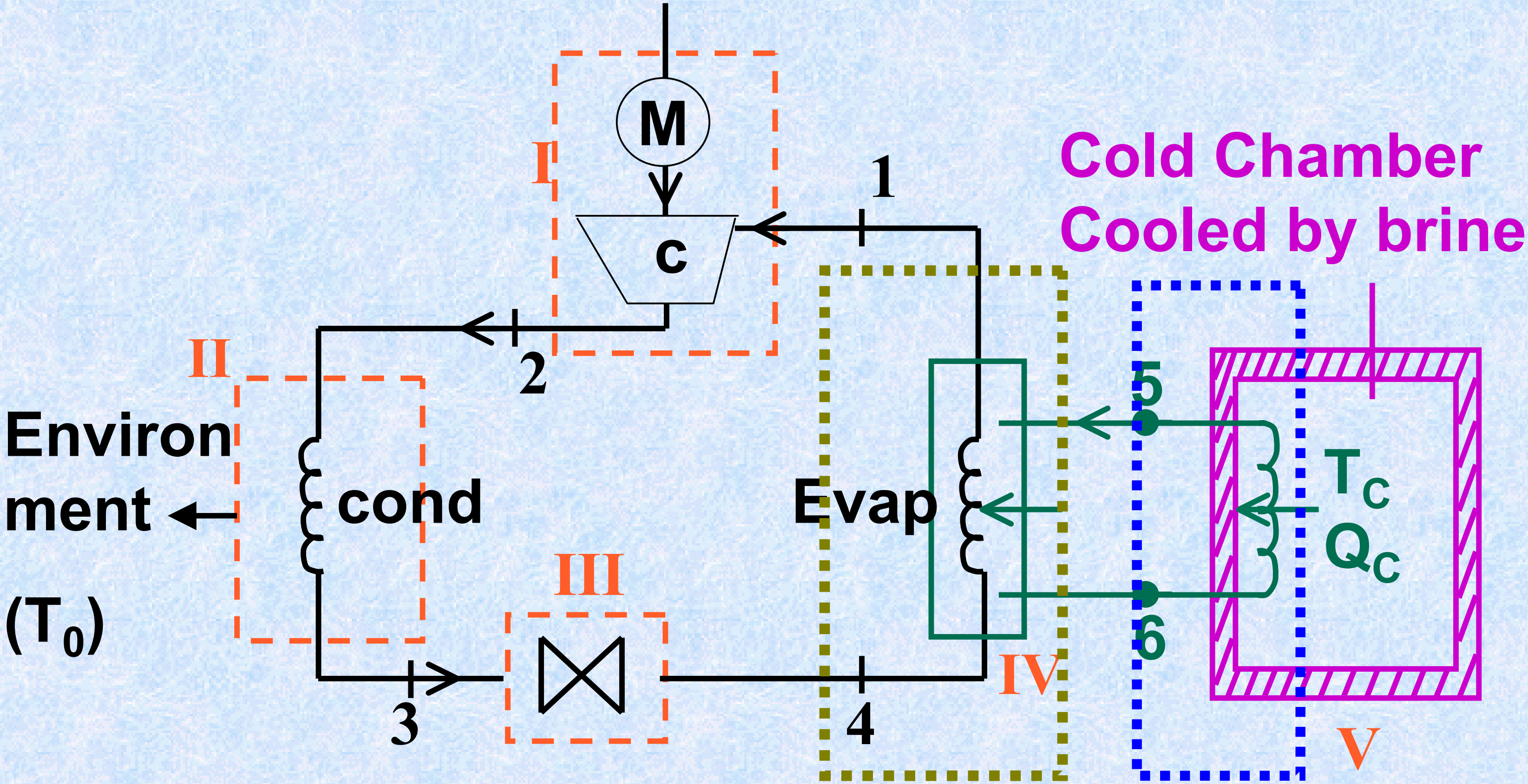
Negligible power input to brine pump

End of Lecture

Lecture 4.8

Exergy Analysis…contd

EXERGY ANALYSIS OF A REFRIG PLANT



Cold Chamber
Cooled by brine

Environment

(T$_0$)

Working fluid : NH$_3$ Coolant Brine

EXERGY ANALYSIS OF A REFRIG PLANT (ref: Kotas)

Design Parameters

Q_C = 93.03kW $T_0 = 20^0C$ $T_c = -1^0C$

Comp inlet, $T_1 = -10^0C$ $T_e = -12^0C$

Comp outlet, $T_2 = 119^0C$

Condenser temp = 28^0C

Condensate outlet temp = 25^0C

EXERGY ANALYSIS OF A REFRIG PLANT

$Compressor,\ \eta_{mech} = .83 \qquad \eta_{\underset{(elect.)}{motor}} = 0.9$

Brine temps.

$T_5 = -5^0C$

$T_6 = -7^0C$

$C_p = 2.85$ kj/kg K

Assumptions

Negligible heat leaks; adiabatic compression

Negligible Press Drop, ΔKE, ΔPE

Negligible power input to brine pump

End of Lecture

LECTURE 4.9

Endo-reversible Engines for maximum power output

ENDOREVERSIBLE ENGINES

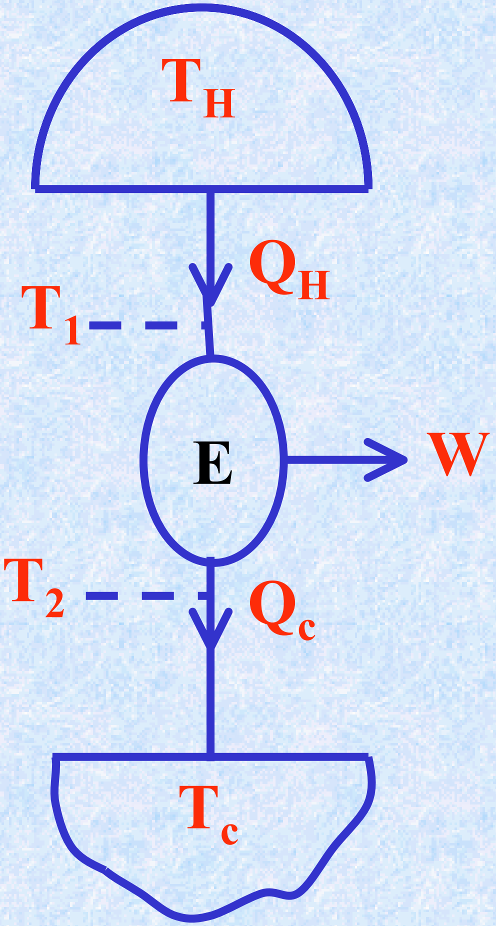
$$\dot{Q}_H = U_H A_H (T_H - T_1)$$

$$\dot{Q}_C = U_C A_C (T_2 - T_c)$$

$$\eta = 1 - \frac{T_2}{T_1} < 1 - \frac{T_c}{T_H}$$

Ist law $\dot{W} = \dot{Q}_H - \dot{Q}_c$

IInd law $-\frac{\dot{Q}_H}{T_1} + \frac{\dot{Q}_c}{T_2} = 0$



ENDOREVERSIBLE ENGINES

$$\therefore \dot{W} = \dot{Q}_H \left(1 - \frac{\dot{Q}_c}{\dot{Q}_H} \right) = \dot{Q}_H \left(1 - \frac{T_2}{T_1} \right)$$

Combine these eqs. to express $\dot{W}$ in terms of $T_H, T_c, U_M A_H$ and the ratio $\frac{T_2}{T_1} = \tau$

$$\rightarrow \frac{Q_H}{Q_c} = \frac{T_1}{T_2} = \frac{1}{\tau}$$

ENDOREVERSIBLE ENGINES

$$T_2 = \frac{\dot{Q}_c}{U_c A_c} + T_c$$

$$T_1 = \frac{\dot{Q}_H}{Q_c} T_2 = \frac{\dot{Q}_H}{\dot{Q}_c} \left\{ \frac{\dot{Q}_c}{U_c A_c} + T_c \right\}$$

$$= \frac{1}{\tau} \left\{ \frac{\dot{Q}_c}{U_c A_c} + T_c \right\}$$

ENDOREVERSIBLE ENGINES

$$\therefore\ Q_H = U_H A_H \left(T_H - T_1\right)$$

$$= U_H A_H T_H - \frac{U_H A_H}{\tau} \left\{ \frac{\dot{Q}_c}{U_c A_c} + T_c \right\}$$

$$= U_H A_H T_H - \frac{U_H A_H \dot{Q}_H}{U_c A_c} - \frac{U_H A_H T_c}{\tau}$$

ENDOREVERSIBLE ENGINES

Rearranging

$$\frac{\dot{Q}_H}{U_H A_H T_H} = \frac{1 - \frac{T_c}{T_H} \cdot \frac{1}{\tau}}{1 + \frac{U_H A_H}{U_c A_c}} = \frac{\tau - \frac{T_c}{T_H}}{\tau \left(1 + \frac{U_H A_H}{U_c A_c} \right)}$$

hence

$$\frac{\dot{W}}{U_H A_H T_H} = \frac{\tau - \frac{T_c}{T_H}}{\tau \left(1 + \frac{U_H A_H}{U_c A_c} \right)} (1 - \tau)$$

ENDOREVERSIBLE ENGINES

For max output with given

$U_H A_H$, $U_c A_c$, T_c & T_H

$$\frac{\partial \dot{W}}{\partial \tau} = 0$$

$$= \frac{U_H A_H T_H}{\left\{1 + \frac{U_H A_H}{U_c A_c}\right\}} \left[\frac{1-\tau}{\tau} + \left(\tau - \frac{T_c}{T_H}\right)\left(-\frac{1}{\tau^2}\right) \right]$$

$$= \frac{(U_H A_H)(U_c A_c) T_H}{(U_H A_H + U_c A_c)} \left(-1 + \frac{T_c}{T_H} \cdot \frac{1}{\tau^2} \right)$$

ENDOREVERSIBLE ENGINES

$$\Rightarrow \quad \tau^2_{opt.} = \frac{T_c}{T_H}$$

i.e. for max. output $\tau = \frac{T_2}{T_1} = \left(\frac{T_c}{T_H}\right)^{\frac{1}{2}}$

$$\therefore \quad \eta_{opt} = 1 - \left(\frac{T_c}{T_H}\right)^{\frac{1}{2}}$$

Optimal allocation of heat transfer areas

→ **i.e. relative values of $U_H A_H$ & $U_c A_c$**

Suppose $U_H A_H + U_c A_c = \text{Const. (given)}$ <K>

$$\dot{W} = \frac{(K - \mathrm{U}_{\mathrm{H}}\mathrm{A}_{\mathrm{H}})(\mathrm{U}_{\mathrm{H}}\mathrm{A}_{\mathrm{H}})}{K} \; T_H \left\{1 - \frac{T_c}{T_H}\frac{1}{\tau}\right\}\{1 - \tau\}$$

Optimal Allocation of Heat Transfer Areas

$$\frac{\partial \dot{W}}{\partial \left(\mathrm{U}_{\mathrm{H}} \mathrm{A}_{\mathrm{H}} \right)} = 0 = \left(K - 2 \ \mathrm{U}_{\mathrm{H}} \mathrm{A}_{\mathrm{H}} \right)$$

$$\therefore \ \left(\mathrm{U}_{\mathrm{H}} \mathrm{A}_{\mathrm{H}} \right)_{opt.} = \frac{K}{2} = \frac{\mathrm{U}_{\mathrm{H}} \mathrm{A}_{\mathrm{H}} + \mathrm{U}_{\mathrm{c}} \mathrm{A}_{\mathrm{c}}}{2}$$

or

$$\rightarrow \ \mathrm{U}_{\mathrm{H}} \mathrm{A}_{\mathrm{H}} = \mathrm{U}_{\mathrm{c}} \mathrm{A}_{\mathrm{c}}$$

End of Lecture