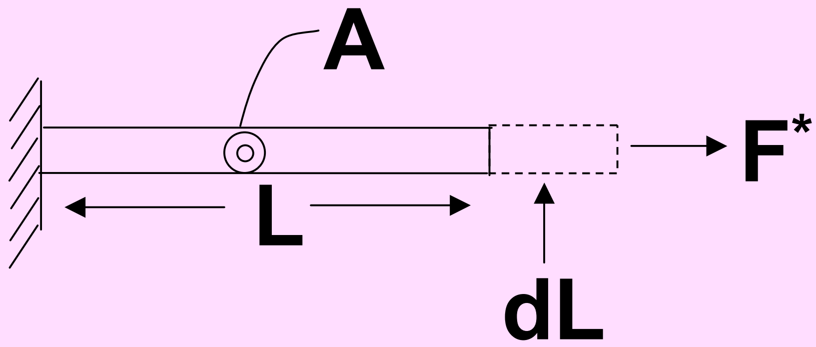
Lecture 3.5

Simple Elastic Systems

SIMPLE ELASTIC SYSTEMS



Basic EQ.

$$Tds \geq dU - f_i{}'' dx_i{}'$$

Rev. process $f_i{}'' = f_i{}' \equiv F^*$ $x_i{}' \equiv L$

$$TdS = dU - F^* dL$$

Per unit vol. basis $\sigma = F^* / A$ $\varepsilon = \dfrac{L - L_o}{L_o}$

$$d\varepsilon = \frac{dL}{L_o} - \frac{L}{L_O^2} dL_O = \frac{dL}{L_o} - (1+\varepsilon)\frac{dL_o}{L_o}$$

SIMPLE ELASTIC SYSTEMS

Basic Eq.

$$dU = TdS + F^{*}dL$$

other potentials

$$dH = TdS - L\,dF^{*} \qquad \{H = U - F^{*}L$$

$$dF = -SdT + F^{*}dL \qquad \{F = U - TS$$

$$dG = -SdT - L\,dF^{*} \qquad \{G = H - TS$$

SIMPLE ELASTIC SYSTEMS

Maxwell's Relations

$$\left(\frac{\partial T}{\partial L}\right)_s = \left(\frac{\partial F^*}{\partial S}\right)_\varepsilon$$

$$\left(\frac{\partial T}{\partial F^*}\right)_s = -\left(\frac{\partial L}{\partial S}\right)_{F^*}$$

$$\left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial F^*}{\partial T}\right)_L$$

$$\left(\frac{\partial S}{\partial F^*}\right)_T = \left(\frac{\partial L}{\partial T}\right)_{F^*}$$

Mnemonic aid :

$$T.S \equiv F^*.L$$

Sign –ve if T & F* together

SIMPLE ELASTIC SYSTEMS

Measurable prop. of Elastic Systems

Sp. heats

$$C_L = \frac{1}{V_o}\left(\frac{\partial U}{\partial T}\right)_L = \frac{T}{V_o}\left(\frac{\partial S}{\partial T}\right)_L$$

$$C_{F^*} = \frac{1}{V_o}\left(\frac{\partial H}{\partial T}\right)_{F^*} = \frac{T}{V_o}\left(\frac{\partial S}{\partial T}\right)_{F^*}$$

Coeff. of Thermal strain

$$\alpha = \frac{1}{L_o}\left(\frac{\partial L}{\partial T}\right)_{F^*}$$

Coeff. of Thermal stress

$$\beta = \left(\frac{\partial \sigma}{\partial T}\right)_L = \frac{1}{A}\left(\frac{\partial F^*}{\partial T}\right)_L$$

Isothermal Young's Modulus

$$\mathrm{Y} = \left(\frac{\partial \sigma}{\partial \varepsilon}\right)_T = \frac{L_o}{A}\left(\frac{\partial F^*}{\partial L}\right)_T$$

SIMPLE ELASTIC SYSTEMS

- Interdependence of various prop.

e.g.

$$\alpha = \frac{1}{L_O}\left(\frac{\partial L}{\partial T}\right)_{F^*} = \frac{1}{L_O}\frac{\partial(L, F^*)}{\partial(T, F^*)}$$

$$\beta = \frac{1}{A}\left(\frac{\partial F^*}{\partial T}\right)_L = \frac{\partial\left(F^*, L\right)}{\partial\left(T, L\right)}$$

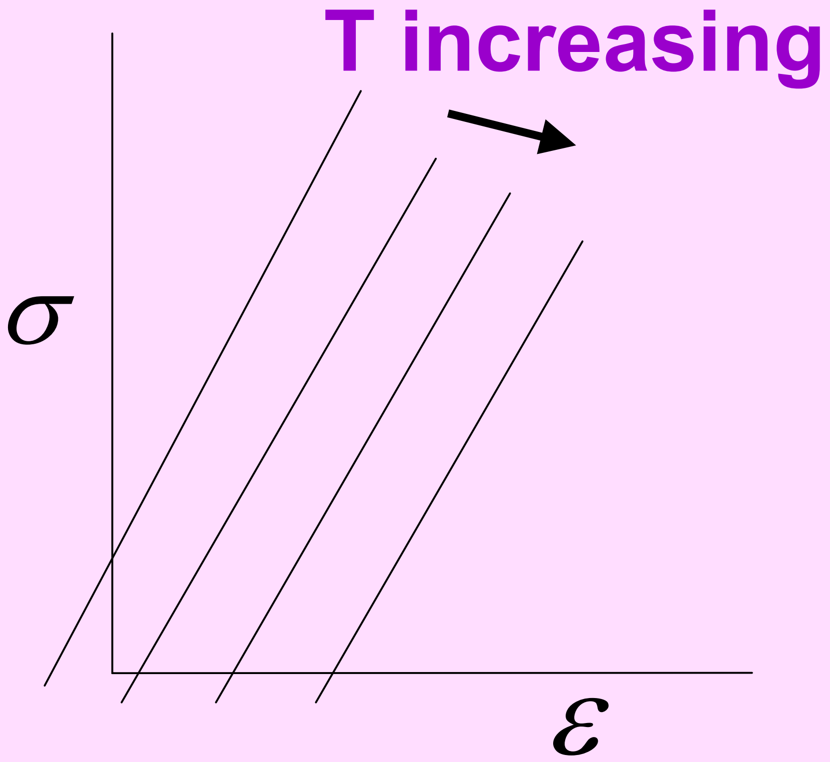
$$\Upsilon = \frac{L_O}{A}\left(\frac{\partial F^*}{\partial L}\right)_T = \frac{L_O}{A}\frac{\partial\left(F^*, T\right)}{\partial\left(L, T\right)}$$

clearly

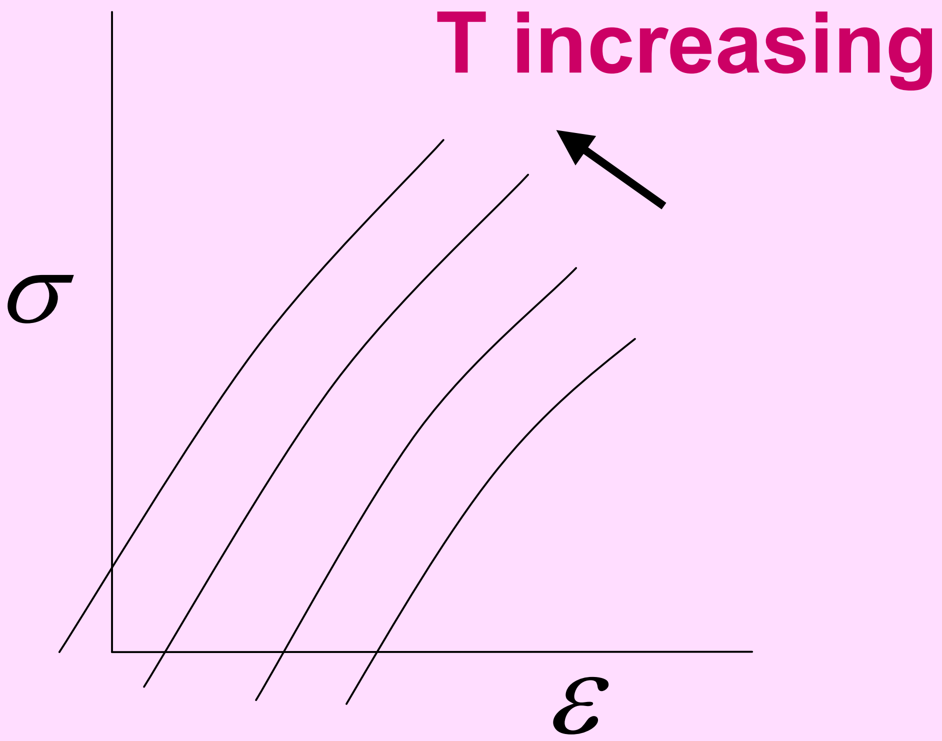
$$\boxed{\beta = -\Upsilon . \alpha}$$

SIMPLE ELASTIC SYSTEMS

Typ. eq. of state $\sigma = \sigma(\varepsilon, T)$



METAL (β –ve)



RUBBER (β +ve)

ANALYSIS OF SIMPLE PROCESSES

Rev. isothermal process

$$S = S(T, F^*)$$

$$dS = \left(\frac{\partial S}{\partial T}\right)_{F^*} dT + \left(\frac{\partial S}{\partial F^*}\right)_T dF^* = \frac{V_O C_{F^*}}{T} dT + \left(\frac{\partial L}{\partial T}\right)_{F^*} dF^*$$

$$= \frac{V_O C_{F^*}}{T} dT + V_O \alpha \, d\sigma$$

Since the process is isothermal

$$ds = \alpha \, d\sigma \quad where \; s = S / V_o$$

ANALYSIS OF SIMPLE PROCESSES

rev isothermal process

$\therefore$ q, Heat Transfer per unit volume is:

$$\int dq = \int T ds = \int T\alpha \, d\sigma$$

{assuming α const.}

$$= T.\alpha(\sigma_2 - \sigma_1)$$

Work done = $\int F^*.dL = \int V_0 \frac{\sigma}{Y} d\sigma$

$$= \frac{V_0}{2Y}\left(\sigma_2^2 - \sigma_1^2\right)$$

{assuming Y const.}

ANALYSIS OF SIMPLE PROCESSES

Rev. adiabatic process

$$dS = 0 = V_O C_{F^*} \frac{dT}{T} + V_O \alpha \, d\sigma$$

$$\therefore \left(\frac{\partial T}{\partial \sigma} \right)_s = - \frac{T.\alpha}{C_{F^*}}$$

If $\alpha, \; C_{F^*}$ **are assumed constant**

$$\Delta T = - \frac{T\alpha}{C_\sigma} . \Delta\sigma$$

Assuming $\Delta T << T$

End of Lecture

Lecture 3.6

Simple Elastic Systems…..contd

Solving a few problems

End of Lecture

Lecture 3.7

Simple Magnetic Systems

Simple Magnetic System

Diamagnetism, Para-magnetism, Ferromagnetism

Generalised Force : H*(magnetic field intensity)

Generalised Coordinate : M (magnetic moment)

M: usually defined / unit volume

Work done $dW = \mu_0 H.V.dM$

$$\mu_0 := 4\pi \; x \, 10^{-7} \; N / amp^2$$

is permitivity of free space

Simple Magnetic System

First Law $dQ = dU - \mu_0 V H^* dM$

Second Law $\mathrm{T}\, dS \geq dQ$

Basic Equation $dU = TdS + \mu_0 V H^* dM$

Other potentials $H = U - \mu_0 V H^* M\,;\;\; F = U - TS;\;\; G = H - TS$

Simple Magnetic System

Maxwell's Relations

$$dU = TdS + \mu_0 VH^* dM \Rightarrow \left(\frac{\partial T}{\partial M}\right)_s = \mu_0 V\left(\frac{\partial H^*}{\partial S}\right)_M$$

$$dH = TdS - \mu_0 VM \; dH^* \Rightarrow \left(\frac{\partial T}{\partial H^*}\right)_s = -\mu_0 V\left(\frac{\partial M}{\partial S}\right)_{H^*}$$

$$dF = -SdT + \mu_0 VH^* dM \Rightarrow \left(\frac{\partial S}{\partial M}\right)_T = -\mu_0 V\left(\frac{\partial H^*}{\partial T}\right)_M$$

$$dG = -SdT - \mu_0 VM \; dH^* \Rightarrow \left(\frac{\partial S}{\partial H^*}\right)_T = \mu_0 V\left(\frac{\partial M}{\partial T}\right)_{H^*}$$

Mnemonic aid ?

Simple Magnetic System

Measurable Properties

Heat Capacities

$$C_{H^*} = \left(\frac{\partial H}{\partial T} \right)_{H^*} = T \left(\frac{\partial S}{\partial T} \right)_{H^*}$$

$$C_M = \left(\frac{\partial U}{\partial T} \right)_M = T \left(\frac{\partial S}{\partial T} \right)_M$$

Simple Magnetic System

Equation of State

Magnetic Susceptibility $\chi = \frac{M}{H^*}$

Simple Equation of State : $\chi V = \frac{C}{T}$

C $\therefore$ Curie's Constant

More accurate : $\chi V = \frac{C}{T - T_0}$

Curie –Weiss equation

(T_o a constant)

Simple Magnetic System

Reversible Isothermal Process

$$S = S\left(T, H^*\right)$$

$\Rightarrow$ **At constant T,**

$$dS = \left(\frac{\partial S}{\partial H^*}\right)_T dH^*$$

Simple Magnetic System

Using Maxwell's relation,

$$dS = \mu_0 V . \left(\frac{\partial M}{\partial T} \right)_{H^*} . dH^*$$

If material obeys

Curie's Equation of State :

$$M = \frac{C}{T} . \frac{H^*}{V} \quad \Rightarrow \quad dS = -\frac{\mu_0 C}{T^2} H^* dH^*$$

SIMPLE MAGNETIC SYSTEMS

Reversible Isothermal…..contd.

Heat Transfer $Q = \int T dS$

$$= -\frac{\mu_0 C}{2T}\left(H_2^{*2} - H_1^{*2}\right)$$

Thus for $H_2^* > H_1^*$, $Q < 0$ **i.e. Heat is rejected during increase in magnetic field**

SIMPLE MAGNETIC SYSTEMS

Reversible Adiabatic Process

$$S(T, H^*) \Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_{H^*} dT + \left(\frac{\partial S}{\partial H^*}\right)_T dH^* = 0$$

Further $\left(\frac{\partial S}{\partial T}\right)_{H^*} = \frac{C_H^*}{T}$; **Substituting this we get**

$$\frac{dT}{T} C_{H^*} = -\left(\frac{\partial S}{\partial H^*}\right) dH^*$$

$$= -\mu_0 V \left(\frac{\partial M}{\partial T}\right)_{H^*} dH^*$$

SIMPLE MAGNETIC SYSTEMS

Reversible Adiabatic Process …..Contd.

If material obeys Curies' Law

$$M = \frac{C}{T}\frac{H^*}{V} \Rightarrow \left(\frac{\partial M}{\partial T}\right)_{H^*} = -\frac{C}{T^2}\frac{H^*}{V}$$

Substituting in earlier equation

$$\frac{dT}{T}C_{H^*} = -\mu_0 V \cdot \left(-\frac{C}{T^2} \cdot \frac{H^*}{V}\right) dH^*$$

$$TdT = \left(\frac{\mu_0 C}{C_H}\right) H^* dH^*$$

SIMPLE MAGNETIC SYSTEMS

Reversible Adiabatic Process …..Contd.

Integrating $$T_2^{\,2} - T_1^{\,2} = \frac{\mu_0 C}{C_{H^*}}\left(H_2^{*2} - H_1^{*2}\right)$$

Thus, if $$H_2 < H_1 \;;\; T_2 < T_1$$

i.e. we can reduce temperature by adiabatic demagnetisation right upto Curie Point.

Used in reaching very low temperatures

Solving a few problems

INTRODUCTION TO COMPLEX SYSTEMS

COMPLEX SYSTEMS

$\Rightarrow$ **More than one mode of work interaction**

Heat Transfer = Change in Int. Energy – Work input

$$TdS = dU - f_1 dx_1 - f_2 dx_2$$

COMPLEX SYSTEMS

Examples

$$TdS = dU + PdV - V_e dQ_e$$

Reversible Cell Liberating Gases

$$TdS = dU + PdV - \mu_0 VH^* dM$$

Magnetization of Compressible Substances

$$TdS = dU - F^* dL - E \; d \; \Pi$$

Piezo-electric System