Module 3

ANALYSIS OF SIMPLE COMPRESSIBLE SYSTEMS

Lecture 3.1

Inter-relationship among properties

Background

- **Simple Systems** : Those having only one mode of (non-thermal) interaction
- **State Postulate :** Two independent properties suffice to specify its state
- **Approach :** Combine the general laws of thermodynamics with specific equation of state and process description

BASIC GOVERNING EQUATIONS

Basic eq. $TdS \geq d\mathrm{E} - f_i{}'' dx_i{}'$

For a reversible process in a compressible fluid

$f_i{}'' = f_i{}' \equiv -P \qquad x_i{}' \equiv V$

$\therefore$ **Basic eq.** $TdS = dE + PdV$

BASIC GOVERNING EQUATIONS…………

$$E = \underbrace{Mg\,Z + \frac{MV^{2}}{2}}_{\text{Macroscopically specifiable}} + \underbrace{U}_{\text{Microscopic}}$$

Often, in analysis of compressible fluids changes in KE & PE are negligible

BASIC GOVERNING EQUATIONS…………

$$\therefore \quad dE \quad = \quad dU$$

$$\therefore \; TdS \quad = \; dU \quad + \; PdV$$

$$dU \quad = \quad TdS \quad - \quad PdV$$

{being relationship solely among the properties, this is valid for all processes, reversible or irreversible}

If process is irreversible

Tds ≠ Heat transfer

PdV ≠ Work done

BASIC GOVERNING EQUATIONS…………..

U = U(S,V)

$$dU = TdS - PdV$$

Fundamental Relation

S,V Canonical Variables

Other thermodynamic properties

H = U+ PV

F = U - TS

G = H - TS

BASIC GOVERNING EQUATIONS…………..

$$dU = TdS - PdV \qquad \boxed{U(S,V)}$$

$$dH = d(U + PV) = TdS + VdP \qquad \boxed{H(S,P)}$$

$$dF = d(U - TS) = -SdT - PdV \qquad \boxed{F(T,V)}$$

$$dG = d(H - TS) = -SdT + VdP \qquad \boxed{G(T,P)}$$

Fundamental Relations

<Measurable> THERMODYNAMIC PROPERTIES OF SIMPLE COMPRESSIBLE FLUIDS

Coeff. of thermal expansion $$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

Isothermal Compr. $$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Isentropic Compr. $$\kappa_s = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S$$

<Measurable> THERMODYNAMIC PROPERTIES OF SIMPLE COMPRESSIBLE FLUIDS

Constant Pressure Heat capacity

$$C_P = \left(\frac{\partial H}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P$$

Constant Volume Heat Capacity

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

RELATIONSHIP BETWEEN THERMODYNAMIC PROPERTIES

Since U, H, F, G are all exact differentials, we can write these as functions of two other properties, in general Z = f (x,y) .

This gives :

$$\left\{ \begin{array}{l} dz = Mdx + Ndy \\ \text{and} \\ \dfrac{\partial M}{\partial Y} = \dfrac{\partial N}{\partial X} = \dfrac{\partial^2 Z}{\partial x \partial \gamma} \end{array} \right.$$

RELATIONSHIP BETWEEN THERMODYNAMIC PROPERTIES

$$\left.\begin{aligned}
\left(\frac{\partial T}{\partial V}\right)_s &= -\left(\frac{\partial P}{\partial S}\right)_v \\
\left(\frac{\partial T}{\partial P}\right)_s &= \left(\frac{\partial V}{\partial S}\right)_P \\
\left(\frac{\partial S}{\partial V}\right)_T &= \left(\frac{\partial P}{\partial T}\right)_v \\
\left(\frac{\partial S}{\partial P}\right)_T &= -\left(\frac{\partial V}{\partial T}\right)_P
\end{aligned}\right\}$$

Maxwell's Relations

JACOBIAN TRANSFORMATION

A method of manipulation of thermodynamic derivatives, which is of much more general nature, is based on the mathematical property of jacobians.

JACOBIAN TRANSFORMATION

Definition : if u, v,……..w are functions of x,y, ……….z, the jacobian is defined as

$$\frac{\partial\left(u,v,\ldots w\right)}{\partial\;\left(x,y,\ldots z\right)} \Longrightarrow \begin{vmatrix} \partial u/\partial x \quad \partial u/\partial y-----\partial u/\partial z \\ \partial v/\partial x \quad \partial v/\partial x-----\partial v/\partial z \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \partial w/\partial x---------\partial w/\partial z \end{vmatrix}$$

JACOBIAN TRANSFORMATION

Useful properties :

1. $(\partial u/\partial x)y,..z \qquad = \partial(u,y,..z)/\partial(x,y,..z)$

2. $\partial(u,v,..w)/\partial(x,y,..z) \qquad = -\partial(v,u,..w)/\partial(x,y,..z)$

3. $\partial(u,v,...w)/\partial(x,y,..z) \qquad = \dfrac{\partial(u,v,...w)}{\partial(r,s,..t)} * \dfrac{\partial(r,s,..t)}{\partial(x,y,..z)}$

4. $\partial(u,v,...w)/\partial(x,y,..z) \qquad = 1/\ \dfrac{\partial(x,y,...z)}{\partial(u,v,...w)}$

JACOBIAN TRANSFORMATION

EXAMPLE

$$C_P = T(\partial S / \partial T)_P = T\ \partial(S,P) / \partial(T,P)$$

$$C_V = T(\partial S / \partial T)_V = T\ \partial(S,V) / \partial(T,V)$$

$$\frac{C_P}{C_V} = \frac{\partial(S,P)}{\partial(S,V)} * \frac{\partial(T,V)}{\partial(T,P)} = \frac{1}{(\partial V / \partial P)_s} * (\partial V / \partial P)T$$

$$= \kappa_T / \kappa_S = \gamma$$

JACOBIAN TRANSFORMATION

$$C_p - C_v = ?$$

$$C_P = \mathrm{T}\left(\frac{\partial S}{\partial \mathrm{T}}\right)_{\mathrm{P}} \qquad C_V = \mathrm{T}\left(\frac{\partial S}{\partial \mathrm{T}}\right)_{v}$$

$$C_P - C_V = \mathrm{T}.\left\{\left(\frac{\partial S}{\partial \mathrm{T}}\right)_{\mathrm{P}} - \left(\frac{\partial S}{\partial \mathrm{T}}\right)_{v}\right\}$$

JACOBIAN TRANSFORMATION

Further since $S = S\left(\mathrm{T}, V\right)$

$$dS = \left(\frac{\partial S}{\partial \mathrm{T}}\right)_{v} d\mathrm{T} + \left(\frac{\partial S}{\partial V}\right)_{\mathrm{T}} dV$$

$$\left(\frac{\partial S}{\partial \mathrm{T}}\right)_{\mathrm{P}} = \left(\frac{\partial S}{\partial \mathrm{T}}\right)_{v} + \left(\frac{\partial S}{\partial V}\right)_{\mathrm{T}} \left(\frac{\partial V}{\partial \mathrm{T}}\right)_{\mathrm{P}}$$

JACOBIAN TRANSFORMATION

$$\therefore C_P - C_V = \mathrm{T}.\left(\frac{\partial S}{\partial V}\right)_{\mathrm{T}} . \left(\frac{\partial V}{\partial \mathrm{T}}\right)_P$$

$$\Big\downarrow \text{ -------}$$ **Maxwell's Rel.**

$$= \mathrm{T}.\left(\frac{\partial \mathrm{P}}{\partial \mathrm{T}}\right)_V . \left(\frac{\partial V}{\partial \mathrm{T}}\right)_P = \mathrm{T}.\left(\frac{\partial \mathrm{P}}{\partial \mathrm{T}}\right)_\nu .\beta V$$

$$= \mathrm{T}.\left\{\frac{\partial\left(P, V\right)}{\partial\left(\mathrm{T}, V\right)} . \frac{\partial\left(\mathrm{T}, \mathrm{P}\right)}{\partial\left(\mathrm{T}, \mathrm{P}\right)}\right\} \beta V$$

JACOBIAN TRANSFORMATION

$$\boxed{C_P - C_V} = \mathrm{T}\beta V\,.\left\{-\frac{\partial(\mathrm{P},V)}{\partial(\mathrm{P},\mathrm{T})}.\right\}\left\{\frac{\partial(\mathrm{T},\mathrm{P})}{\partial(\mathrm{T},V)}\right\}$$

$$= \mathrm{T}\beta V\,.\left\{-\left(\frac{\partial V}{\partial \mathrm{T}}\right)_{\mathrm{P}}\Big/\left(\frac{\partial V}{\partial \mathrm{P}}\right)_{\mathrm{T}}\right\}$$

$$= \mathrm{T}\beta V \qquad \frac{\beta\,.V}{\kappa_T V} \boxed{= \frac{\mathrm{T}\beta^2 V}{\kappa_T\,.}}$$

End of Lecture

Lecture 3.2

Analyzing Typical processes-Equation of state

ISOTHERMAL COMPRESSION

$s = s(T, P) = s(P)$ **for const T**

$\therefore$ **For isothermal process**

$$ds = \left(\frac{\partial s}{\partial P}\right)_T d\mathrm{P} = -\left(\frac{\partial v}{\partial T}\right)_\rho d\mathrm{P} \quad \{Maxwell\; rel\}$$

$$= -\beta v\, dP$$

ISOTHERMAL COMPRESSION

Heat Transfer $= \mathrm{T}\,ds$

$$= -\beta\nu\ \mathrm{T}\ d\,\mathrm{P}$$

For finite pr. Change $Q = -\mathrm{T}\int_{\mathrm{P}_i}^{\mathrm{P}_f} \beta.\nu.dP$

ISOTHERMAL COMPRESSION

Energy Change

$$dU = \mathrm{T}ds - \mathrm{P}d\nu$$

$$= \mathrm{T}ds - \mathrm{P}\left\{\left(\frac{\partial \nu}{\partial \mathrm{P}}\right)_{\mathrm{T}} d\mathrm{P} + \left(\frac{\partial \nu}{\partial \mathrm{T}}\right)_{\mathrm{P}} d\mathrm{T}\right\}$$

ISOTHERMAL COMPRESSION

$$= \mathrm{T}ds - \mathrm{P}\left(\frac{\partial v}{\partial \mathrm{P}}\right)_{\mathrm{T}} d\mathrm{P} \qquad \{\because \mathrm{T} = const.$$

$$= -\beta v \mathrm{T} d\mathrm{P} + v\kappa_T \ \mathrm{P} \ d\mathrm{P}$$

$$\therefore \left(dU\right)_{\mathrm{T}} = \left(-\ \beta v \mathrm{T} + \kappa_T v \ \mathrm{P}\right) d\mathrm{P}$$

REV. ADIABATIC COMPRESSION
≡ S constant

T= T(P,S)

$$d\mathrm{T} = \left(\frac{\partial \mathrm{T}}{\partial P}\right)_s dP$$

$$\left(\frac{\partial \mathrm{T}}{\partial P}\right)_s = \frac{\partial(\mathrm{T},s)}{\partial(P,s)} = \frac{\partial(\mathrm{T},s)}{\partial(P,\mathrm{T})} \cdot \frac{\partial(P,\mathrm{T})}{\partial(P,s)}$$

REV. ADIABATIC COMPRESSION
≡ S constant

$$= -\left(\frac{\partial s}{\partial P}\right)_{\mathrm{T}} \bigg/ \left(\frac{\partial s}{\partial \mathrm{T}}\right)_{P}$$

Maxwell's relation

$$= \left(\frac{\partial \upsilon}{\partial \mathrm{T}}\right)_{P} \bigg/ \left(\frac{\partial s}{\partial \mathrm{T}}\right)_{P}$$

$$= \beta v \;/ (C_P / \mathrm{T}) = \frac{\mathrm{T}\,\beta v}{C_P}$$

Hence

$$\boxed{d\mathrm{T} = \frac{\mathrm{T}\,\beta v}{C_P}\, dP}$$

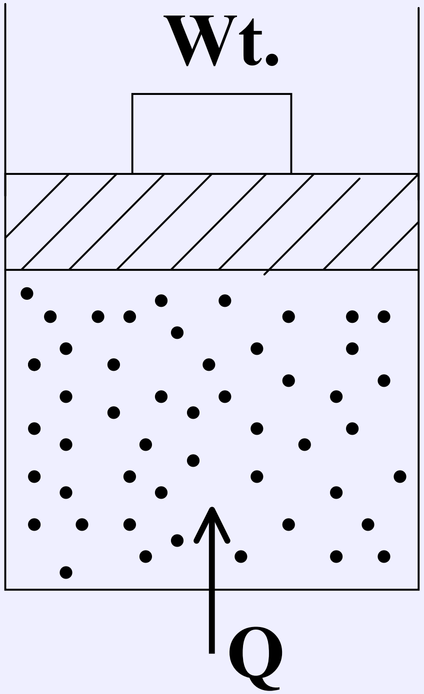
Need to find properties of the fluid like

$\beta_T\ \beta_s\ \kappa_T\ \kappa_s$

EQUATIONS OF STATE

EQUATION OF STATE

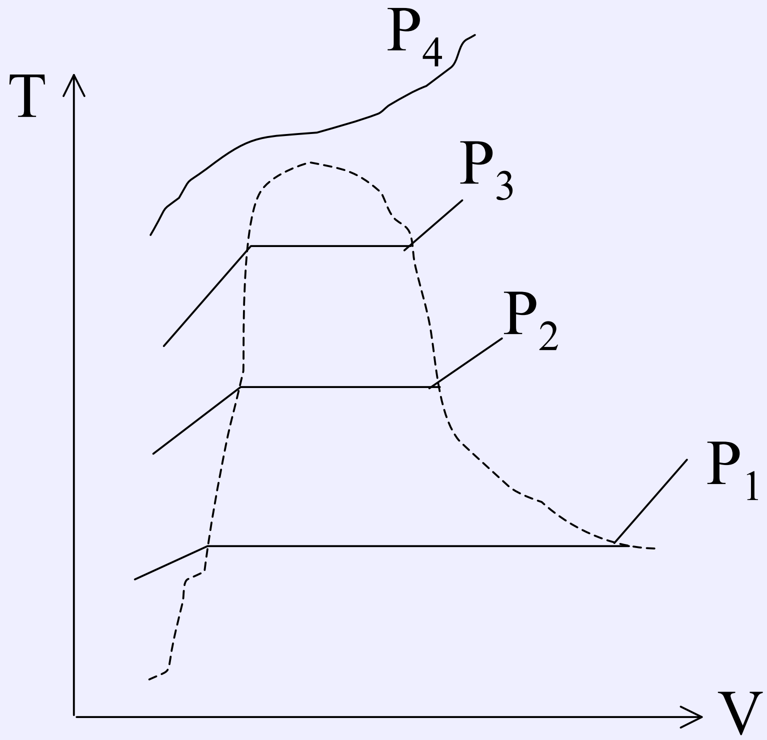
Gathering $P-v-\mathrm{T}$ Data



Expts. at const. P

EQUATION OF STATE

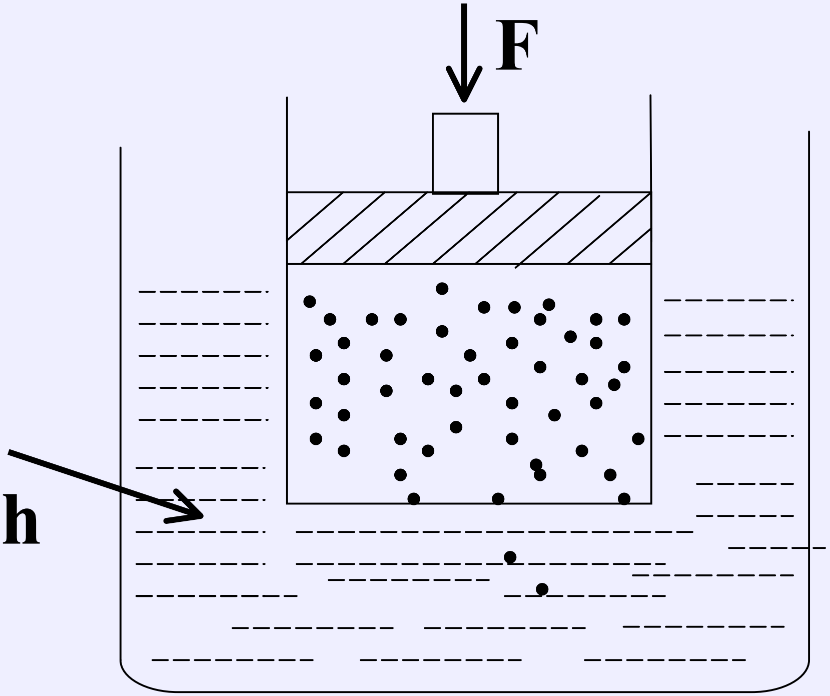
Gathering $P-\nu-\mathrm{T}$ Data



$$\mathrm{P}_4 > \mathrm{P}_3 > \mathrm{P}_2 > \mathrm{P}_1$$

EQUATION OF STATE

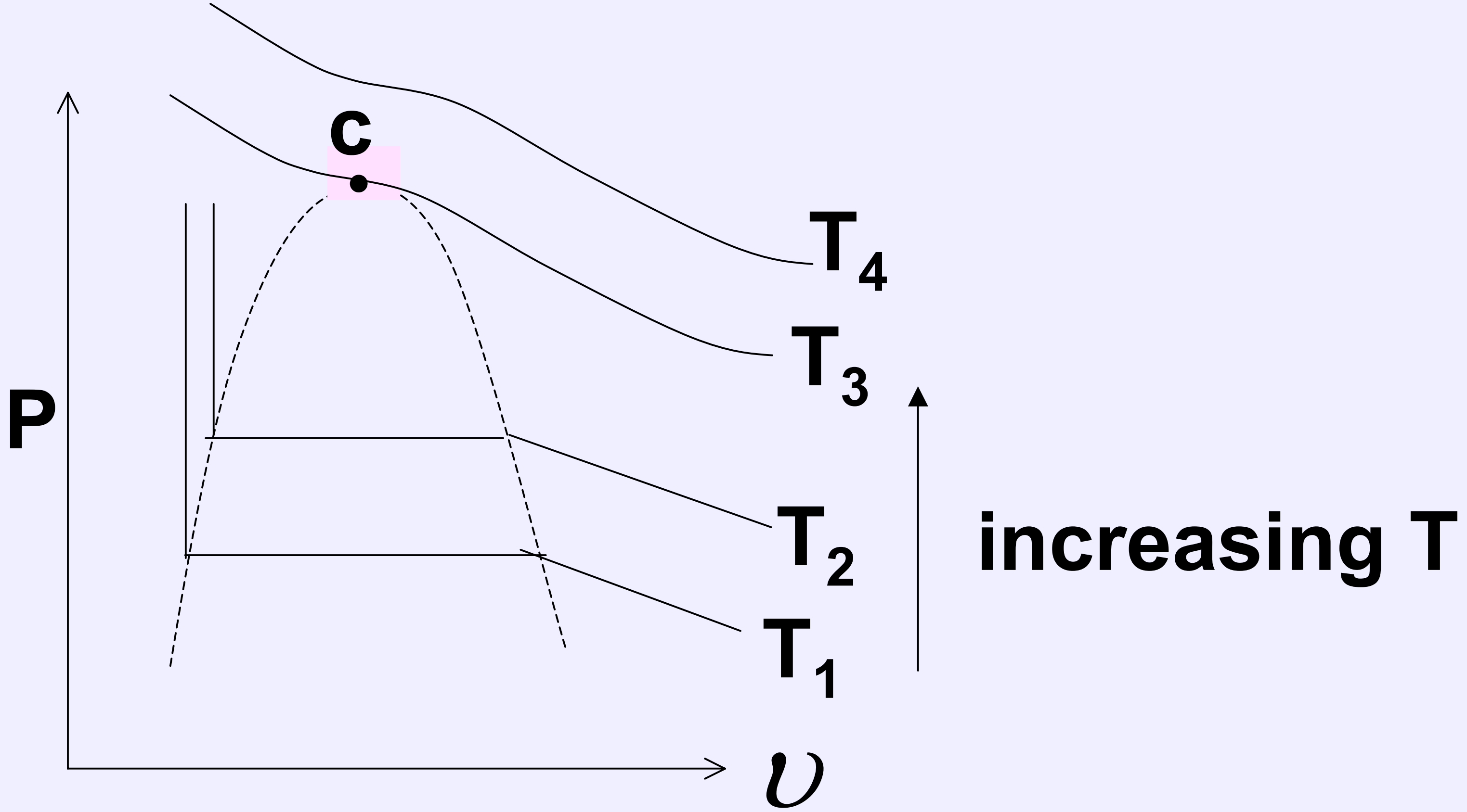
Gathering $P-\nu-T$ Data



Thermo-static bath

Expts. at const. T

REPRESENTATION OF P-V-T DATA – EQ. OF STATE



REPRESENTATION OF P-V-T DATA – EQ. OF STATE

- No single eq. is able to accurately predict P-V-T data of all known substances.
- Simplest IDEAL – GAS Relationship

$$\boxed{\frac{p.\upsilon}{T} = const. \quad = R}$$

the Gas Const

REPRESENTATION OF P-V-T DATA – EQ. OF STATE

υ→vol / k mole ;

R→ Univ Gas Const. = 8314.3 $\dfrac{\text{J}}{\text{kmol K}}$

deduced from kinetic theory on the basis of simplifying assumptions

(i) Vol. occupied by gas mols is negligible

(ii) Intermolecular forces are negligible

REPRESENTATION OF P-V-T DATA – EQ. OF STATE

VANDER WAAL’S EQ. OF STATE

$$p = \frac{RT}{\upsilon - b} - \frac{a}{\upsilon^{2}}$$

or

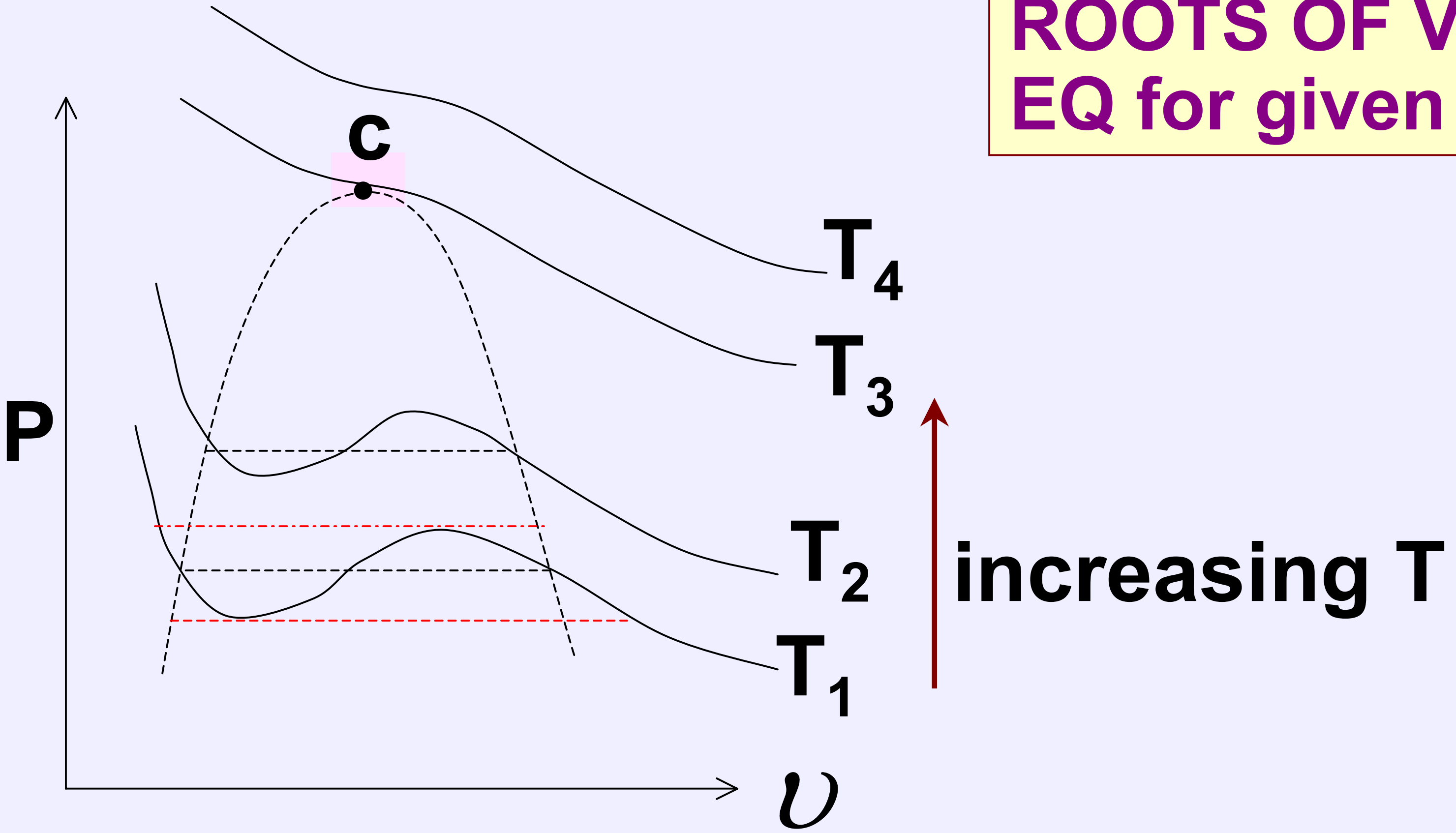
$$\left(p + \frac{a}{V^{2}}\right)\left(V - b\right) = RT$$

At large $V\ ;\ V \rangle\rangle\ b;\quad \frac{a}{V^{2}} \langle\langle\ p$

& V.W. eq. → Ideal Gas Law

REPRESENTATION OF P-V-T DATA – EQ. OF STATE

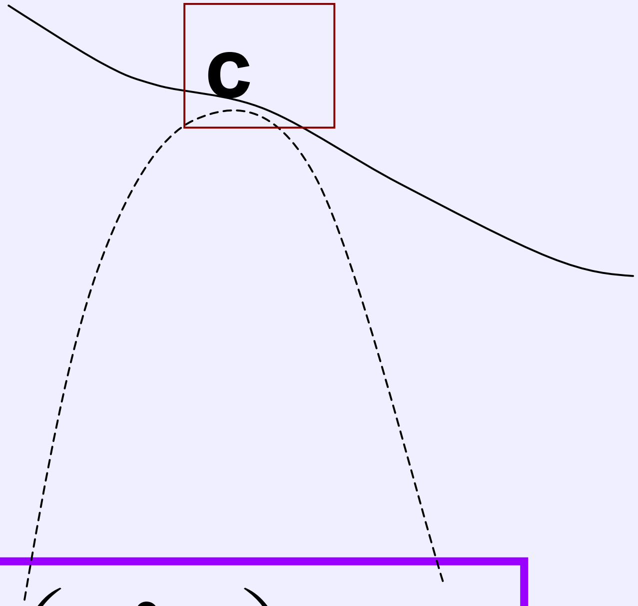
ROOTS OF V – W EQ for given P, T



REPRESENTATION OF P-V-T DATA – EQ. OF STATE

Determining constants a and b

CRITICAL ISOTHERM : C is a point of inflexion



$$\therefore \left(\frac{\partial p}{\partial \upsilon}\right)_{T_c} = 0 \quad ; \quad \left(\frac{\partial^2 p}{\partial \upsilon^2}\right)_{T_c} = 0$$

REPRESENTATION OF P-V-T DATA – EQ. OF STATE

Now from V-W EQ

$$\left(\frac{\partial p}{\partial V}\right)_{T_c} = -\frac{RT_c}{\left(V_c - b\right)^2} + \frac{2a}{\upsilon_C{}^3} = 0$$

$$\left(\frac{\partial^2 p}{\partial V^2}\right)_{T_c} = -\frac{2RT_c}{\left(V_c - b\right)^3} + \frac{6a}{\upsilon_c{}^4} = 0$$

REPRESENTATION OF P-V-T DATA – EQ. OF STATE

Solving these two equations along with V-W eq. at T_C gives

$$p_c = \frac{a}{27b^2} \qquad T_c = \frac{8a}{27Rb} \qquad \upsilon_c = 3b$$

OR

$$a = \frac{27}{64}\frac{(RT_c)^2}{P_c} \qquad b = \frac{RT_c}{8P_c}$$

REPRESENTATION OF P-V-T DATA – EQ. OF STATE

NB : We don't use $b = \frac{\upsilon_c}{3}$

since measurements of υ_C are usually less reliable than those of p_C & T_{C}.

$$\left.\begin{array}{l}\text{Shortcoming}\\ \text{V-W EQ.}\end{array}\right\}$$ From above $\frac{p_c \upsilon_c}{RT_c} = 0.375$

actual values are much smaller

OTHER EMPIRICAL EQS. OF STATE

REDLICH – KWONG

$$P = \frac{RT}{v-b} - \frac{a}{T^{1/2}v(v+b)}$$

Using conditions of criticality

$$a = .42748 \;\; R^2 T_c^{2.5} / P_c$$

$$b = .08664 \;\; RT_c / P_c = .2599 \;\; \upsilon_c$$

$$Z_c = \frac{P_c V_c}{RT_c} = \frac{1}{3}$$

OTHER EMPIRICAL EQS. OF STATE

Beattie – Bridgeman - 5 constants

Benedict – Webb-Rubin - 8 constants

Martin – Hou - 9 constants

Virial Eq. of state

Generalised Compressibility Charts

$$\boxed{Z = \frac{P\upsilon}{RT} = f\left(T, P\right)}$$

Basis of Generalised Charts

Principle of Corresponding States :

There exists a unique relationship between the reduced properties P_R, V_R, T_R for all fluids.

$$P_R = \frac{P}{P_c} \qquad T_R = \frac{T}{T_c} \qquad V_R = \frac{V}{V_c}$$

$$Z = \frac{P\upsilon}{RT} = f\left(T_r , P_r\right)$$

Vander – Waal

Unique relationship

Gen. Charts & Vander-Waal's Equation

V.W eq. :

$$\left\{p+\frac{\left(3p_cV_c{}^2\right)}{\upsilon^2}\right\}\left\{V-\frac{V_c}{3}\right\}=RT$$

or

$$\left\{\frac{P}{P_c}+3\left(\frac{V_c}{V}\right)^2\right\}\left\{\frac{V}{V_c}-\frac{1}{3}\right\}=\frac{RT}{P_cV_c}=\frac{8}{3}\frac{T}{T_c}$$

$$\left\{\because V_c=3b=\frac{3RT_c}{8p_c}\right.$$

or

$$\left(P_r+\frac{3}{V_r{}^2}\right)\left(3V_R-1\right)=8T_r$$

← **Unique relationship**

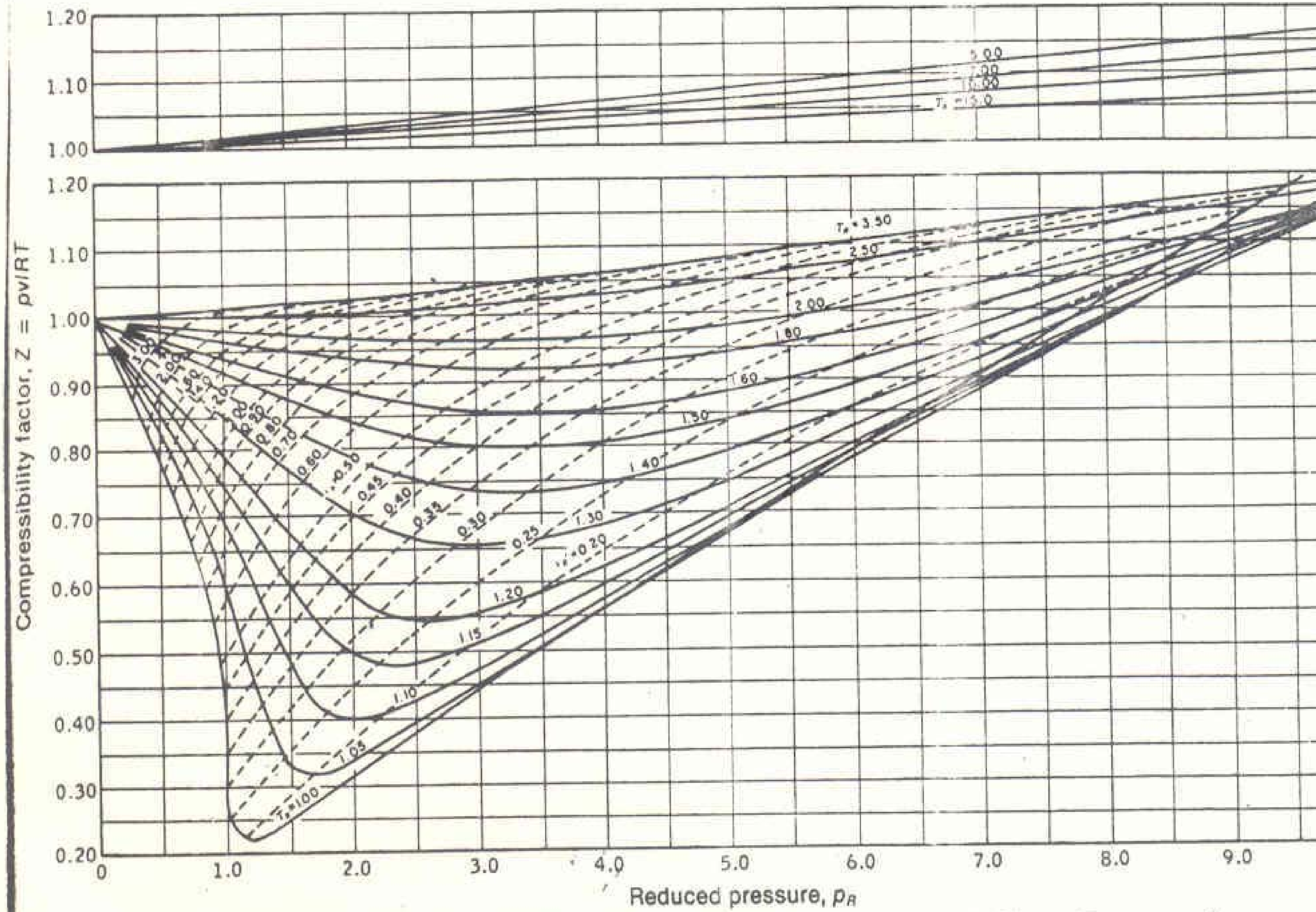


Figure A-2 Generalized compressibility chart, $p_R \leq 10.0$. *Source:* E. F. Obert, *Concepts of Thermodynamics*, McGraw-Hill, New York, 1960.

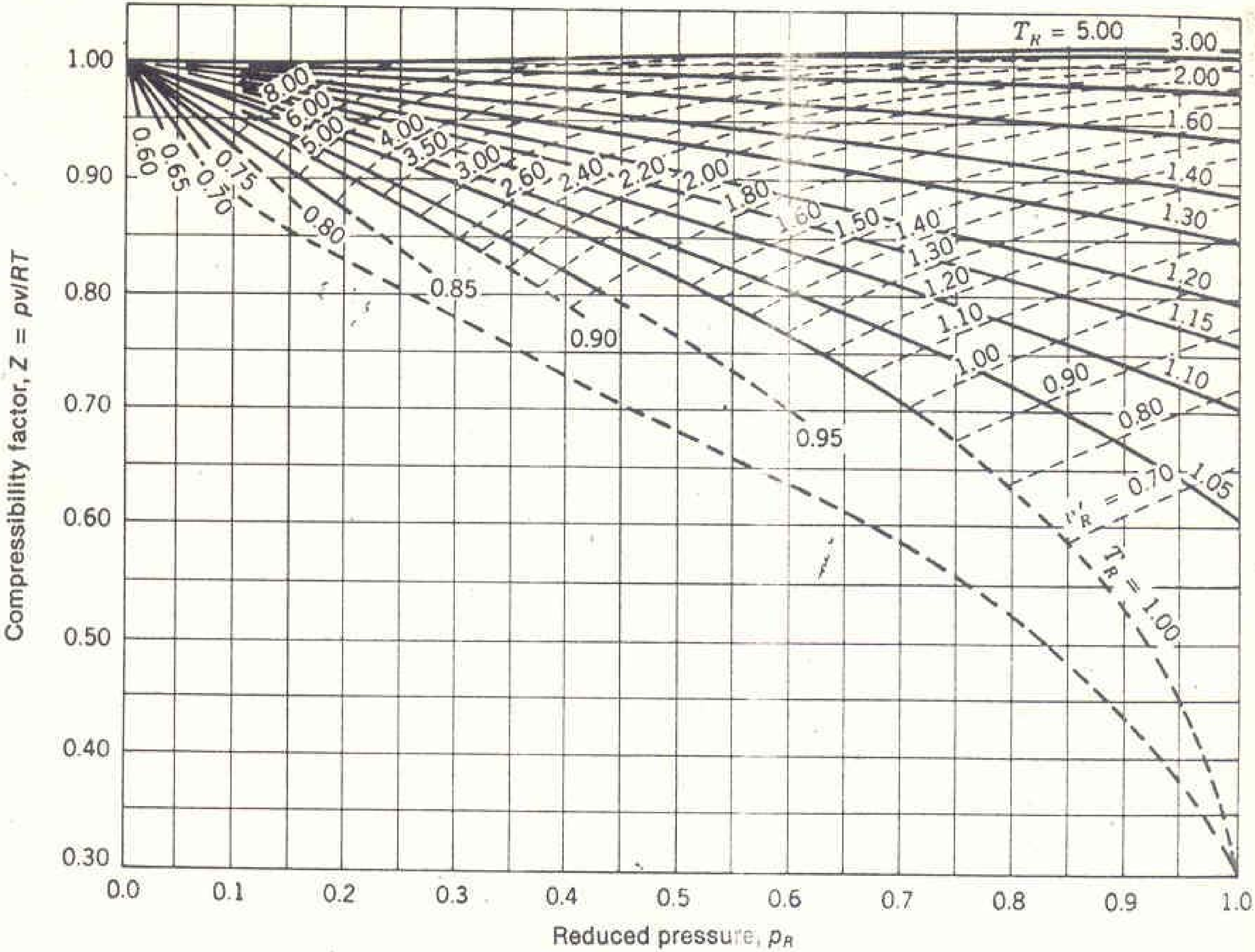


Figure A-1 Generalized compressibility chart, $p_R \leq 1.0$. *Source:* E. F. Obert, *Concepts of Thermodynamics*, McGraw-Hill, New York, 1960.

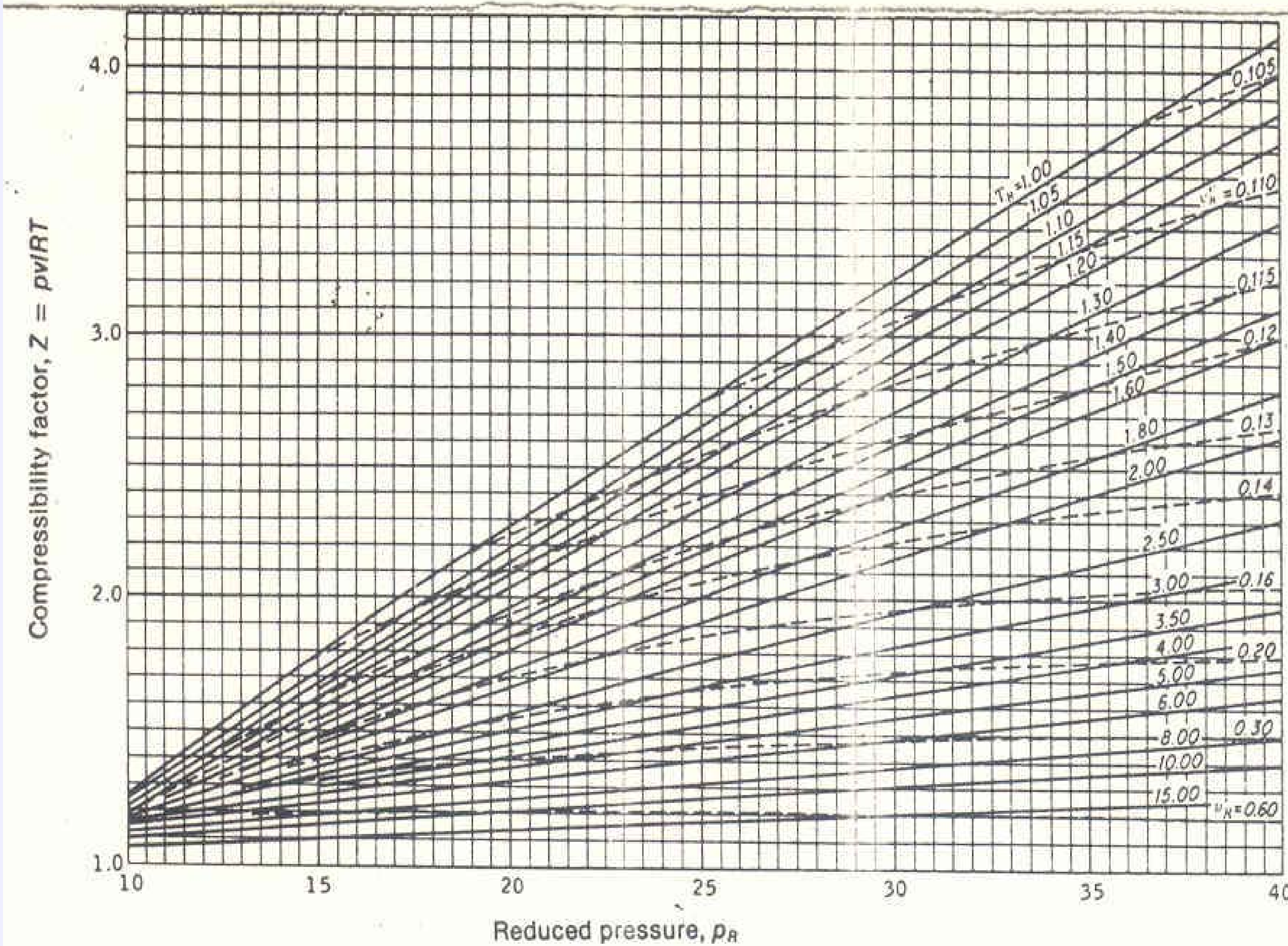


Figure A-3 Generalized compressibility chart, $10 \leq p_R \leq 40$. *Source:* E. F. Obert, *Concepts of Thermodynamics*, McGraw-Hill, New York, 1960.

End of Lecture

Lecture 3.3

Thermodynamic property tables and charts

Recap…..

- Basic governing equations
- Measurable properties of compressible fluids
    - interrelationship among properties
- Analyzing typical processes
- Equations of state
    - Generalised compressibility charts

Establishing 'Derived' Thermodynamic Properties

Exptl. Data needed : p-V-T + C_p, C_V data

ENTROPY $s = s(T, P)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_P dT + \left(\frac{\partial s}{\partial P}\right)_T dP$$

$$\therefore\ Tds = C_p dT + T\left(\frac{\partial s}{\partial P}\right)_T dP$$

Establishing ‘Derived’ Thermodynamic Properties of Simple Compressible Substances

$$Tds = C_p dT + T\left\{-\left(\frac{\partial v}{\partial T}\right)_P\right\}dP$$

$$\therefore ds = \frac{C_p}{T}dT - \left(\frac{\partial v}{\partial T}\right)_P dP$$

ENTHALPY

From above : $Tds = C_p dT - T\left(\frac{\partial \upsilon}{\partial T}\right)_P dP$

$$= dh - vdP$$

$$\therefore dh = C_p dT + \left[v - T\left(\frac{\partial \upsilon}{\partial T}\right)_P\right] dP$$

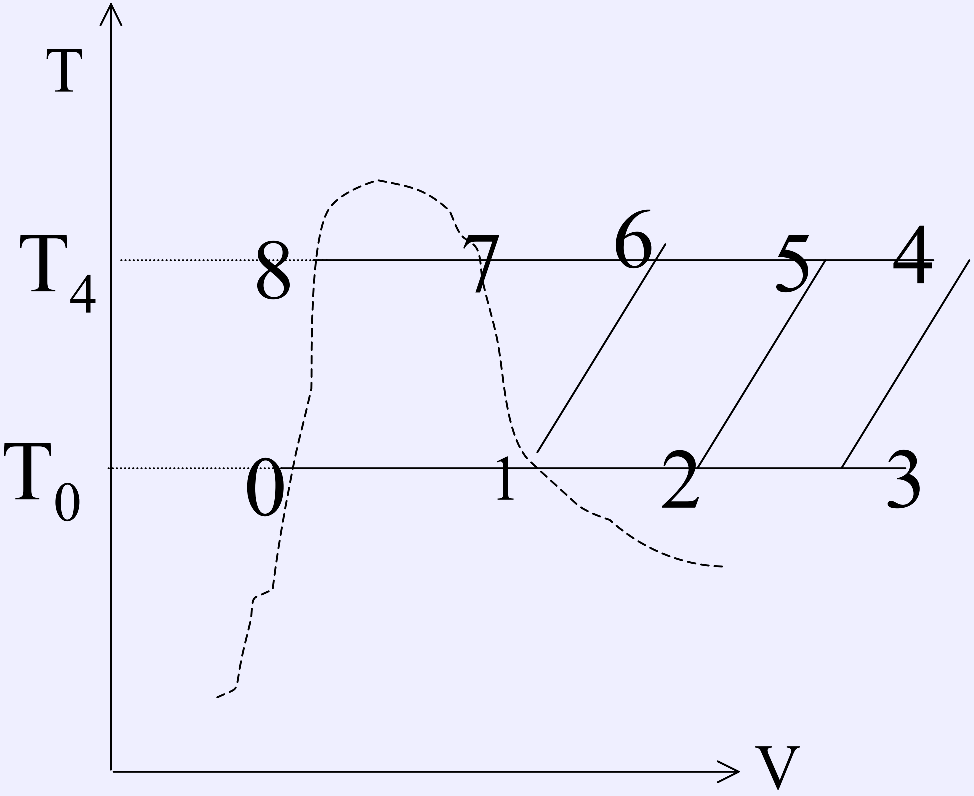
Establishing 'Derived' Thermodynamic Properties

Reference Point 0

Process 0-1

$$\left(\frac{\partial s}{\partial v}\right)_T = \left[\frac{\partial p}{\partial T}\right]_v$$

$$= \left[\frac{dp}{dT}\right]_{Sat}$$



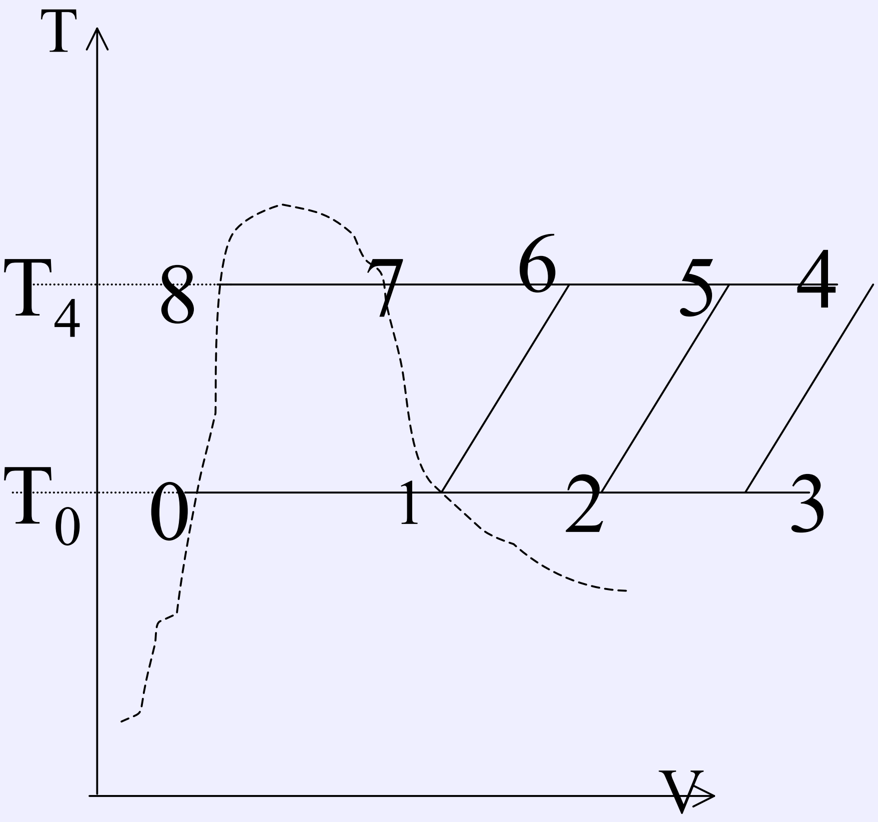
Establishing 'Derived' Thermodynamic Properties

Process 0-1

$$s_1 - s_0 = (v_1 - v_0)\left(\frac{dp}{dT}\right)_{Sa}$$

$$from \quad dh = Tds$$

$$h_1 - h_0 = T(s_1 - s_0)$$

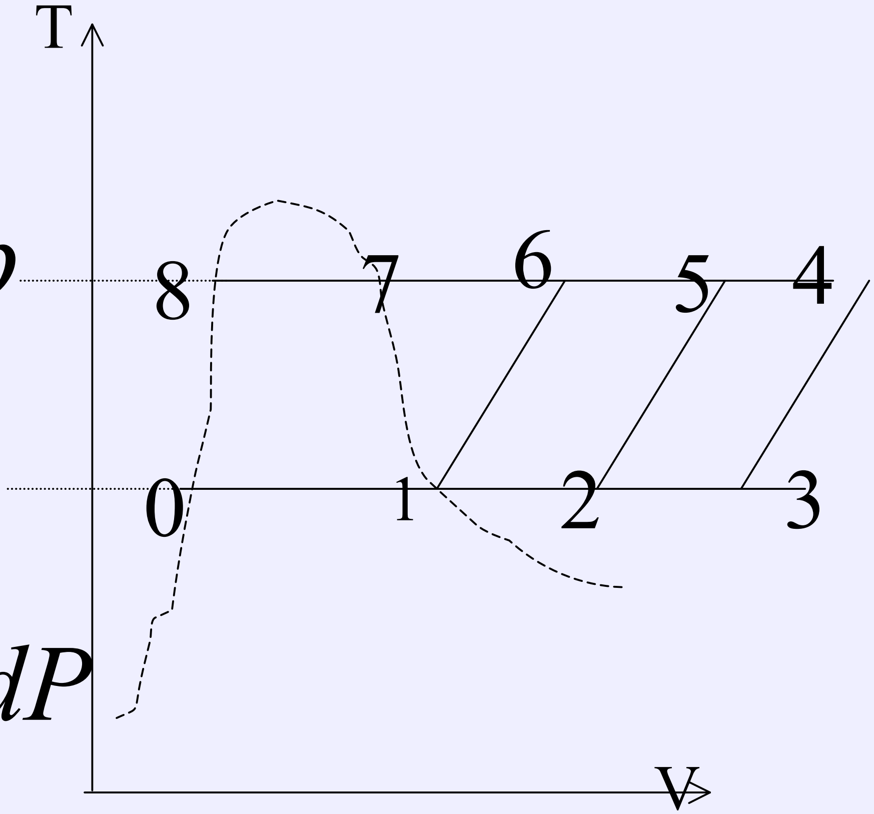


Establishing 'Derived' Thermodynamic Properties

Process 1-2

$$s_2 - s_1 = \int_{p_1}^{p_2} -\left(\frac{\partial v}{\partial T}\right)_P dp$$

$$h_2 - h_1 = \int_{P_1}^{P_2} \left[v - T\left(\frac{\partial v}{\partial T}\right)_P\right] dP$$

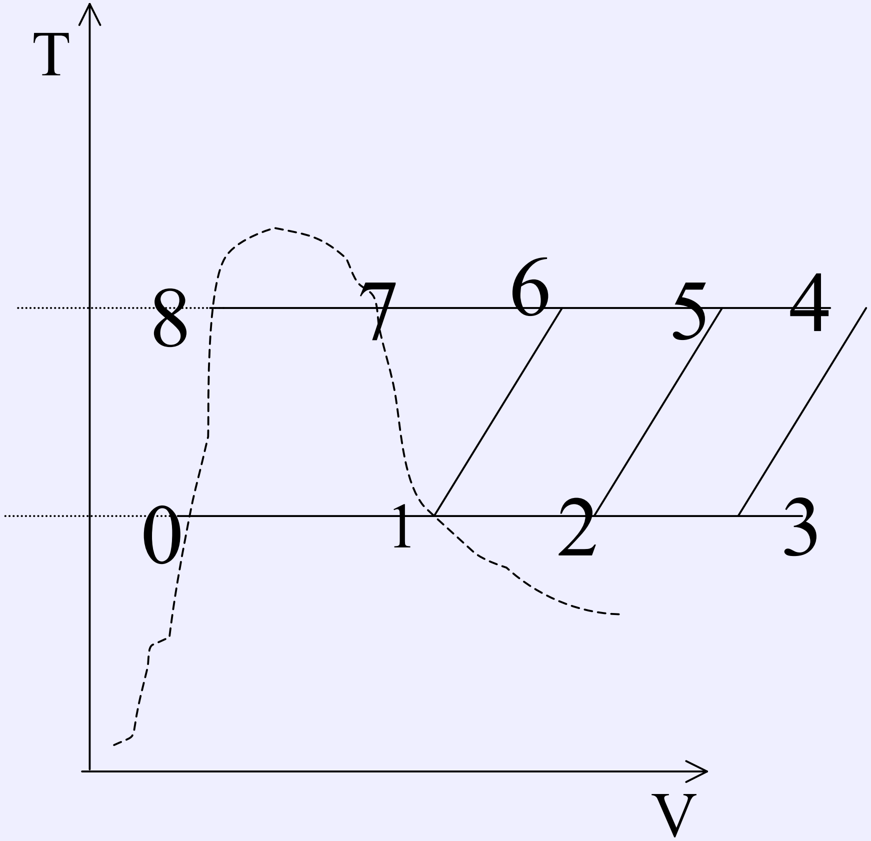


Establishing 'Derived' Thermodynamic Properties

Process 3-4 along a low pressure isobar

$$s_4 - s_3 = \int_3^4 \left(\frac{C_{po}}{T} \right)_P dT$$

$$h_4 - h_3 = \int_3^4 \left[C_{P0} \right] dT$$



Find prop at 5,6.7…. Starting from 4

Establishing ‘Derived’ Thermodynamic Properties

→Integrating various terms

Need for ‘proper’ eq. of state

$$V = f(P,T)$$

If it is $p = f(V,T)$ we have to transform the integrand to facilitate integration.

……….Transforming integrand

$$\int_{p_1}^{p_2} \upsilon dp = \int_{p_1}^{p_2} \left[d(p\upsilon) - p d\upsilon \right]$$

$$= (p_2 v_2 - \ p_1 v_1) - \left[\int_{\upsilon_a}^{\upsilon_2} p d\upsilon \right]_T$$

and

$$\left(\partial\upsilon / \partial T\right)_P = -\left(\partial\upsilon / \partial p\right)_T \cdot \left(\partial p / \partial T\right)_v$$

Property Tables &Charts

- Property values thus calculated have been tabulated for various regions
- properties in liquid vapour region

$$\pi = \pi_l \,(1-x) + \pi_v \,x$$

* Need for interpolation

* Linear interpolation for small increments

* More accuracy : logarithmic for entropy and proportional to (1/p) for sp. vol. ?

End of Lecture

Lecture 3.4

Generalised Charts for Other properties--Applications

GENERALISED CHARTS FOR ENTROPY & ENTHALPY

Enthalpy Plots of $\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c}$ vs p_r with T_r as parameter ?

Entropy $\frac{\bar{s}^* - \bar{s}}{R}$

$$\therefore ds = \frac{C_p}{T} dT - \left(\frac{\partial v}{\partial T} \right)_P dP$$

$$\therefore dh = C_p dT + \left[v - T \left(\frac{\partial \upsilon}{\partial T} \right)_P \right] dP$$

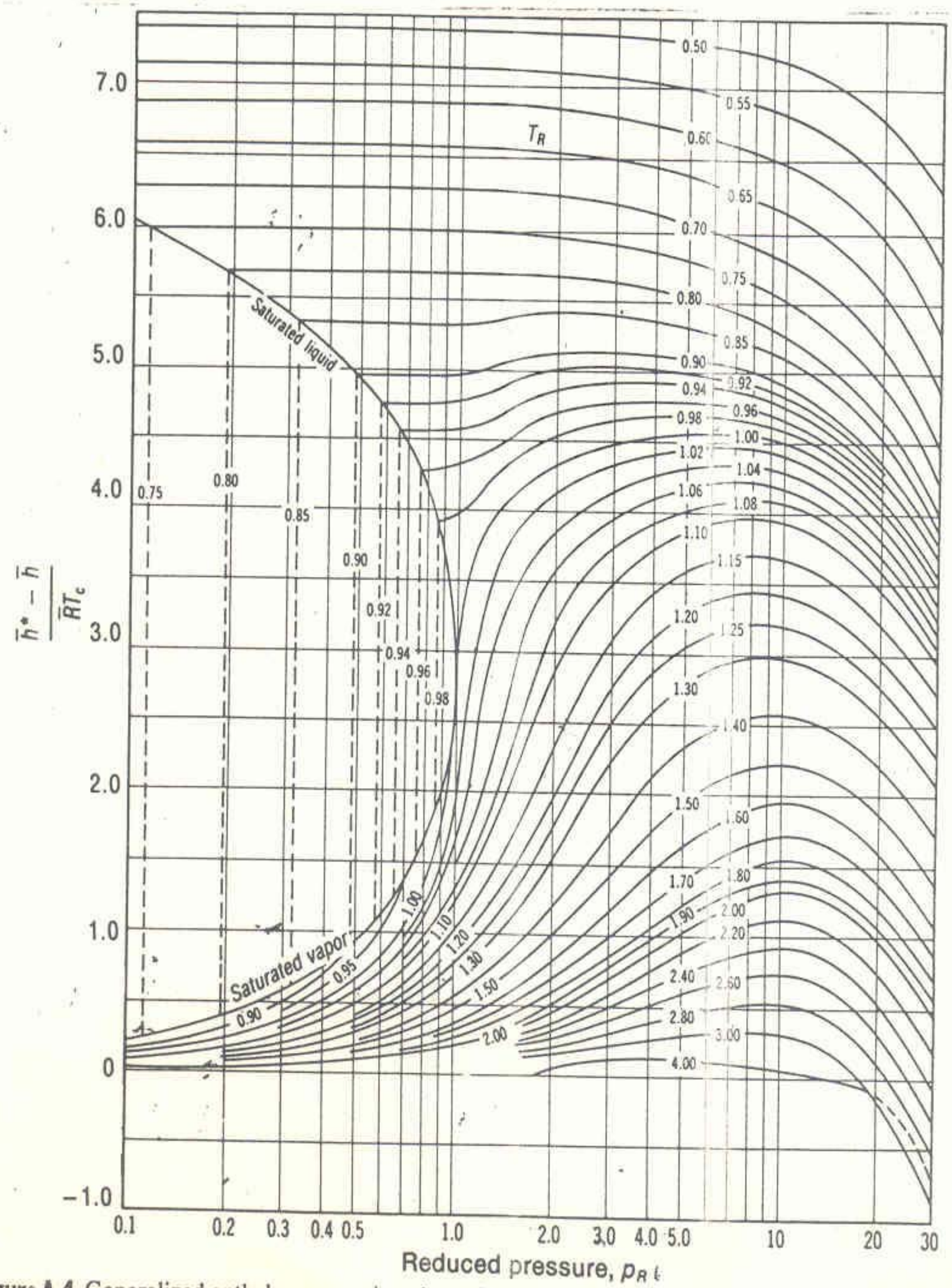


Figure A-4 Generalized enthalpy correction chart. *Source:* Adapted from G. J. Van Wylen and R. E. Sonntag, *Fundamentals of Classical Thermodynamics,* 3rd. ed., English/SI, Wiley, New York, 1986.

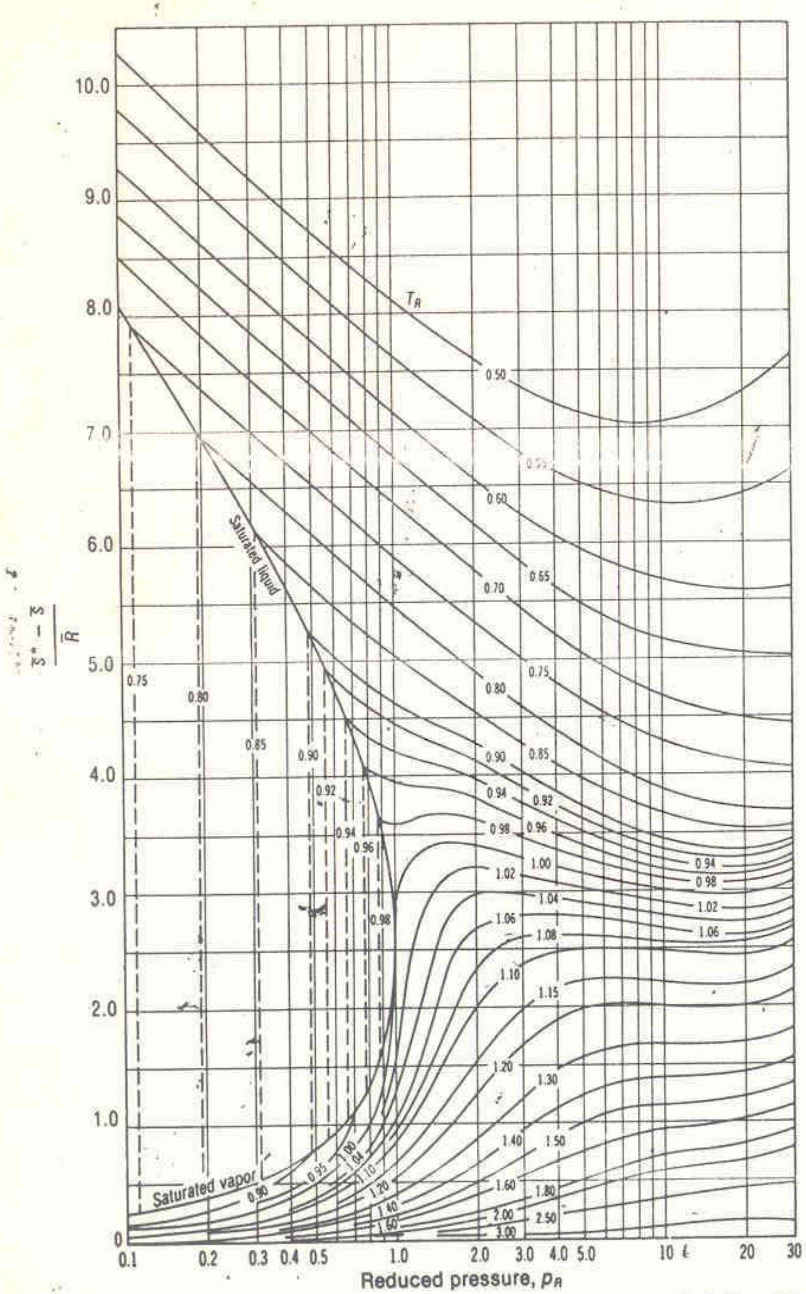


Figure A.5 Generalized entropy correction chart. *Source:* Adapted from G. J. Van Wylen and R. E. Sonntag, *Fundamentals of Classical Thermodynamics,* 3rd ed., English/SI, Wiley, New York, 1986

GENERALISED CHARTS FOR ENTROPY & ENTHALPY

To find enthalpy diff. between any two states

$$\bar{h}_2 - \bar{h}_1 = \left(\bar{h}^* - \bar{h}\right)_1 - \left(\bar{h}^* - \bar{h}\right)_2 + \left(\bar{h}_2{}^* - \bar{h}_1{}^*\right)$$

where $\bar{h}_2{}^* - \bar{h}_1{}^* = \int_{T_1}^{T_2} \bar{C}_p{}^* dT$ **= enthalpy change in ideal gas behaviour**

GENERALISED CHARTS FOR OTHER PROP.

To find Entropy Difference between any two states

$$\bar{s}_2 - \bar{s}_1 = \left(\bar{s}^* - \bar{s}\right)_1 - \left(\bar{s}^* - \bar{s}\right)_2 + \left(\bar{s}^*{}_2 - \bar{s}^*{}_1\right)$$

$$\bar{s}^*{}_2 - \bar{s}^*{}_1 = \int_{T_1}^{T_2} \frac{C^*_{P0}}{T} dT - \bar{R}\, \ell n \frac{P_2}{P_1}$$

Entropy change with ideal gas behaviour

APPLICATIONS

Processes involving Ideal gases

$$P\upsilon = RT \qquad \beta = \frac{1}{\nu}\left(\frac{d\nu}{d\mathrm{T}}\right)_P = \frac{1}{T}$$

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = \frac{1}{P}$$

$$C_P - C_V = \frac{\mathrm{T}\beta^2 V}{\kappa_T .} = \frac{TV}{1/P}\left(\frac{1}{T}\right)^2 = R$$

Processes involving Ideal gases

$$u = u(T, v)$$

$$\boxed{du =} \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$= C_v dT + \left(T\left(\frac{\partial s}{\partial v}\right)_T - P\right) dv$$

$$\boxed{= C_v dT}$$

$$\boxed{dh = C_p dT}$$

Processes involving Ideal gases

Rev adiabatic process: ds=0

$$\therefore \frac{dT}{T} = \frac{\beta}{C_P} . \frac{RT}{P} dP = \frac{R}{C_P} \frac{dP}{P} = \frac{C_P - C_v}{C_P} \frac{dP}{P}$$

$$= \frac{\gamma - 1}{\gamma} \frac{dP}{P} \qquad \{\gamma = C_P / C_v$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} \Rightarrow P_2 v_2^{\gamma} = P_1 v_1^{\gamma}$$

USING GENERALISED CHARTS---EXAMPLE

One Kg Freon-12 at 5 bar, 50^0 C is compressed isothermally in a cylinder to 10 bar. Find ΔW, ΔQ Using G-Charts.

Given

$$M = 121 \quad \mathrm{T}_c = 384.7 K$$

$$\mathrm{P}_c = 40.1\, bar \quad C_{Po} = 0.56 \; kJ / kg K$$

USING GENERALISED CHARTS---EXAMPLE

$$P_{r_1} = \frac{5}{40.1} = .1245$$

$$T_{r_1} = 323/384.7 = .84$$

$$Z_1 = .92 \qquad \frac{\bar{s}_1^* - \bar{s}_1}{R} = .25$$

$$\frac{\bar{h}^* - \bar{h}}{\bar{R}\, T_c} = .25$$

$$P_{r_2} = \frac{10}{40.1} = .249$$

$$T_{r_2} = .84$$

$$Z_2 = .83 \qquad \frac{\bar{s}_2^* - \bar{s}_2}{R} = 0.4$$

$$\frac{\bar{h}_2^* - \bar{h}_2}{RT_c} = 0.45$$

USING GENERALISED CHARTS---EXAMPLE

$$Q = \mathrm{T}\left(s_2 - s_1\right)$$

$$\bar{s}_2 - \bar{s}_1 = \left(\bar{s}^* - \bar{s}\right)_1 - \left(\bar{s}^* - \bar{s}\right)_2 + \left(s_2^* - \bar{s}_1^*\right)$$

$$= .25\mathrm{x}8.314 - 0.4\mathrm{x}8.3413 + \int \frac{C_{po}}{\mathrm{T}} d\mathrm{T} - \overline{R}\,\ell n \frac{P_2}{P_1}$$

$$= -7.26\,kJ/kmol\,K$$

$$\therefore Q = \frac{323 \times \left(-7.26\right) \times 1}{121} = -19.4kJ$$

USING GENERALISED CHARTS---EXAMPLE

$$\bar{h}_2 - \bar{h}_1 = \left(\bar{h}^* - \bar{h}\right)_1 - \left(\bar{h}^* - \bar{h}\right)_2 + \left(h_2^* - h_1^*\right)$$

$$= -6.36\ kJ$$

= 0, for ideal gas

$$u_2 - u_1 = \left(h_2 - p_2 v_2\right) - \left(h_1 - p_1 v_1\right)$$

$$= \left(h_2 - h_1\right) + \left(p_1 v_1 - p_2 v_2\right)$$

USING GENERALISED CHARTS---EXAMPLE

$$\upsilon_1 = \left(\frac{ZRT}{\mathrm{P}}\right)_1 = \frac{.92 \times 8.3143 \times 323}{500} \times \frac{1}{121}$$

$$= .0409 \quad m^3 / \kappa g$$

$$\upsilon_2 = (ZRT/P)_2 = .0183 \ m^3 / kg$$

$$\therefore (U_2 - U_1) = -6.36 + (500 \times .0409) - (1000 \times .0183) \, kJ$$

$$= -4.21 \ kJ$$

$$\Delta Q = \Delta U - \Delta W \quad \therefore \Delta W = \Delta U - \Delta Q = -4.21 + 19.4 = 15.19 \, kJ$$

End of Lecture