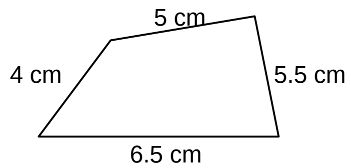
G9 More About Areas and Volumes

G9.1 Perimeter and Area of a Polygon

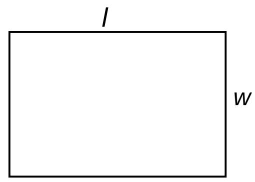
Recall that the perimeter of a polygon is the sum of the lengths of the dimensions.
For example,



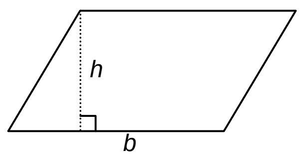
the perimeter of the quadrilateral = 4 + 5 + 5.5 + 6.5 = 21 cm.

Also recall that we have learnt the following formulae for finding the area of some particular polygons:

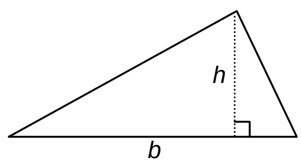
1. Area of rectangle = lw



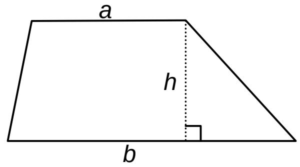
2. Area of parallelogram = bh



3. Area of triangle $= \frac{1}{2}bh$



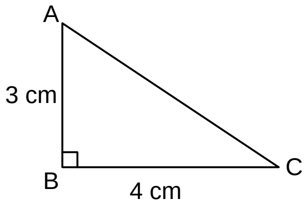
4. Area of trapezium $= \frac{1}{2}(a+b)\times h$



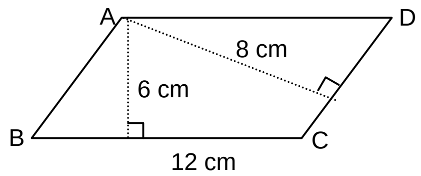
Example 1

Find the perimeter and area of each of the following polygons.

(a)

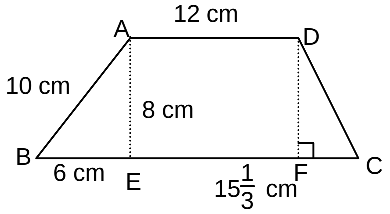


(b)



ABCD is a parallelogram.

(c)



Solution

(a) $AC^2 = 3^2 + 4^2$ (Pyth. Theorem)

$AC = \sqrt{25} = 5$ cm

Perimeter of $\Delta ABC = 3 + 4 + 5 = 12$ cm

Area of $\Delta ABC = \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$

(b) Area of parallelogram $ABCD = 6 \times 12 = 72 \text{ cm}^2$

$CD = \frac{72}{8} = 9$ cm

Perimeter of parallelogram ABCD = (12 + 9) × 2 = 42 cm.

(c) Consider ΔABE.

$BE^2 + AE^2 = 6^2 + 8^2 = 100$

$= 10^2 = AB^2$

∴ ΔABE is right-angled at E (Converse of Pyth. Theorem).

DF = AE = 8 cm

$DC^2 = \left(15\frac{1}{3} - 12\right)^2 + 8^2$ (Pyth. Theorem)

$DC = \sqrt{75\frac{1}{9}} = 8\frac{2}{3}$ cm

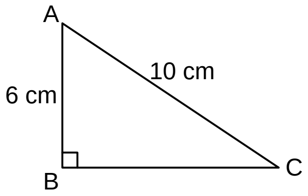
Perimeter of $ABCD = 12 + 10 + 6 + 15\frac{1}{3} + 8\frac{2}{3} = 52$ cm

Area of $ABCD = \frac{1}{2} \times \left[12 + \left(6 + 15\frac{1}{3}\right)\right] \times 8 = 133\frac{1}{3} \text{ cm}^2$

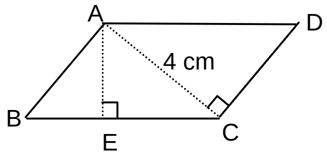
Checkpoint 1

Find the perimeters and areas of the polygons below.

(a)



(b)



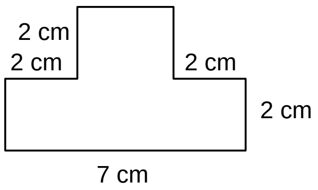
ABCD is a parallelogram and AE = 2.4 cm.

[*Hint: Let CD = x cm, and find BC in terms of x using the area of ABCD.*]

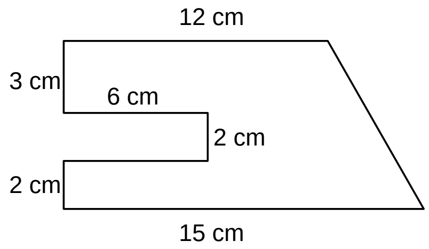
Example 2

Find the shaded area of the following figures.

(a)



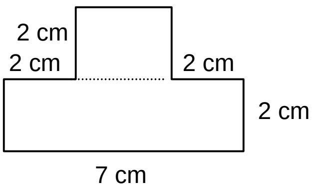
(b)



Solution

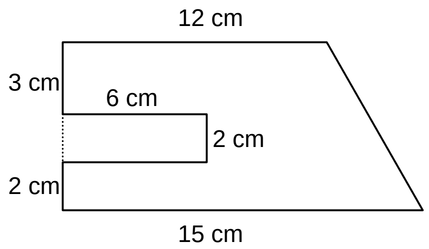
(a) $\text{Area} = (7-2-2)\times 2 + 7\times 2$

$= 20\text{ cm}^2$



(b) $\text{Area} = \frac{1}{2}(15+12)\times(3+2+2) - 6\times 2$

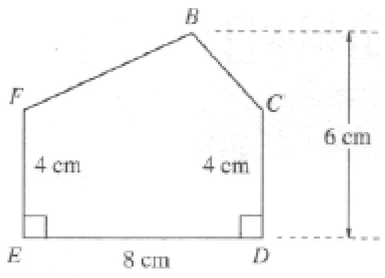
$= 82.5\text{ cm}^2$



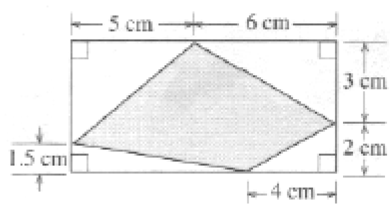
Checkpoint 2

In the following figures, find the area of (a) BCDEF; (b) shaded region.

(a)

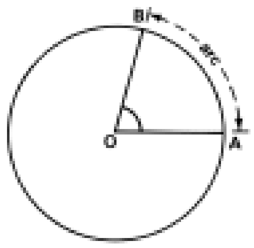


(b)



G9.2 Length of Arc

The figure below is a circle with centre O and radius OA.

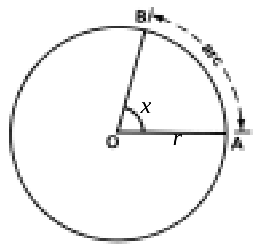


The part AB along the circumference is called an arc of the circle. It is written as arc AB or denoted by $\overset{\frown}{AB}$.

In fact, the arc length is directly proportional to the angle subtended at the centre by the arc.

i.e. $\dfrac{\text{Arc length}}{\text{Circumference}} = \dfrac{\text{Angle subtended at the centre}}{360°}$

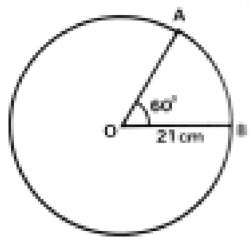
Hence, for any circle with radius *r*, if the angle subtended at the centre by an arc is *x*, then the arc length is given by the formula:



$$\text{Arc length} = \frac{x}{360°} \times 2\pi\, r$$

Example 3

In the figure, O is the centre of the circle. Find the length of arc AB. (Take $\pi = \dfrac{22}{7}$)



Solution

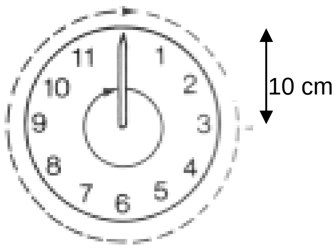
We have $x = 60^{\circ}$, $r = 21$ cm,

$$\begin{aligned}\text{Length of arc AB} &= \frac{x}{360°} \times 2\pi\, r \\ &= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21 \\ &= 22 \text{ cm}\end{aligned}$$

Example 4

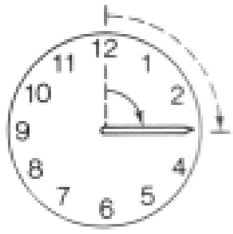
The length of the minute-hand of a clock is 10 cm. Find the distance, which the tip of the minute-hand moves in 15 minutes.

(Take π = 3.14 and correct the answer to 1 decimal place if necessary.)



Solution

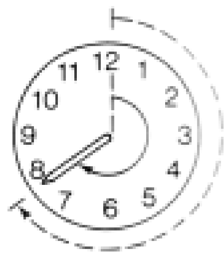
[The tip of the minute hand has moved 15 out of 60 minutes, i.e. $\frac{15}{60}$ of the circumference of the clock.]



$$\text{Distance moved} = \frac{15}{60} \times 2\pi(10)$$
$$= 15.7 \text{ cm}$$

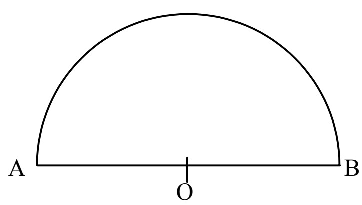
Checkpoint 3

Refer to **Example 4**, find the distance, which the tip of the minute-hand moves in 38 minutes.



Checkpoint 4

The figure shows a semi-circle with centre O and diameter AOB. If the perimeter of the figure is 36 cm, find the radius of the circle. (Take $\pi = \frac{22}{7}$)

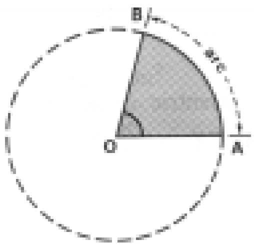


G9.3 Area of a Sector

In the figure,

(1) The region enclosed by the two radii OA, OB and the arc AB is called a sector of the circle.

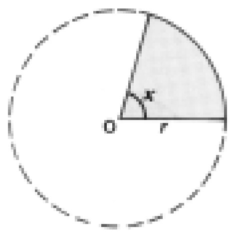
(2) In a sector, the angle subtended at the centre by an arc is called the angle of the sector. Here ∠AOB is called the angle of the sector AOB.



In fact, the area of sector is directly proportional to the angle of the sector.

i.e. $$\frac{\text{Area of sector}}{\text{Area oa a circle}} = \frac{\text{Angle of the sector}}{360°}$$

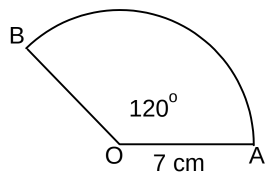
Hence, for any angle with radius *r*, if the angle of a sector is *x*, then the area of the sector is given by the formula:



$$\text{Area of sector} = \frac{x}{360°} \times \pi\, r^2$$

Example 5

In the figure, find the area of the sector AOB. (Take $\pi = \frac{22}{7}$)



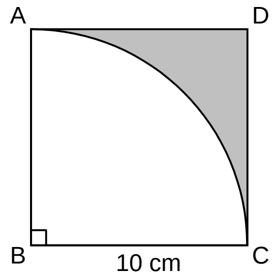
Solution

We have $x = 120^{o}$, $r = 7$ cm.

$$\begin{aligned}\text{Area of sector AOB} &= \frac{x}{360°} \times \pi\, r^2 \\ &= \frac{120°}{360°} \times \frac{22}{7} \times 7 \times 7 \\ &= 51\frac{1}{3}\text{cm}^2\end{aligned}$$

Example 6

In the figure, ABCD is a square of side 10 cm and AC is an arc of a circle with centre at B, find the area of the shaded region. (Take $\pi = 3.14$)



Solution

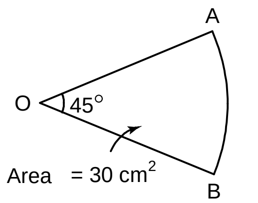
$$\begin{aligned}\text{Area of shaded region} &= \text{Area of square ABCD} - \text{Area of sector BAC}\\ &= 10\times 10-\frac{90^\circ}{360^\circ}\times\pi(10)^2\\ &= 100-78.5\\ &= 22.5\text{ cm}^2\end{aligned}$$

Checkpoint 5

In the figure, AOB is a sector with centre O. If the area of the sector is 30 cm^2, and $\angle AOB = 45^\circ$,

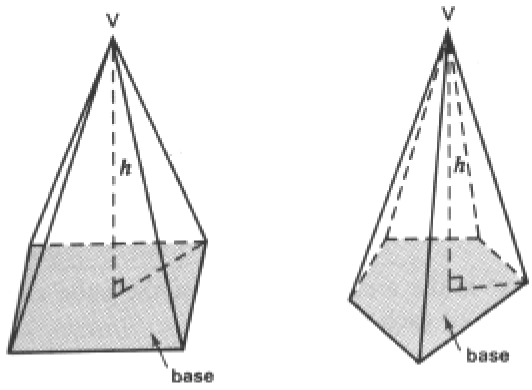
(a) find the radius of the sector;

(b) find the length of $\overset{\frown}{AB}$.



G9.4 The Pyramid

The solids shown in the following figures are some pyramids.



The shaded part can be any polygon and is called *the base* of the pyramid.

Definitions:

(1) *Vertex* (V): The common point that all the faces other than the base meet in

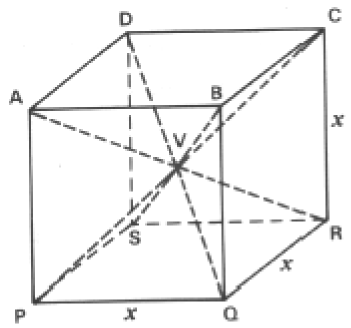
(2) *Height* (h): The perpendicular distance of the vertex from the base that opposite the vertex.

(3) *Slant edges*: The lines joining the vertex and the corners of the base.

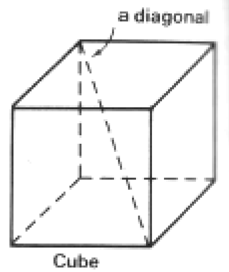
(4) *Right pyramid*: A pyramid that all the slant edges are equal in length.

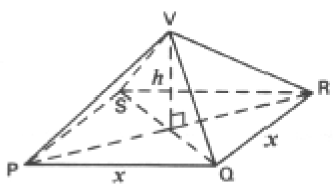
Volume of a Pyramid

The following figure shows a cube with ABCD as the top face and PQRS as the bottom face. The length of each side of the cube is x.



It is clear that all the four *diagonals of the cube* meet at a point V, dividing the cube into six congruent pyramids. The following figure shows one of the six pyramids.





Obviously, if each pyramid, h represents its height and v its volume, then

$$6v = x \cdot x \cdot x = x \cdot x \cdot (2h) = 2x^2 h$$

$$v = \frac{2x^2 h}{6} = \frac{1}{3}x^2 h$$

Note that x^2 is the area of one face of the cube and it is also the area of the base of each pyramid.

In general,

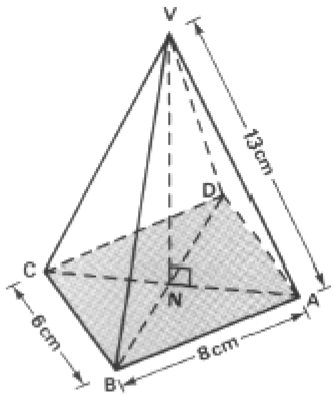
$$\textbf{Volume of any pyramid} = \frac{1}{3} \times \textbf{Base area} \times \textbf{Height}$$

Example 7

The figure shows a right pyramid with a rectangular base ABCD. If AB = 8 cm, BC = 6 cm and VA = 13 cm, find

(a) the height of the pyramid,

(b) the volume of the pyramid.



Solution

(a) From the right-angled triangle ABC,

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{8^2 + 6^2} \\ &= \sqrt{100} = 10\text{ cm} \end{aligned}$$

$\because$ $AN = NC = \frac{1}{2}AC$ (property of rectangle)

$\therefore$ $AN = 5\text{ cm}$

From the right-angled triangle VAN,

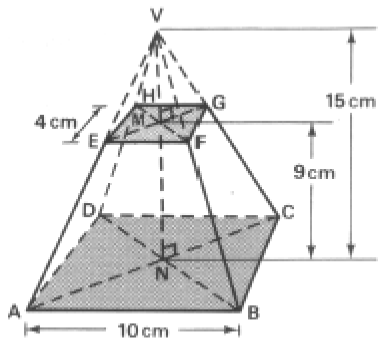
$$\begin{aligned} VN &= \sqrt{VA^2 - AN^2} \\ &= \sqrt{13^2 - 5^2} \\ &= \sqrt{144} = 12\text{ cm} \end{aligned}$$

$\therefore$ The height of the pyramid is 12 cm.

(b) $$\text{Volume of pyramid} = \frac{1}{3} \times 8 \times 6 \times 12 = 192\text{ cm}^3$$

Example 8

The figure shows a frustum formed by cutting a right square pyramid along a plane parallel to the base of the pyramid. The base ABCD and the upper face EFGH of the frustum are squares of sides 10 cm and 4 cm respectively. If VN = 15 cm and MN = 9 cm, find



(a) the volume of the original pyramid VABCD;

(b) the volume of the pyramid VEFGH removed;

(c) the volume of the frustum.

Solution

(a) Volume of the original pyramid VABCD

$$= \frac{1}{3} \times 10 \times 10 \times 15 = 500 \text{ cm}^3$$

(b) Volume of pyramid VEFGH

$$= \frac{1}{3} \times 4 \times 4 \times (15 - 9)$$

$$= 32 \text{ cm}^3$$

(c) Volume of frustum

$$= 500 - 32$$

$$= 468 \text{ cm}^3$$

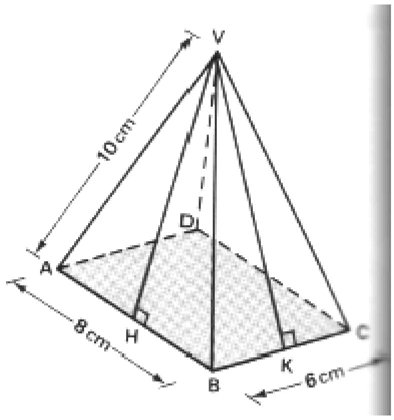
Example 9

In the figure, VABCD is a right pyramid with a rectangular base ABCD. If AB = 8 cm, BC = 6 cm and VA = 10 cm, find

(a) the areas of ΔVAB and ΔVBC,

(b) the total surface area of the pyramid.

(*Correct the answers to 1 d.p.*)



Solution

(a) Let VH be the height of the isosceles triangle VAB.

$$VH = \sqrt{VA^2 - AH^2}$$
$$= \sqrt{10^2 - 4^2}$$
$$= \sqrt{84} = 9.165 \text{ cm}$$

AH = 4 cm since VABCD is right.

$\therefore$ Area of ΔVAB

$$= \frac{1}{2} \times 8 \times 9.165$$
$$= 36.7 \text{ cm}^2 \qquad \text{(corr. to 1 d.p.)}$$

Similarly,

Area of ΔVBC

$$= \frac{1}{2} \times 6 \times \sqrt{10^2 - 3^2}$$
$$= \frac{1}{2} \times 6 \times \sqrt{91}$$
$$= 28.6 \text{ cm}^2 \qquad \text{(corr. to 1 d.p.)}$$

(b) Area of four lateral surfaces

$$= 2 \times (36.7 + 28.6)$$
$$= 130.6 \text{ cm}^2$$

Area of base ABCD

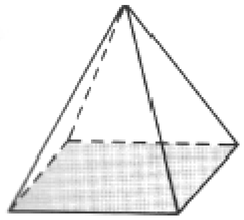
$$= 8 \times 6$$
$$= 48 \text{ cm}^2$$

Total surface area

$$= 130.6 + 48$$
$$= 179.6 \text{ cm}^2 \qquad \text{(corr. to 1 d.p.)}$$

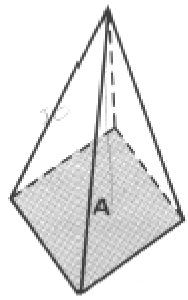
Checkpoint 6

In the figure, the base of the pyramid is a rectangle of dimensions 2 cm by 5 cm, and the height is 3 cm. Find the volume of the pyramid.



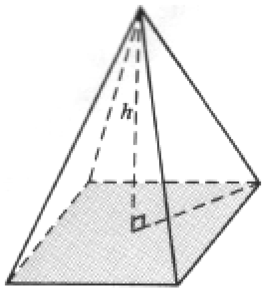
Checkpoint 7

In the figure, the volume of the pyramid is 72 cm^3 and the height is 12 cm. Find the base area (A) of the pyramid.

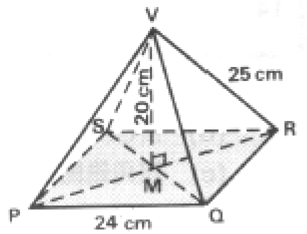


Checkpoint 8

In the figure, the base of the pyramid is a rectangle 4 cm by 7 cm and the volume is 84 cm^3. Find the height (h) of the pyramid.



Checkpoint 9



The figure shows a right pyramid with a rectangular base PQRS. The diagonals PR and QS intersect at M; PQ = 24 cm, VM = 20 cm and VR = 25 cm. Find

(a) MR,

(b) PR,

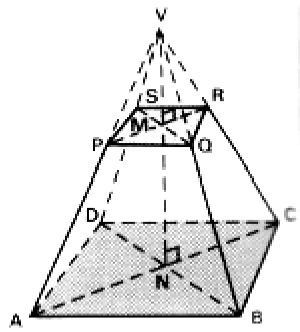
(c) QR,

(d) the volume of the pyramid.

Checkpoint 10

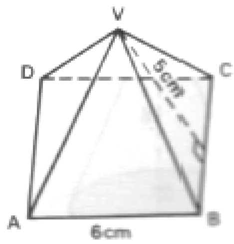
The figure shows a frustum with a rectangular base ABCD and a rectangular top PQRS. AB = 12 cm, BC = 9 cm, PQ = 4 cm, QR = 3 cm, VM = 3 cm and MN = 6 cm. Calculate the volume of the frustum.

(*312 cm*^{3})



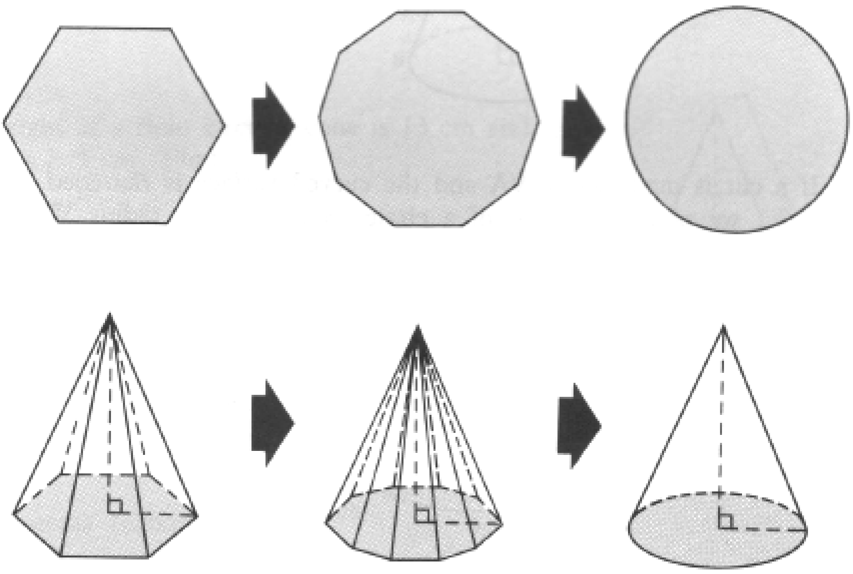
Checkpoint 11

The figure shows a right pyramid VABCD with a square base ABCD. Find the total surface area of the pyramid.



G9.5 The Right Circular Cone

Consider a right pyramid, if the number of sides of the base (a polygon) increases infinitely, the base will eventually become a circle and the right pyramid will then become a right circle cone.



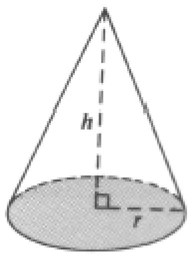
Right circular cone

Thus a right circular cone may be regarded as a special type of right pyramid and its volume is also given by:

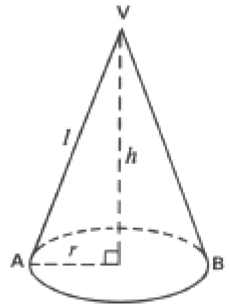
$$\text{Volume} = \frac{1}{3} \times \text{Base area} \times \text{Height}$$

In fact, for a right circular cone, if the radius of the base is r and the height is h, the volume is given by the formula

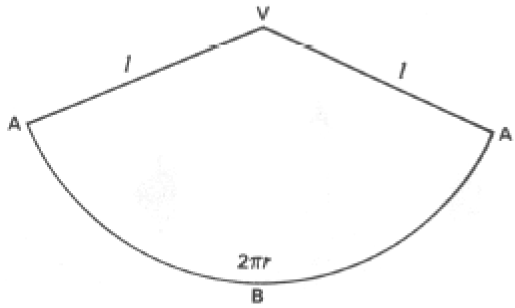
$$\textbf{Volume of right circular cone} = \frac{1}{3}\pi r^2 h$$



The figure below shows a hollow right circular cone opened at the base with base radius *r* and height *h*. VA is called a ***slant side*** and its length *l* is called the ***slant height*** of the cone.



If a cut is made along VA and the curved surface is flattened out (放平), we obtain a sector of a circle, centre V and radius VA as shown in the figure below.



In fact, $\dfrac{\text{area of sector}}{\text{area of complete circle}} = \dfrac{\text{arc length of sector}}{\text{circumference of complete circle}} = \dfrac{2\pi r}{2\pi l}$

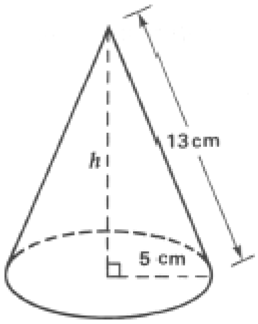
$$\begin{aligned}\text{Area of sector} &= \text{Area of complete circle} \times \frac{2\pi r}{2\pi l} \\ &= \pi l^2 \times \frac{2\pi r}{2\pi l} \\ &= \pi rl\end{aligned}$$

Thus, **Curved surface area of right circular cone = πrl**

Note that $r^2 + h^2 = l^2$.

Example 10

The slant height of a right circular cone is 13 cm and its base radius is 5 cm.



(a) Find the curved surface area of the cone;

(b) Find the volume of the cone.

(Take $\pi = 3.14$)

Solution

(a) $$\text{Curved surface area} = \pi rl = 3.14 \times 5 \times 13 = 204.1\ \text{cm}^2$$

(b) Let h be the height of the circular cone.

By Pythagoras' Theorem,

$$h = \sqrt{l^2 - r^2} = \sqrt{13^2 - 5^2} = \sqrt{144} = 12\ \text{cm}$$

$$\therefore \quad \text{Volume of the cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 5^2 \times 12 = 314\ \text{cm}^3$$

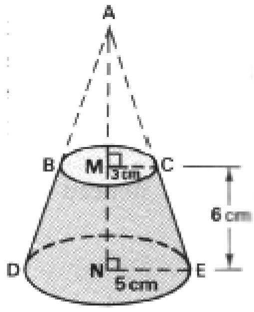
Example 11

The figure shows a frustum formed by cutting a right circular cone along a plane parallel to the base of the cone. The radii of the upper face and the base of the frustum are 3 cm and 5 cm respectively. If the height of the frustum is 6 cm, find

(a) the height (AM) of the removed cone ABC;

(b) the height (AN) of the original cone ADE;

(c) the volume of the frustum in terms of π.



Solution

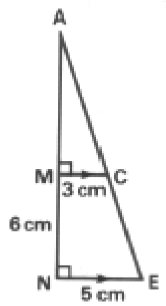
(a) $\because$ $\Delta AMC \sim \Delta ANE$ (AAA)

$\therefore$ $\dfrac{AM}{AN} = \dfrac{MC}{NE}$ (corr. sides, ~Δs)

$\dfrac{AM}{AM+6} = \dfrac{3}{5}$

$5AM = 3AM + 18$

$AM = 9$



$\therefore$ The height of the removed cone ABC is 9 cm.

(b) $AN = AM + MN$

$= 9 + 6$

$= 15 \text{ cm}$

$\therefore$ The height of the original cone ADE is 15 cm.

(c) Volume of the original cone ADE

$= \dfrac{1}{3} \times \pi \times 5^2 \times 15$

$= 125\pi \text{ cm}^3$

Volume of the removed cone ABC

$= \dfrac{1}{3} \times \pi \times 3^2 \times 9$

$= 27\pi \text{ cm}^3$

$\therefore$ Volume of the frustum

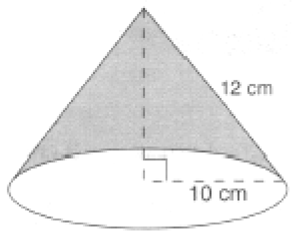
$= 125\pi - 27\pi$

$= 98\pi \text{ cm}^3$

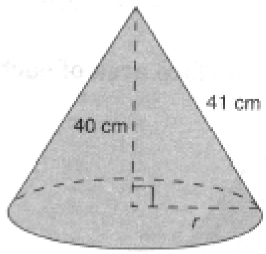
Checkpoint 12

In each of the following circular cones, find the volume and its curved surface area. Take $\pi = 3.14$ and give the answers correct to 1decimal place.

(a)



(b)

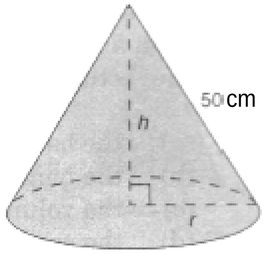


Checkpoint 13

The right circular cone's curved surface area is 1500π cm^2.

(a) Find the values of r and h.

(b) Find, in terms of π, the volume of the cone.



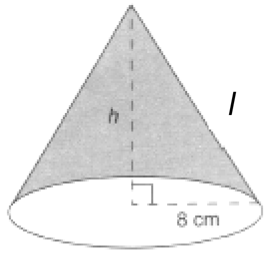
Checkpoint 14

The right circular cone has a volume of 128π cm^3.

(a) Find the values of h and l.

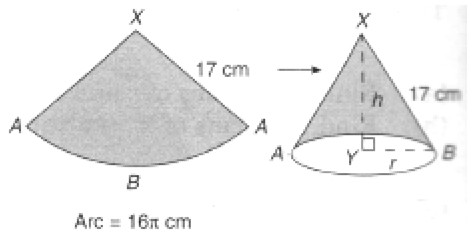
(b) Find, in terms of π, the curved surface area of the cone.

($80\pi\ cm^2$)



Checkpoint 15

A piece of paper in the form of a sector is folded to form a right circular cone as shown. The radius of the sector is 17 cm and the arc ABA is 16π cm. Find



(a) the base radius (r);

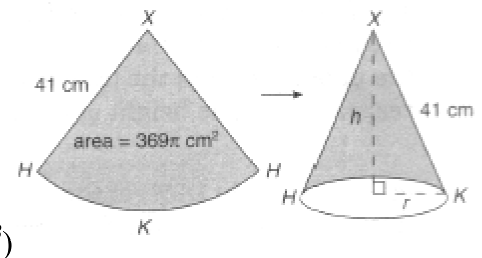
(b) the height (h);

(c) the volume of the cone in terms of π;

(d) the curved surface area in terms of π.

Checkpoint 16

A piece of metal sheet in the form of a sector is bent to form a right circular cone as shown. The radius of the sector is 41 cm and its area is 369π cm^2. Find



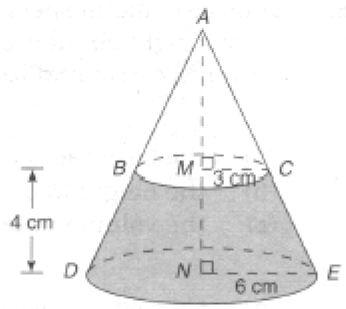
(a) the base radius (r);

(b) the height (h);

(c) the volume of the cone in terms of π. ($1080\pi\ cm^3$)

Checkpoint 17

The figure shows a frustum formed by removing the right circular cone ABC from the right circular cone ADE. The radii of the upper face and the base of the frustum are 3 cm and 6 cm respectively. The height of the frustum is 4 cm. Find



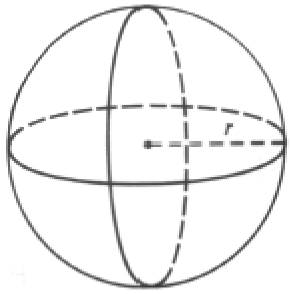
(a) the height (AM) of the removed cone ABC;

(b) the volume of the frustum in terms of π; $(84\pi\, cm^3)$

(c) the curved surface area of the frustum in terms of π. $(45\pi\, cm^2)$

G9.6 The Sphere

Footballs and tennis balls are common examples of spheres.



The surface area of a sphere and the volume of a sphere are given by the formulae

$$\textbf{Surface area of sphere} = 4\pi r^2$$

and

$$\textbf{Volume of sphere} = \frac{4}{3}\pi r^3$$

Example 6

The surface area of a sphere is 314 cm^2. Find, to the nearest cm^3, the volume of the sphere. (Take $\pi = 3.14$)

Solution

Let the radius of the sphere be r cm.

$$4\pi r^2 = 314$$

$$r^2 = \frac{314}{4 \times 3.14} = 25$$

$$r = \sqrt{25} = 5$$

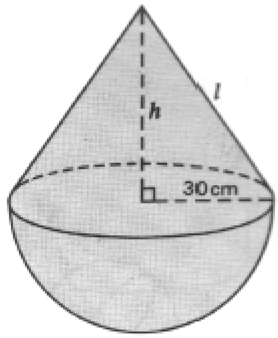
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.14 \times 5^3$$

$$= 523 \text{ cm}^3 \qquad \text{(corr. to the neaest cm}^3\text{)}$$

Example 13

The figure shows a solid consisting of a right circular cone and a hemisphere with a common base which is a circle of radius 30 cm. The volume of the cone is equal to $\frac{2}{3}$ the volume of the sphere. Find



(a) the height of the cone;

(b) the surface area of the whole solid in terms of π.

Solution

(a) Let the height of the cone be *h*.

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of the sphere} = \frac{1}{2} \times \left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$$

$$\therefore \quad \frac{1}{3}\pi r^2 h = \frac{2}{3}\left(\frac{2}{3}\pi r^3\right)$$

$$h = \frac{4}{3}r$$

$$= \frac{4}{3} \times 30$$

$$= 40 \text{ cm}$$

∴ The height of the cone is 40 cm.

(b) If the slant height of the cone is *l*, we have

$$l = \sqrt{40^2 + 30^2} = 50 \text{ cm}$$

$$\therefore \quad \text{Area of the surface of the cone} = \pi rl$$

$$= \pi \times 30 \times 50$$

$$= 1500\pi \text{ cm}^2$$

$$\text{Area of curved surface of hemisphere} = \frac{1}{2}(4\pi r^2) = 2\pi r^2$$

$$= 2\pi \times 30^2$$

$$= 1800\pi \text{ cm}^2$$

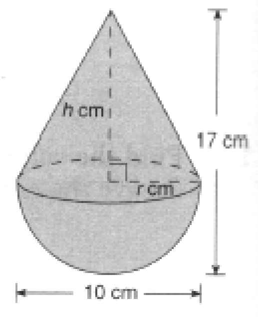
∴ Surface area of the whole solid

$$= 1500\pi + 1800\pi$$

$$= 3300\pi \text{ cm}^2$$

Checkpoint 18

The figure shows a right circular cone and a hemisphere with a common base. Find, in terms of π,

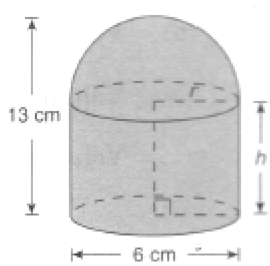


(a) the volume of the solid;

(b) the surface area of the solid. ($115\pi\ cm^2$)

Checkpoint 19

The solid consists of a cylinder and a hemisphere. Find in terms of π,

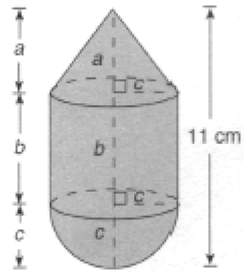


(a) the volume of the solid; ($108\pi\ cm^3$)

(b) the curved surface area of the solid. ($78\pi\ cm^2$)

Checkpoint 20

The solid consists of a right circular cone, a cylinder and a hemisphere. The volumes of these three parts are equal.



(a) Find the ratio of a:b:c. (6:2:3)

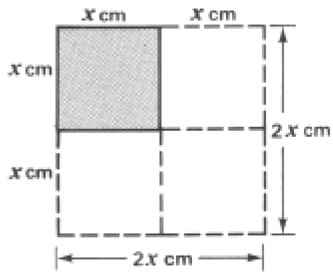
(b) If the total length of the solid is 11 cm, find the volume of the solid in terms of π. ($54\pi\ cm^3$)

G9.7 Similar Figures and Solids

Two plane figures or solids are similar if the larger can be regarded as a magnification of the smaller.

Consider the following cases:

(1) In the figure, each side of the shaded square is x cm. If each side of a square is doubled, a new square of side $2x$ cm is formed. This new square is a magnification of the shaded square and thus the squares are similar.



$$\frac{\text{Side of shaded square}}{\text{Side of new square}} = \frac{x}{2x} = \frac{1}{2}$$

$$\frac{\text{Area of shaded square}}{\text{Area of new square}} = \frac{x^2}{(2x)^2} = \left(\frac{1}{2}\right)^2$$

$$\therefore \quad \frac{\text{Area of shaded square}}{\text{Area of new square}} = \left(\frac{\text{Side of shaded square}}{\text{Side of new square}}\right)^2$$

In fact, for two similar figures, the ratio of the area is equal to the square of the ratio of any two corresponding lengths.

Therefore, given two similar figures, if A_1, A_2 denote their areas and l_1, l_2 denote any two of their corresponding lengths, then we have

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

Example 14

The corresponding sides of two similar triangles are in the ratio 3:2. If the area of the larger triangle is 36 cm^2, find the area of the smaller one.

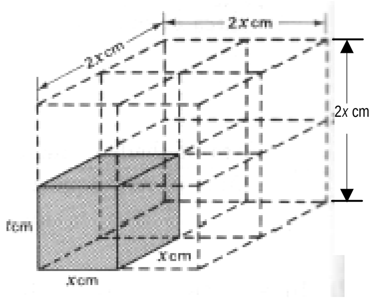
Solution

$$\frac{\text{Area of larger triangle}}{\text{Area of smaller triangle}} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\frac{36}{\text{Area of smaller triangle}} = \frac{9}{4}$$

$$\therefore \quad \text{Area of smaller triangle} = 36 \times \frac{4}{9} = 16\ \text{cm}^2$$

(2) In the figure, each side of the shaded cube is x cm. If each side of the cube is doubled, a new cube of $2x$ cm is formed. This new cube is a magnification of the shaded cube and thus the two cubes are similar.



$$\frac{\text{Side of shaded cube}}{\text{Side of new cube}} = \frac{x}{2x} = \frac{1}{2}$$

$$\frac{\text{Surface area of shaded cube}}{\text{Surface area of new cube}} = \frac{6 \cdot x^2}{6 \cdot (2x)^2} = \left(\frac{1}{2}\right)^2$$

$$\frac{\text{Volume of shaded cube}}{\text{Volume of new cube}} = \frac{x^3}{(2x)^3} = \left(\frac{1}{2}\right)^3$$

$$\therefore \quad \frac{\text{Surface area of shaded cube}}{\text{Surface area of new cube}} = \left(\frac{\text{Side of shaded cube}}{\text{Side of new cube}}\right)^2$$

$$\frac{\text{Volume of shaded cube}}{\text{Volume of new cube}} = \left(\frac{\text{Side of shaded cube}}{\text{Side of new cube}}\right)^3$$

In fact, for any two similar solids,

(1) the ratio of the areas is equal to the square of the ratio of any two corresponding sides;

(2) the ratio of the volumes is equal to the cube of the ratio of any two corresponding sides.

Therefore, given two similar solids, if A_1, A_2 denote their areas, V_1, V_2 denote their volumes and l_1, l_2 denote any two of their corresponding lengths, then we have

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \quad \text{and} \quad \frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

Example 15

Two similar cylinders have base diameters 4 cm and 6 cm. If the volume of the smaller cylinder is 64 cm^3, find the volume of the larger cylinder.

Solution

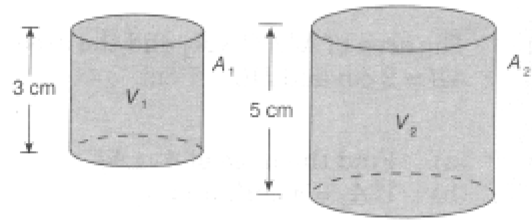
$$\frac{\text{Volume of larger cylinder}}{\text{Volume of smaller cylinder}} = \left(\frac{6}{4}\right)^2 = \frac{27}{8}$$

$$\frac{\text{Volume of larger cylinder}}{64} = \frac{27}{8}$$

$$\therefore \quad \text{Volume of larger cylinder} = 64 \times \frac{27}{8} = 216\ \text{cm}^3$$

Checkpoint 21

Let A_1, A_2 denote the curved surface areas of two similar cylinders respectively, while V_1, V_2 denote their volumes. Find



(a) $A_1:A_2$;

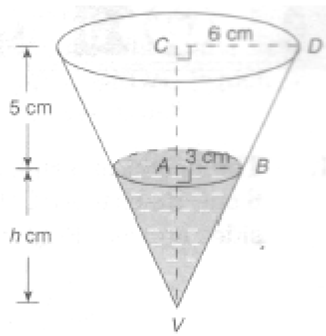
(b) $V_1:V_2$;

(c) V_2 and A_2 if $V_1 = 54$ cm^3. (*250 cm^3, 70.7 cm^2*)

(*Give the answers correct to 1 decimal place where necessary.*)

Checkpoint 22

The figure shows the dimensions of a right conical vessel standing vertically and containing some water.



(a) Find the depth (h cm) of the water.

(b) Find the volume of the water. ($15\pi\ cm^3$)

(c) Find the water:vessel volume ratio.

(d) Find the amount of water needed to fill the cone. ($105\pi\ cm^3$)

(*Give the answers in terms of* π *where necessary.*)

Example 16

If the surface area of a soap bubble is increased by 21%, find the percentage increase in

(a) its diameter;

(b) its volume.

Solution

(a) $$\left(\frac{\text{new diameter}}{\text{old diameter}}\right)^2 = \frac{\text{Surface area of new bubble}}{\text{Surface area of old bubble}} = \frac{121}{100}$$

$$\frac{\text{new diameter}}{\text{old diameter}} = \sqrt{\frac{121}{100}} = \frac{110}{100} = 110\%$$

∴ Percentage increase in diameter is 10%.

(b) $$\frac{\text{new volume}}{\text{old volume}} = \left(\frac{\text{new diameter}}{\text{old diameter}}\right)^3$$

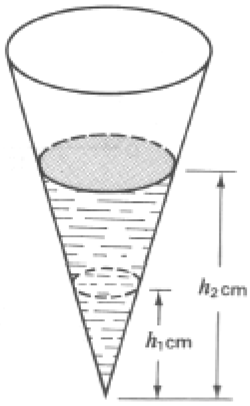
$$= \left(\frac{11}{10}\right)^3$$

$$= \frac{1331}{1000}$$

$$= 133.1\%$$

∴ Percentage increase in volume is 33.1%

Checkpoint 23



Referring to the figure, when 100 cm^3 of water is poured into the right conical vessel which stands vertically, the depth of water is h_1 cm and the surface area of the part that contains water is 96 cm^2. If 700 cm^3 more water is added, the depth of water becomes h_2 cm. Find

(a) $h_1 : h_2$;

(b) the increase in the surface area that contains water. (*288 cm^2*)

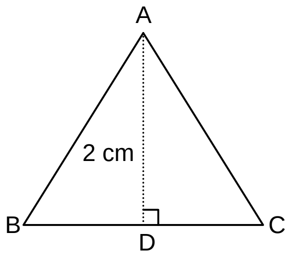
Exercise G9
More About Areas and Volumes

In this exercise, if an answer is not exact, it should be given correct to 1 decimal place unless otherwise stated.

G9.1

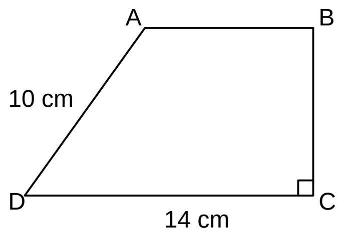
1. Find the perimeter and area of the following figures.

(a)



ΔABC is equilateral.

(b)

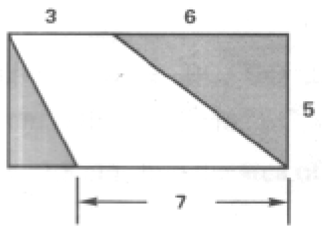


ABCD is a trapezium with AB = BC.

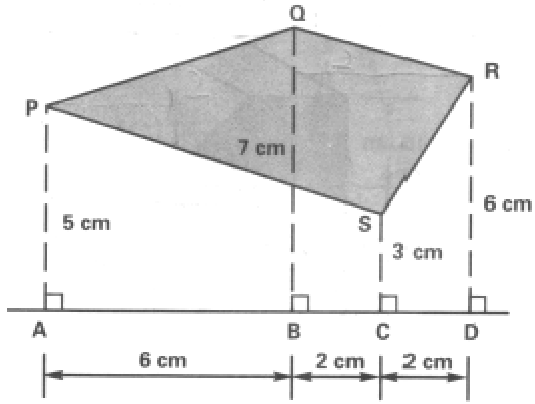
[*Hint: Let AB = BC = x cm.*]

2. In each of the following figures, find the area of the shaded region.

(a)

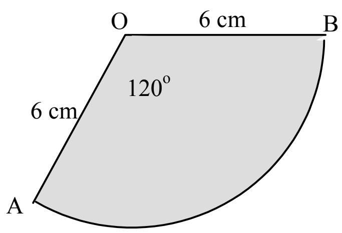


(b)



G9.2

3. Find the arc length AB of the following figure.

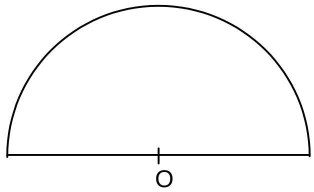


4. Copy and complete the following table.

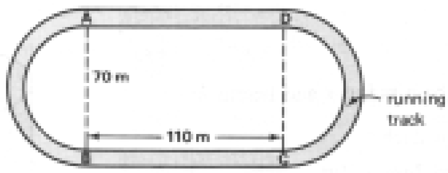
Radius	Angle subtended at centre	Arc length
5 cm	90^{o}	
	450^{o}	5π cm
3 cm		π cm

5. An arc of a circle of radius 10 cm is 10 cm long. What angle does the arc subtend at the centre correct to the nearest 0.1 degree?

6. In the figure, a wire 54 cm long is bent into a semi-circle and its diameter. Find the radius of the semi-circle. (Take $\pi = \frac{22}{7}$)

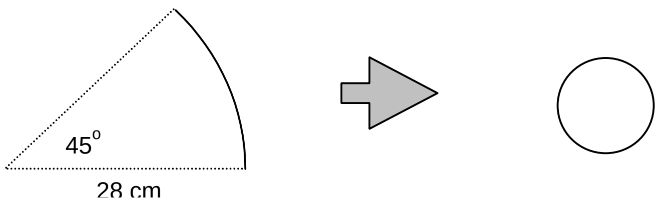


7. The diagram shows with dimensions a running track. The inside edge of the track is in the shape of a rectangle (ABCD) joined at each end by a semi-circle. The width of the track is constant.



(a) Find the distance round the inside edge of the track for one lap.
(b) If the track is 7 m wide, find the distance round the outside edge of the track for one lap.

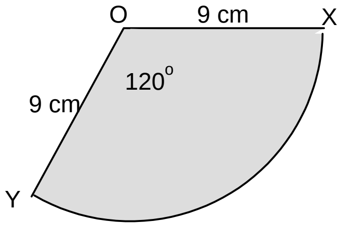
8. A piece of wire in the form of an arc of a circle of radius 28 cm, subtends an angle of 45^{o} at the centre of the circle. It is bent into the form of a complete circle. Find the radius of this circle.



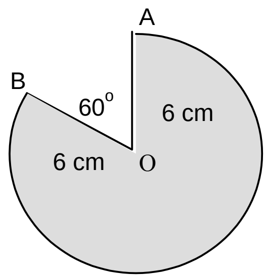
G9.3

9. Find the area for each of the following sectors.

(a)



(b)

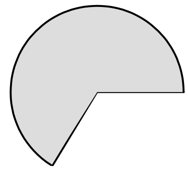


10. Use the given measures of a circle to complete the table below.

Radius	Angle of the sector	Area of sector
5 cm		27.5 cm^2
	10^o	154 cm^2

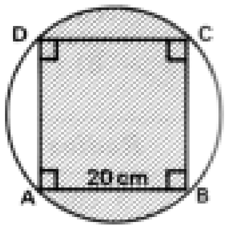
11. In the figure, the area of the shaded sector is $\frac{3}{5}$ of the whole circle.

Find the angle of the shaded sector.

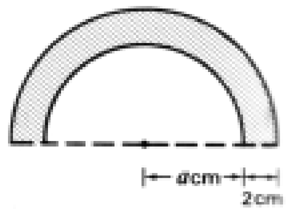


12. The length of the minute-hand of a clock is 20 cm. Find the area swept by the minute-hand in 25 minutes.

13. In the figure, ABCD is a square of side 20 cm. Find the area of the shaded region.

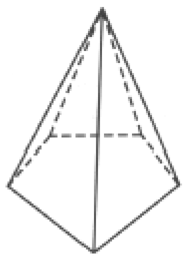


14. The diagram shows two concentric semi-circular arcs. The radius of the smaller semi-circle is a cm. The radius of the larger semi-circle is 2 cm longer than that of the smaller semi-circle. Find the area of the region in terms of a and π.

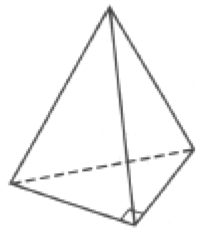


G9.4

15. In the figure, the base of the pyramid is a pentagon of area 30 cm^2 and height 5 cm. Find the volume of the pyramid.

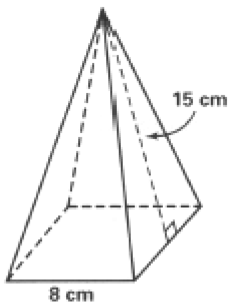


16. In the figure, the base of the pyramid is a right-angled triangle whose two perpendicular sides are 6 cm and 8 cm. The height of the pyramid is 15 cm. Find the volume of the pyramid.

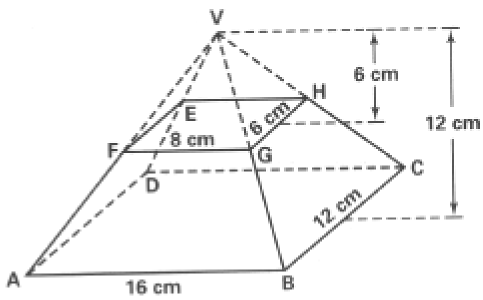


17. The volume of a pyramid is 180 cm^3. It has a pentagonal base of area 27 cm^2. What is its height?

18. The figure shows the dimensions of a right pyramid with a square base. Find its total surface area.



19. The figure shows a rectangular-based frustum ABCDEFGH which is formed by removing a small right pyramid VEFGH from a larger right pyramid VABCD. The removed pyramid is 6 cm high and its base is a rectangle with dimensions 6 cm by 8 cm. The original larger pyramid is 12 cm high and its base is a rectangle with dimensions 12 cm by 16 cm. Find the volume of the frustum.

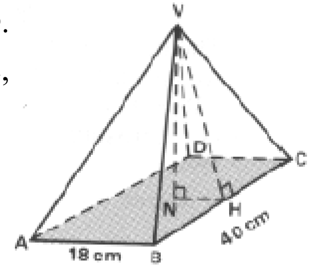


20. The figure shows a right pyramid with a rectangular base ABCD. AB = 18 cm and BC = 40 cm. If H is the mid-point of BC, NH = 9 cm and VH = 15 cm, find

(a) the height VN,

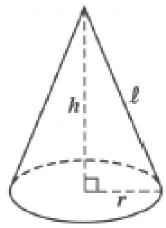
(b) slant edge VC,

(c) the volume of the pyramid.



G9.5

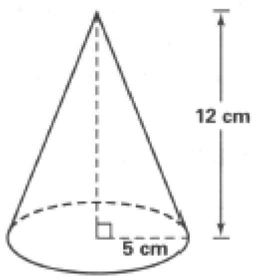
21. Referring to the given figure which shows a right circular cone of radius *r*, height *h* and slant height *l*, complete the following table. You may tear off the last page which has given the same table. (*Give the answers in terms of π if necessary.*)



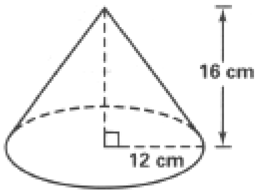
r	***h***	***l***	**Curved surface area**	**Volume**
	4 cm	5 cm		
5 cm	12 cm		65π cm^2	
6 cm				96π cm^3
8 m		17 m		
	40 m	41 m	369π cm^2	

22. Find the height of the cone if it is given that the volume of the cone is 486π m^3 and its base diameter is 12 m.

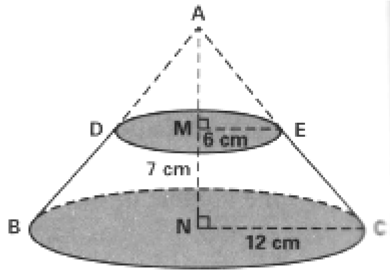
23. The figure shows a right circular cone. Find its curved surface area.



24. The figure shows a solid right circular cone. Find its total surface area.



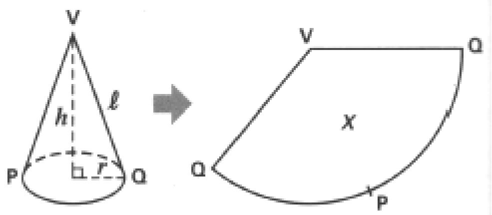
25. The figure shows a frustum formed by removing the right circular cone ADE from the right circular cone ABC. If ME = 6 cm, NC = 12 cm and MN = 7 cm.



(a) Find the length of AM.

(b) Find the volume of the frustum.

26. The sector in the following figure is formed by cutting the hollow paper cone along VQ and laying in flat.



Find the area (X) of the sector with the measurements of the cone given below.

(a) Slant height $l = 8$ cm; base radius $r = 6$ cm;

(b) Slant height $l = 7$ cm; base area = 25π cm^2;

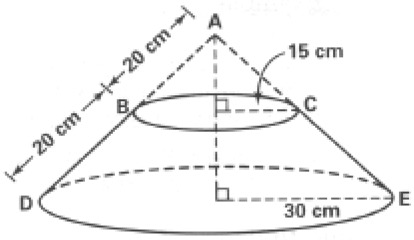
(c) Slant height $l = 12$ cm; base circumference = 6π cm;

(d) Height $h = 40$ cm; base radius $r = 9$ cm.

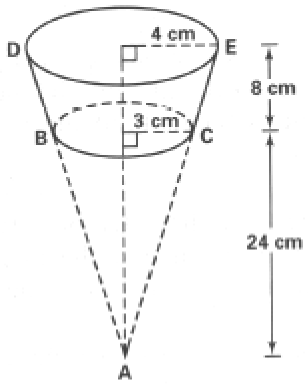
27. The base radius of a right circular cone is 10 cm and the height of the cone is 8 cm. Find the volume of the cone.

28. The base circumference of a right circular cone is 16π cm and the height of the cone is 30 cm. Find the volume of the cone.

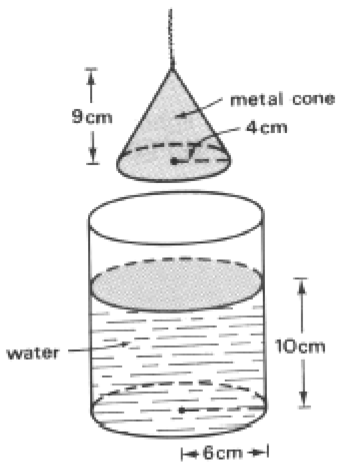
29. The figure shows a platform which is in the form of a frustum of a right circular cone. Find the total surface area of the platform.



30. The figure shows a cup which is in the form of an inverted frustum of a right circular cone. Find the capacity of the cup.



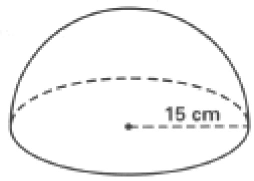
31. Water stands at a height of 10 cm in a cylindrical vessel, the base radius of which is 6 cm. A metal cone of height 9 cm and base radius 4 cm is lowered completely into the water and there is no water overflow. Find the new depth of water in the vessel.



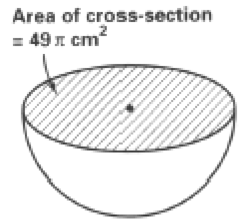
G9.6

32. Find the volume of each of the following hemispheres.

(a)



(b)



33. Find the surface area of a sphere if its diameter is 24 cm.

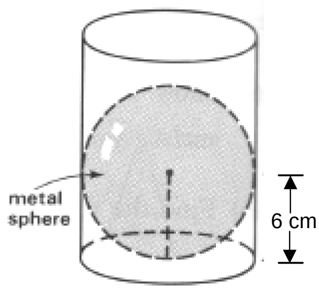
34. Find the curved surface area of a hemisphere if its radius is 10 cm.

35. The volume of a volleyball is 4849 cm^3. Find the diameter of the volleyball.

36. The radius of a tennis ball is 3 cm. Find the surface area of the tennis ball.

37. A sphere of radius 3 cm is melted down and recast into the form of a right circular cone whose height is equal to twice the diameter of the sphere. Calculate the radius of its base.

38. A metal sphere with radius 6 cm is fit inside a cylinder.



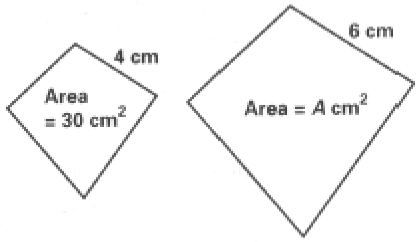
(a) Find the volume of the sphere.

(b) Find the minimum volume of water required to cover the sphere.

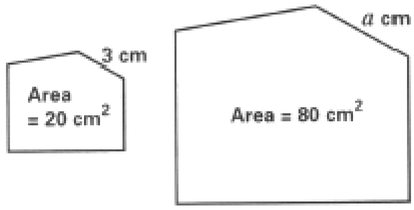
G9.7

39. Find the unknown in each of the following pairs of similar plane figures or solids.

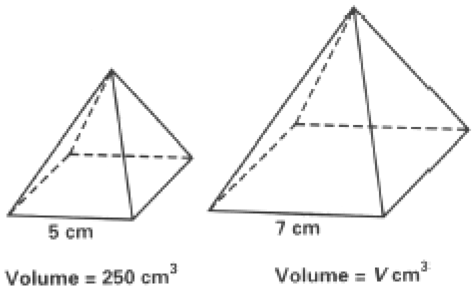
(a)



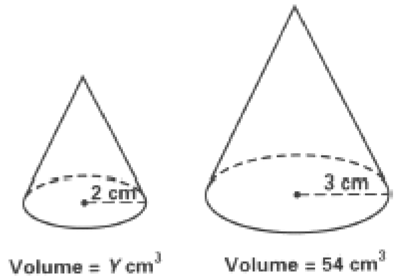
(b)



(c)



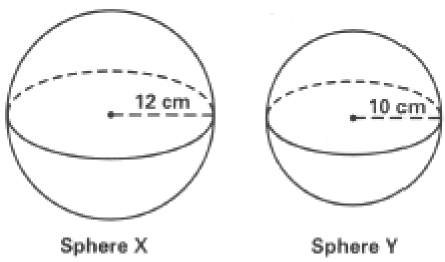
(d)



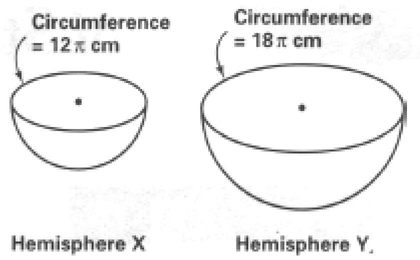
40. The heights of two similar cones are 6 cm and 10 cm. Find the ratio of their volumes.

41. In each of the following, two solids X and Y are similar. Find the ratio of the total surface area of X to that of Y.

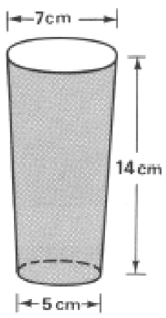
(a)



(b)

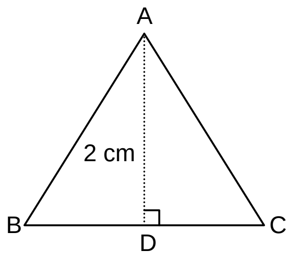


42. In the figure, the cup is in the shape of a frustum formed from a right circular cone. Find its capacity.

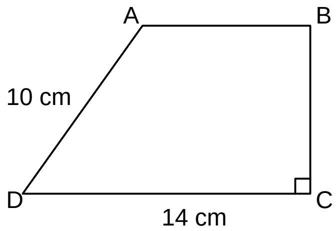


Figures for Question 1

(a)

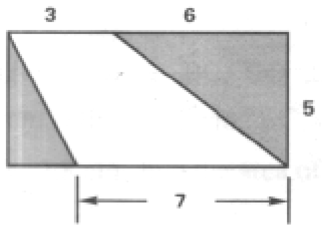


(b)



Figures for Question 2

(a)



(b)

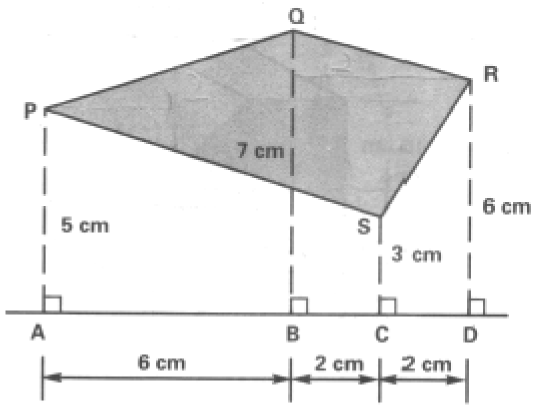


Table for Question 4

Radius	Angle subtended at centre	Arc length
5 cm	90^o	
	450^o	5π cm
3 cm		π cm

Table for Question 10

Radius	Angle of the sector	Area of sector
5 cm		27.5 cm^2
	10^o	154 cm^2

Figure for Question 13

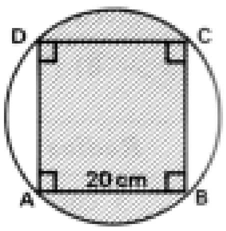


Figure for Question 18

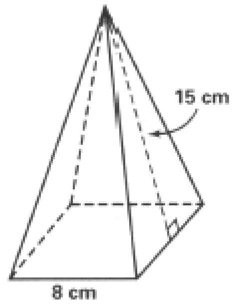


Figure for Question 19

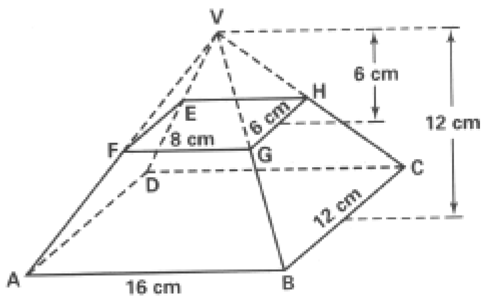


Table for Question 21

r	*h*	*l*	Curved surface area	Volume
	4 cm	5 cm		
5 cm	12 cm		65π cm^2	
6 cm				96π cm^3
8 m		17 m		
	40 m	41 m	369π cm^2	

Figure for Question 23

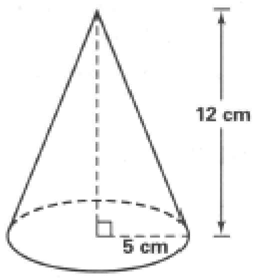


Figure for Question 24

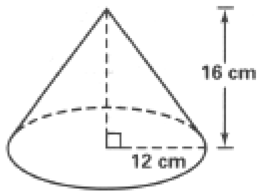


Figure for Question 29

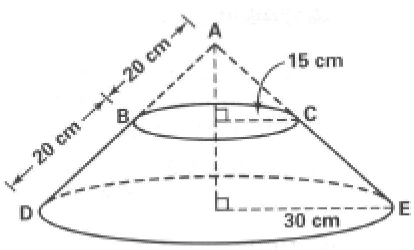


Figure for Question 42