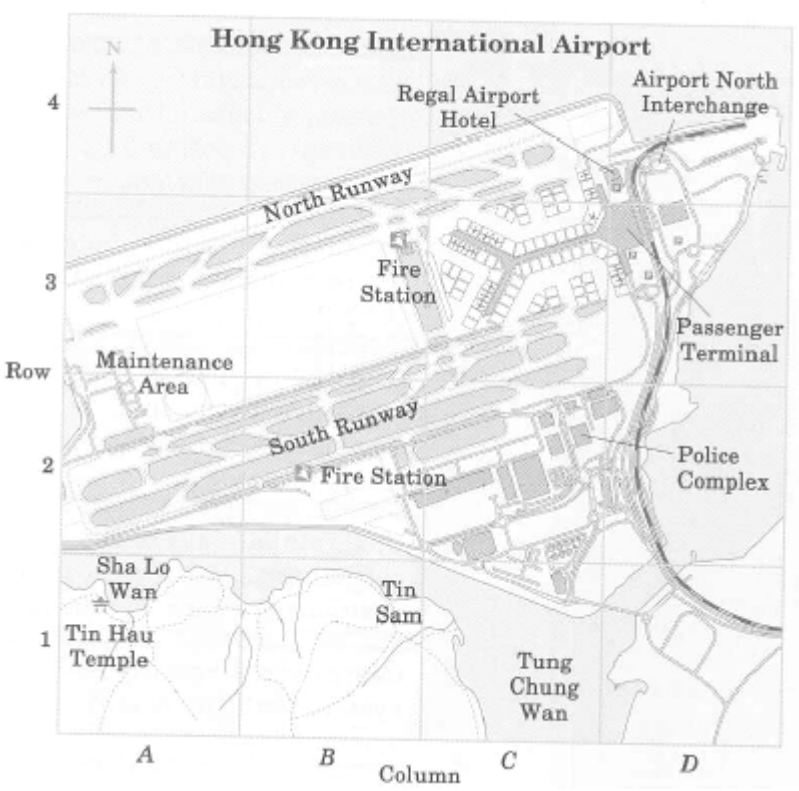
G7 Introduction to Coordinates

G7.1 Rectangular Coordinates

A. Introduction to Coordinate Systems

How can we represent positions in a plane?

Let us consider the map in the figure, which is split(被切開) by a grid into columns A, B, C, D and rows 1, 2, 3, 4.



Consider the position of the Passenger Terminal in the figure. Since it is located in column D and row 3, we can use D3 to represent its position on the map. This kind of method for representing positions is called a coordinate system (坐標系統).

Checkpoint 1

Refer to the figure, write down the positions of the following places.

(a) Airport North Interchange _____

(b) Sha Lo Wan _____

(c) Police Complex _____

(d) The fire station near the South Runway _____

Discussion

Refer to the figure again. What is the position of the Regal Airport Hotel, C4 or D4? Discuss the limitations of representing positions in this way.

_____

_____

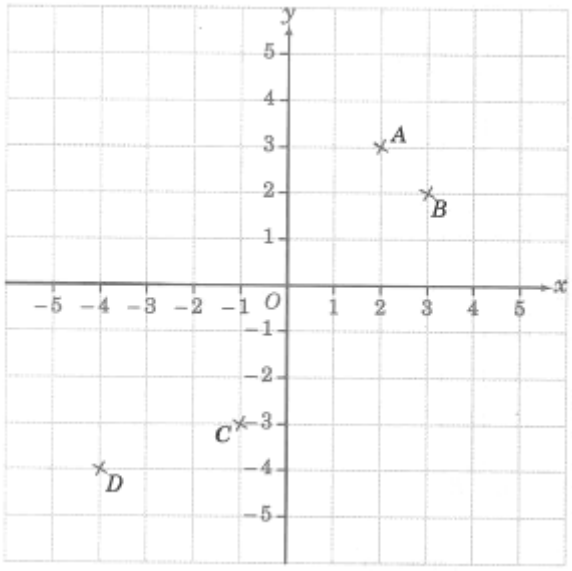
_____

We have seen that the system of representing positions as discussed have its limitations. Obviously it is not suitable for representing positions precisely, since the system intends to locate the position of a region (範圍) but not a point.

Can we have a coordinate system that allows people to represent the position of *any* "point" in the plane accurately?

B. Rectangular Coordinate System

In the 17th century, the French mathematician Descatres invented a coordinate system that locates the position of a point by considering its distances from two axes put in the plane. This plane is called the **Cartesian plane** (笛卡兒平面) or the **rectangular coordinate plane** (直角坐標平面). This coordinate system can represent the position of any point on the plane accurately. It is very useful in advanced study on geometry.



Some special terms in the rectangular coordinate plane:

x-axis: the horizontal number line

y-axis: the vertical number line

origin (O): the intersection of the two axes

Note that the *x*-axis and *y*-axis are perpendicular to each other.

The point A is the intersection of the vertical line through the number 2 on the *x*-axis and the horizontal line through the number 3 on the *y*-axis. This means A is 2 units from the *y*-axis and 3 units from the *x*-axis. We use the ordered pair (2, 3) to represent the position of A. The number 2 is called the *x*-coordinate of A and 3 is called the *y*-coordinate of A. The ordered pair (2, 3) is referred as the coordinates of A.

Similarly,

the coordinates of B are (3, 2),

the coordinates of C are (–1, –3), and

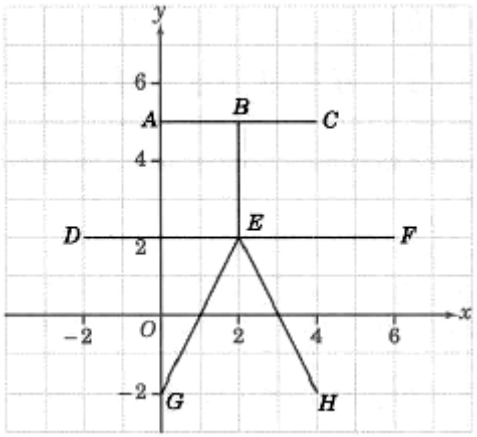
the coordinates of D are (–4, –4).

Note: (1) The ordered pairs (2, 3) and (3, 2) are not the same because they represent different positions A and B respectively.

(2) A point in the rectangular coordinate plane is often denoted by a capital letter together with the coordinates written in brackets, e.g. A(2, 3), B(3, 2).

Checkpoint 2

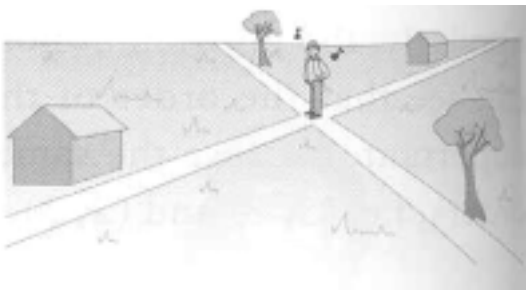
Write down the coordinates of the points A, B, …, H below which form the Chinese character for sky (天).



Example 1

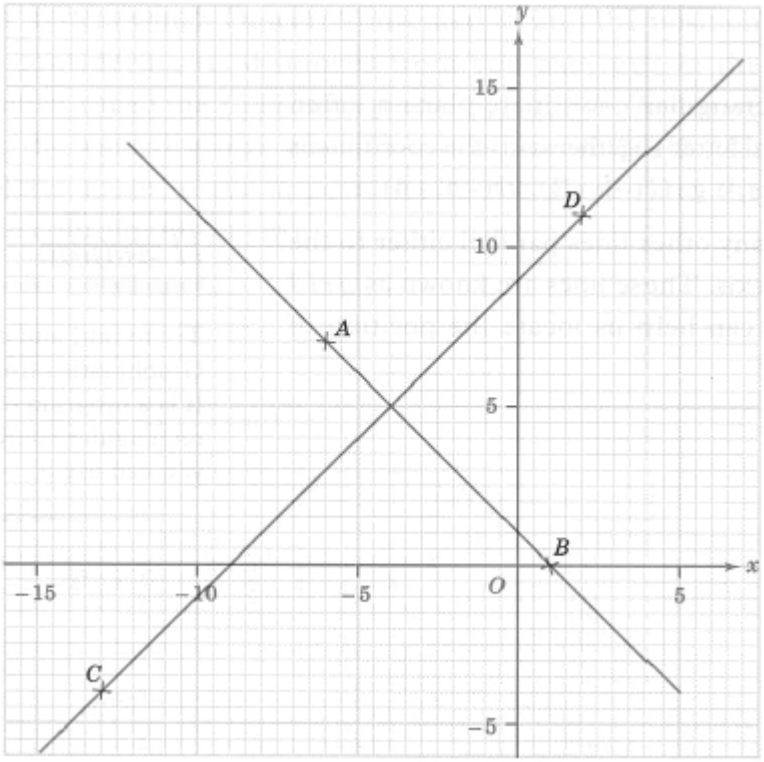
(a) Mark the four points A(–6, 7), B(1, 0), C(–13, –4) and D(2, 11) in the rectangular coordinate plane.

(b) Suppose the situation shown in the figure can be represented by the coordinate plane in part (a), such that A and B represent the trees while C and D represent the houses. Draw a line through A and B and another line through C and D to represent the roads.



Solution

(a), (b)



The singing man is at the intersection of the roads. So the coordinates of the singing man are (–4, 5).

Checkpoint 3

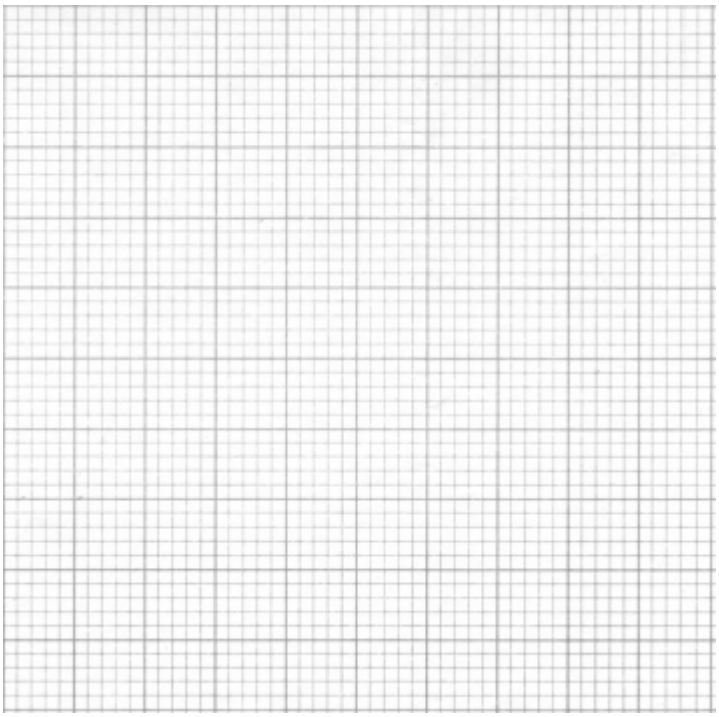
(a) Draw a quadrilateral (四邊形) with vertices M(–4, 1), N(1, 6), P(4, 5) and Q(5, –2) in a rectangular coordinate plane. Is the origin O inside this quadrilateral?

(b) Find the coordinates of the point of intersection of MN and the *y*-axis.

(c) Find the coordinates of the point of intersection of MQ and the *x*-axis.

(d) Find the coordinates of the point of the intersection of the two diagonals.

(a)

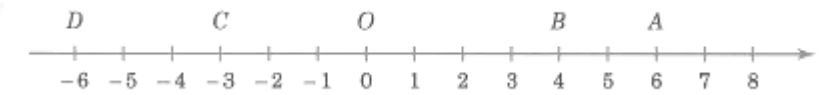


G7.2 Lengths and Areas in the Rectangular Coordinate System

In a rectangular coordinate plane, we can find the length of two points if we know the coordinates of the two points.

At this stage, we will only consider the simple case that the points are on the same horizontal line or vertical line.

The figure shows the horizontal number line.



On the number line, the position of A is 6 while that of B is 4; thus the length of AB = 6 – 4 = 2.
The position of B is 4 and that of C is –3; thus the length of BC = 4 – (–3) = 7.
The position of C is –3 and that of D is –6; thus the length of CD = –3 – (–6) = 3.

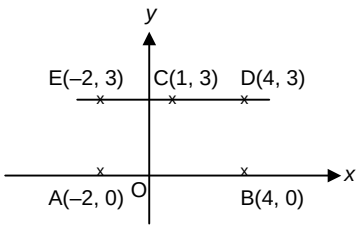
We see that, on a horizontal number line,

the distance between two points = the larger number – smaller number.

This principle can be applied to find the distance between two points on a horizontal number line in the rectangular coordinate plane.

Refer to the figure,

AB = 4 – (–2) = 6
CD = 4 – 1 = 3
CE = 1 – (–2) = 3
DE = 4 – (–2) = 6

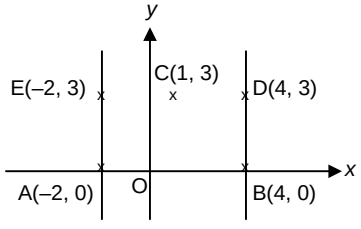


Observe that the points C, D and E have the same *y*-coordinate, but this *y*-coordinate does not play any role in the calculation of distances.

Similarly, the distance between two points on a vertical number line in the rectangular coordinate plane can also be found.

Refer to the figure,

AE = 3 – 0 = 3
DB = 3 – 0 = 3



Observe that the points (1) A and E; (2) D and B, have the same *x*-coordinate, but these *x*-coordinates do not play any role in the calculation of distances.

Checkpoint 4

Find the distances of the following pairs of points.

(a) A(–2, 3); B(5, 3)

(b) P(0, –5); Q(–6, –5)

(c) X(–1, –3); Y(–1, –7)

Checkpoint 5

Suppose P(–1, 4) and Q(–1, b) are two points on the same vertical line and PQ = 10. Find the two possible values of b.

Example 2

A(–15, 30), B(–15, –20), C(55, –20) and D(55, 30) are four points in a rectangular coordinate plane. Given that ABCD is a rectangle, what is its perimeter?

Solution

AB = 30 – (–20) = 50 units

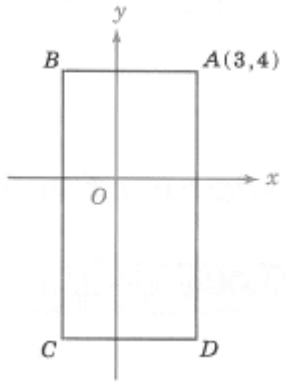
BC = 55 – (–15) = 70 units

$\therefore$ The perimeter of ABCD = $(50 + 70) \times 2$

= 240 units

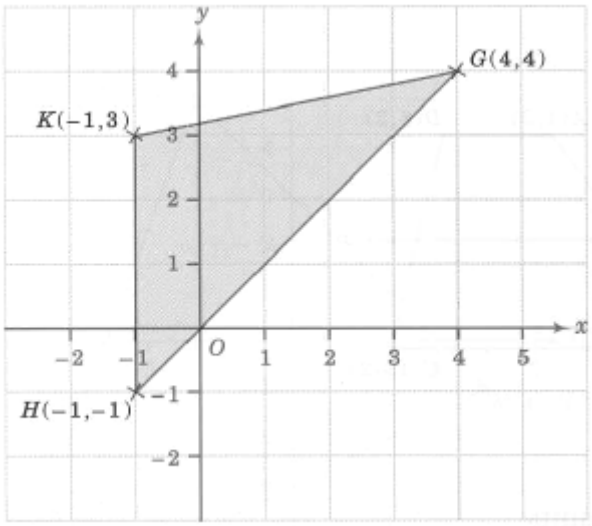
Checkpoint 6

The rectangle ABCD as shown has its sides either horizontal or vertical. The coordinates of A are (3, 4), AB = 5 units and the perimeter of ABCD is 30 cm. Find the coordinates of B, C and D.



Example 3

The figure shows a triangle with vertices at H(–1, –1), K(–1, 3) and G(4, 4).

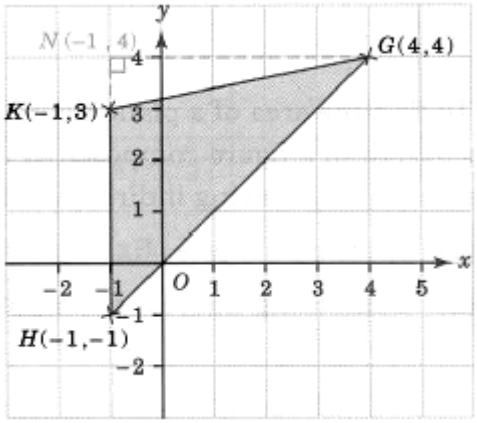


(a) Find the length of HK.

(b) Find the height of ΔHKG with respect to the base HK.

(c) Calculate the area of ΔHKG.

Solution



(a) $HK = 3 - (-1) = 4$ units

(b) Through G, construct a perpendicular to HK to meet HK produced at N. The coordinates of N are (–1, 4).

$\therefore$ height $= GN$

$= 4 - (-1)$

$= 5$ units

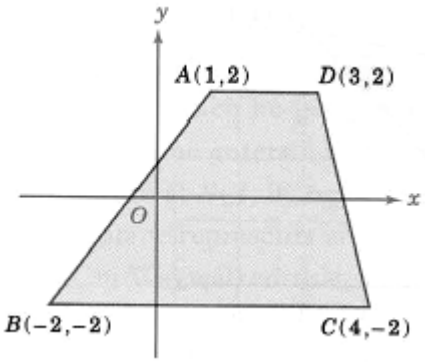
(c) Area of $\Delta HKG = \frac{1}{2} HK \times GN$

$= \frac{1}{2} \times 4 \times 5$

$= 10$ sq. units

Example 4

In the figure, A(1, 2), B(–2, –2), C(4, –2) and D(3, 2) are the four vertices of a trapezium. Find the area of ABCD.



Solution

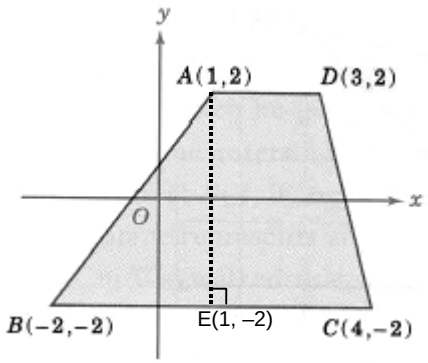
$AD = 3 - 1 = 2$ units

$BC = 4 - (-2) = 6$ units

Construct a perpendicular AE to BC.

Then the coordinates of E are (1, –2).

$AE = 2 - (-2) = 4$ units



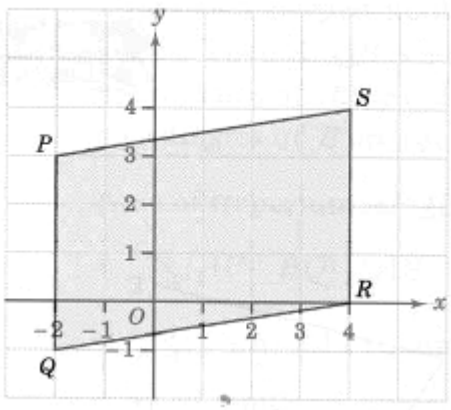
$\therefore$ Area of $ABCD = \frac{1}{2}(AD + BC) \times AE$

$= \frac{1}{2}(2 + 6) \times 4$

$= 16$ sq. units

Checkpoint 7

The figure shows the figure of a parallelogram PQRS. Find its area.



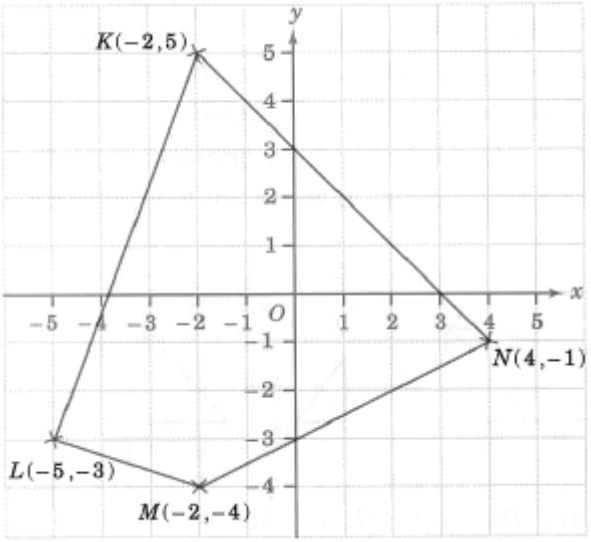
To find the area of a plane figure which has no horizontal or vertical sides, or a figure composed of two or more simple figures, we have to use the following indirect methods:

(1) Splitting a figure;

(2) Combining figures.

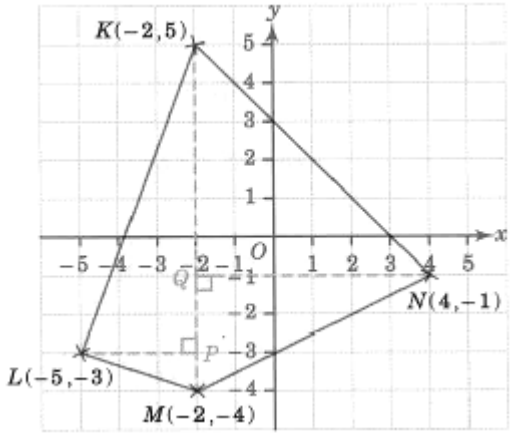
Example 5

In the figure, the vertices of the quadrilateral are K(–2, 5), L(–5, –3), M(–2, –4) and N(4, 1). Find the area of the quadrilateral.



Solution

Join K and M. With the notation, the coordinates of P and Q are (–2, –3) and (–2, –1).



$$\text{KM} = 5-(-4) = 9 \text{ units}$$

$$\text{LP} = -2-(-5) = 3 \text{ units}$$

$$\text{NQ} = 4-(-2) = 6 \text{ units}$$

$$\text{Area of } \Delta\text{KLM} = \frac{1}{2}\text{KM}\times\text{LP}$$

$$= \frac{1}{2}\times 9\times 3$$

$$= 13.5 \text{ sq. units}$$

$$\text{Area of } \Delta\text{KMN} = \frac{1}{2}\text{KM}\times\text{NQ}$$

$$= \frac{1}{2}\times 9\times 6$$

$$= 27 \text{ sq. units}$$

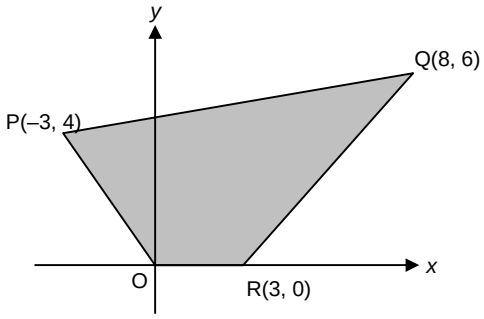
$$\therefore \quad \text{Area of KLMN} = 13.5 + 27$$

$$= 40 \text{ sq. units}$$

Can we find the area of KLMN easily by joining L and N instead of joining K and M? Why?

Example 6

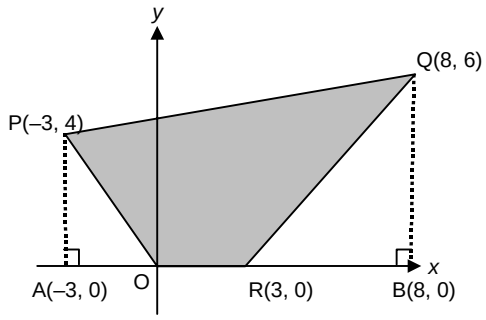
Find the area of the quadrilateral in the figure.



Solution

Draw two perpendiculars PA and QB to the *x*-axis. Then APQB is a trapezium.

The coordinates of A and B are (–3, 0) and (8, 0) respectively.



$$\text{AP} = 4 - 0 = 4 \text{ units}$$
$$\text{OA} = 0 - (-3) = 3 \text{ units}$$
$$\text{OR} = 3 - 0 = 3 \text{ units}$$
$$\text{BR} = 8 - 3 = 5 \text{ units}$$
$$\text{BQ} = 6 - 0 = 6 \text{ units}$$

$$\text{Area of trapezium APQB} = \frac{1}{2}(\text{AP} + \text{BQ}) \times \text{AB}$$
$$= \frac{1}{2} \times (4 + 6) \times 11$$
$$= 55 \text{ sq.units}$$

$$\text{Area of } \Delta\text{OAP} = \frac{1}{2} \times \text{OA} \times \text{AP}$$
$$= \frac{1}{2} \times 3 \times 4$$
$$= 6 \text{ sq.units}$$

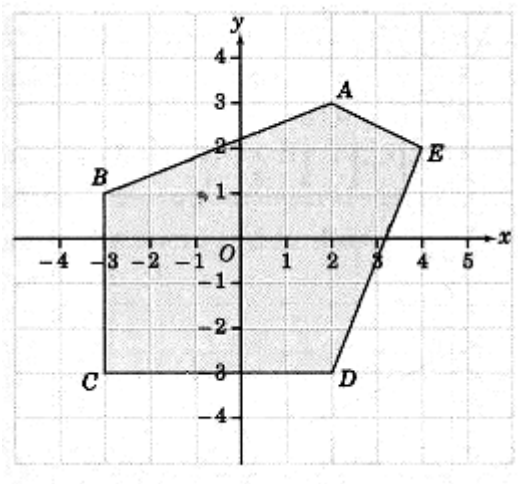
$$\text{Area of } \Delta\text{RBQ} = \frac{1}{2} \times \text{RB} \times \text{BQ}$$
$$= \frac{1}{2} \times 5 \times 6$$
$$= 15 \text{ sq.units}$$

$\therefore$ $\text{Area of OPQR} = \text{area of trapezium APQB}$
$- \text{area of } \Delta\text{OAP} - \text{area of } \Delta\text{RBQ}$
$= 55 - 6 - 15$
$= 34 \text{ sq.units}$

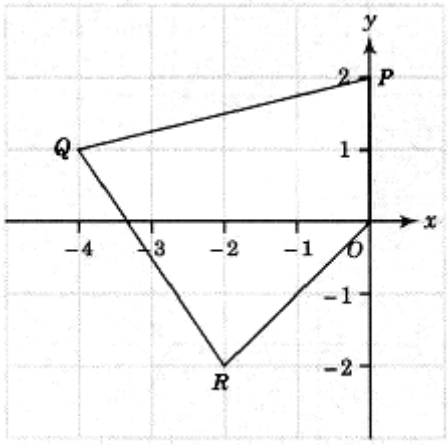
Checkpoint 8

Find the areas of the following polygons.

(a)



(b)

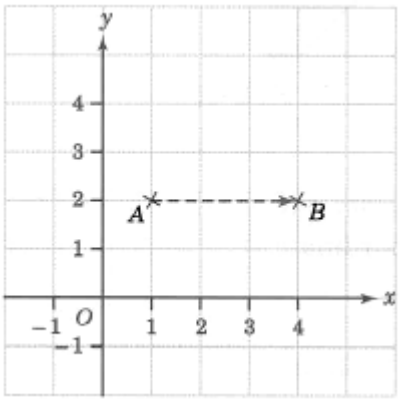


G7.3 Transformations in a Rectangular Coordinate Plane

A. Translation

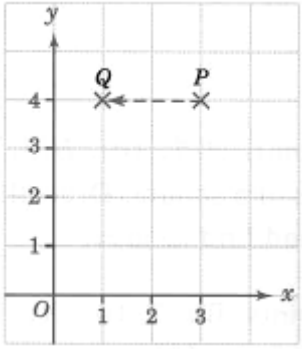
Checkpoint 9

(a)



In the figure, A(1, 2) translate 3 units to the right to reach B. What are the coordinates of B?

(b)



In the figure, P(3, 4) translate 2 units to the left to reach Q. What are the coordinates of Q?

From the above practice, we can see that if P(x, y) is translated n units to the right (i.e. in the direction of the positive x-axis), then

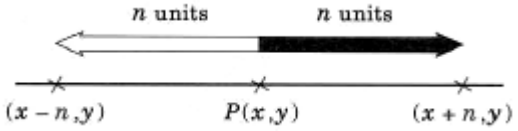
(1) the y-coordinate stays the same, and

(2) the x-coordinate is increased by n.

On the other hand, if P(x, y) is translated n units to the left (i.e. in the direction of the negative x-axis), then

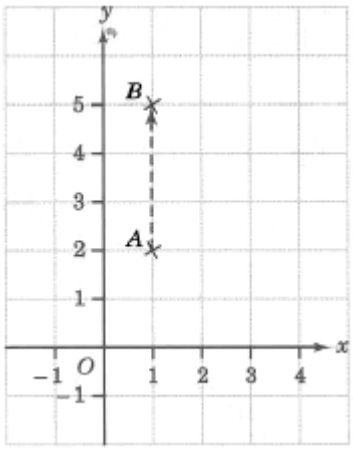
(1) the y-coordinate stays the same, and

(2) the x-coordinate is decreased by n.



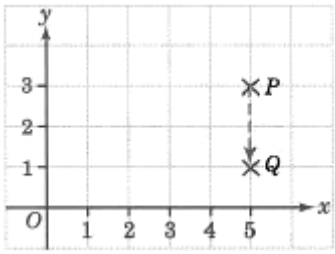
Checkpoint 10

(a)



In the figure, A(1, 2) translate 3 units upward to reach B. What are the coordinates of B?

(b)

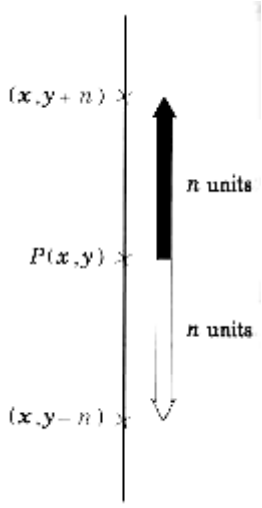


In the figure, P(5, 3) translate 2 units downward to reach Q. What are the coordinates of Q?

From the practice, we can see that, if P(x, y) is translated n units upward or downward, then the x-coordinate remains unchanged, while

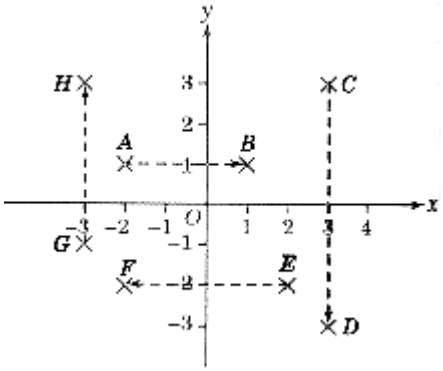
(1) the y-coordinate is increased by n units for an upward translation.

(2) the y-coordinate is decreased by n units for a downward translation.



Checkpoint 11

Refer to the figure on the right, complete the following sentences.



(a) A is translated _____ units _____ to B.

The coordinates of B are _____.

(b) C is translated _____ units _____ to D.

The coordinates of D are _____.

(c) E is translated _____ units _____ to F.

The coordinates of F are _____.

(d) G is translated _____ units _____ to H.

The coordinates of H are _____.

Checkpoint 12

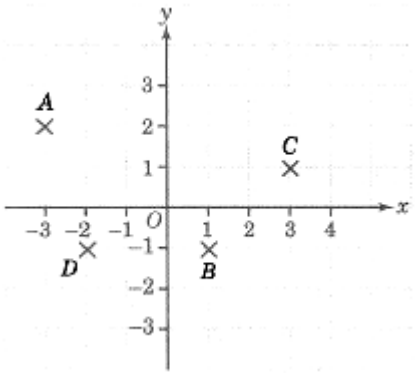
Plot in the figure the images A'. B', etc. of the following translation.

(a) Point A is translated 5 units downward to A'.

(b) Point B is translated 3 units upward to B'.

(c) Point C is translated 1 unit to the left to C'.

(d) Point D is translated 4 units to the right to D'.



B. Reflection

Reflection in the Axes

Checkpoint 13

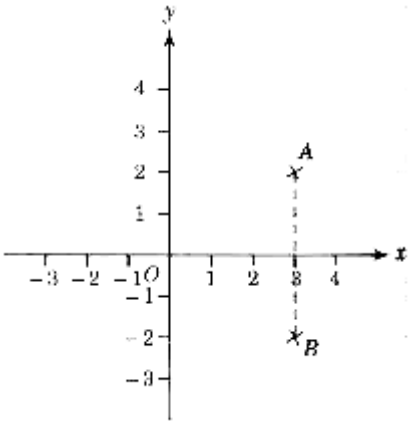
(a) In the figure, A is reflected to B with the *x*-axis as the axis of reflection. We also say that B is the image of A when A is reflected in the axis.

Distance of A from the *x*-axis = _____ units.

Distance of B from the *x*-axis = _____ units.

The coordinates of A are _____.

The coordinates of B are _____.



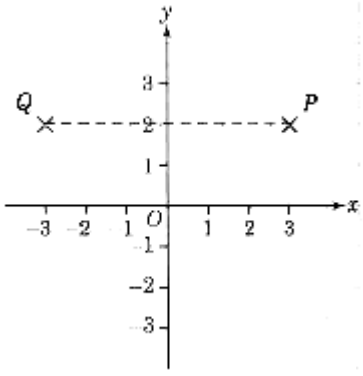
(b) In the figure, P is reflected to Q with the *y*-axis as the axis of reflection.

Distance of P from the *x*-axis = _____ units.

Distance of Q from the *x*-axis = _____ units.

The coordinates of P are _____.

The coordinates of Q are _____.

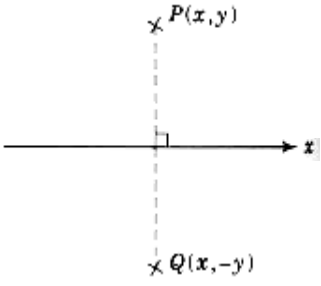


From the practice above, we can see that if P(x, y) is reflected in the *x*-axis, then

(1) the *x*-coordinate stays the same, and

(2) the *y*-coordinate is numerically equal to the original, but its sign is opposite.

Thus P(x, y) is reflected in the *x*-axis to Q(x, $-y$).

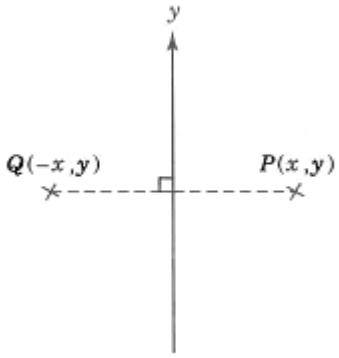


On the other hand, if P(x, y) is reflected in the y-axis, then

(1) the y-coordinate stays the same,

(2) the x-coordinate is numerically equal to the original, but its sign is opposite.

Thus P(x, y) is reflected in the y-axis to Q($-x$, y).

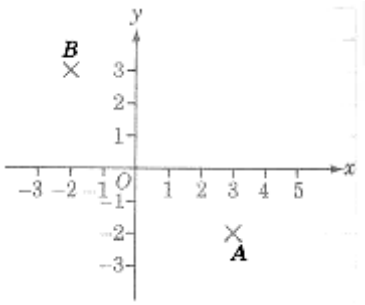


Note that the line joining the original point and its image is perpendicular to the axis of reflection.

Checkpoint 14

Complete the following by referring to the figure.

	Original point	Reflection	Image
(a)	A	about x-axis	(___, ___)
(b)	A	about y-axis	(___, ___)
(c)	B	about x-axis	(___, ___)
(d)	B	about y-axis	(___, ___)



Reflection in a Horizontal Line and a Vertical Line

Checkpoint 15

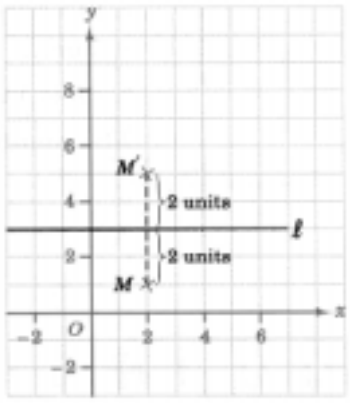
(a) In the figure, M is reflected to M' with the horizontal line l as the axis of reflection.

Distance of M from the line l = _____ units.

Distance of M' from the line l = _____ units.

The coordinates of M are _____.

The coordinates of M' are _____.



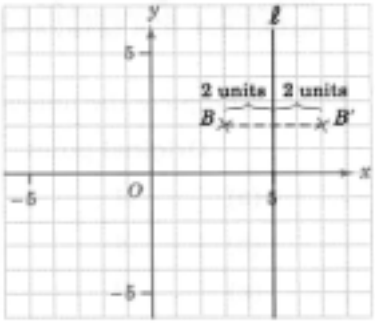
(b) In the figure, B is reflected to B' with the vertical line l as the axis of reflection.

Distance of B from the line l = _____ units.

Distance of B' from the line l = _____ units.

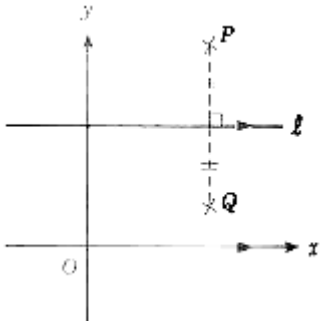
The coordinates of B are _____.

The coordinates of B' are _____.

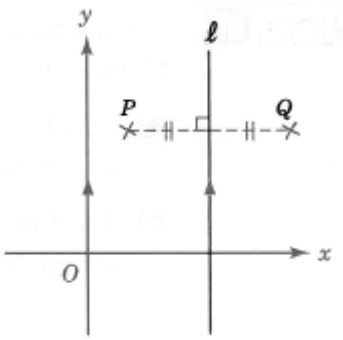


From the practice above, we can see that:

(1) If the point P is reflected in a horizontal line l to the point Q, then P and Q have the same *x*-coordinate and they are equidistance from the line l.



(2) If the point P is reflected in a vertical line l to the point Q, then P and Q have the same *y*-coordinate and they are equidistance from the line l.



Example 7

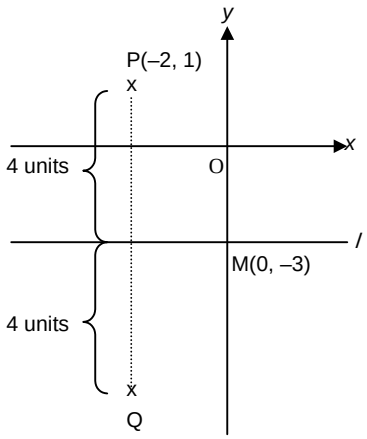
l is a line parallel to the *x*-axis and it passes through M(0, –3).

(a) Find the coordinates of the point Q if it is the image when the point P(–2, 1) is reflected in l.

(b) Find the coordinates of the point R if it is the image when M is reflected in a vertical line through Q.

Solution

[A rough diagram may be useful.]



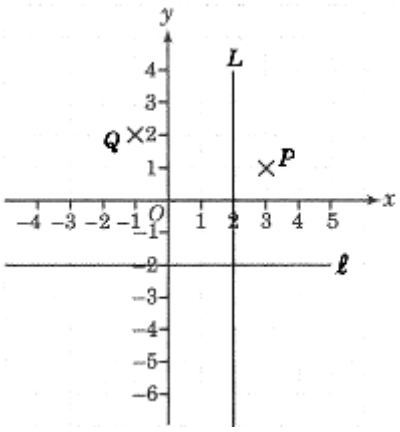
(a) Distance of P(–2, 1) from l = 1 – (–3) = 4 units
Thus, Q is 8 units below P.
The coordinates of Q are (–2, 1 – 8), i.e. (–2, –7).

(b) Distance of M(0, –3) from PQ = 0 – (–2) = 2 units
Thus R is 4 units to the left of M.
The coordinates of R are (0 – 4, –3), i.e. (–4, –3).

Checkpoint 16

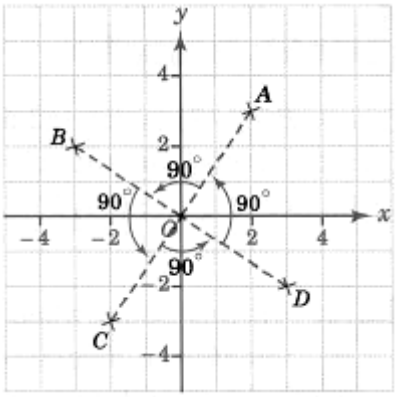
In the figure, l is a horizontal line while *L* is vertical. Complete the following by referring to the figure.

	Original point	Reflection	Image
(a)	P	about *L*	(___, ___)
(b)	P	about l	(___, ___)
(c)	Q	about *L*	(___, ___)
(d)	Q	about l	(___, ___)



C. Rotation

The figure shows four points A, B, C and D in the rectangular coordinate plane. When point A is rotated anti-clockwise (↶) about the origin O through 90^o, it reaches B. In symbols, this transformation can be represented by:



$$A(2, 3) \xrightarrow{90^o} B(-3, 2)$$

Checkpoint 17

Referring to the figure again, complete the following:

(a) $B(-3, 2) \xrightarrow{90^o} C(\_\_\_\_\_, \_\_\_\_\_)$

(b) $C(\_\_\_\_\_, \_\_\_\_\_) \xrightarrow{90^o} D(\_\_\_\_\_, \_\_\_\_\_)$

(c) $D(\_\_\_\_\_, \_\_\_\_\_) \xrightarrow{90^o} A(2, 3)$

(d) $A(2, 3) \xrightarrow{180^o} \_\_\_ (\_\_\_\_\_, \_\_\_\_\_)$

(e) $B(-3, 2) \xrightarrow{180^o} \_\_\_ (\_\_\_\_\_, \_\_\_\_\_)$

(f) Can you discover some patterns concerning the coordinates of a point and its image after it rotated anti-clockwise about O through (i) 90^o and (ii) 180^o?

In fact, the results of the practice verify that: When the point P(x, y) is rotated anti-clockwise about the origin O through 90^o to the point Q, then the coordinates of Q will be ($-y$, x).

i.e. $P(x, y) \xrightarrow{90^o} Q(-y, x)$

Also, if P is rotated anti-clockwise about O through 180^o to the point R, then the coordinates of R will be ($-x$, $-y$).

i.e. $P(x, y) \xrightarrow{180^o} R(-x, -y)$

Checkpoint 18

Write down, in coordinates, the transformations T_1, T_2 and T_3 such that

(a) T_1 is a transformation that rotates a point clockwise about O through 180^o to a new point.

(b) T_2 is a transformation that rotates a point anti-clockwise about O through 270^o to a new point.

(c) T_3 is a transformation that rotates a point clockwise about O through 90^o to a new point.

(a) T_1: $(x, y) \xrightarrow{180^o} (-x, -y)$

(b)

(c)

Checkpoint 19

Refer to the figure and complete the following.

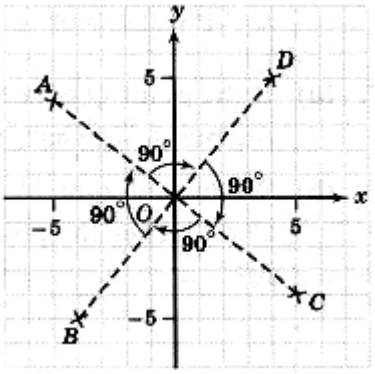
[*Part (a) is done for you as an example.*]

(a) $A(-5, 4) \xrightarrow{90^o}$ D(4, 5)

(b) B(_____, _____) $\xrightarrow{90^o}$ _____

(c) C(_____, _____) $\xrightarrow{270^o}$ _____

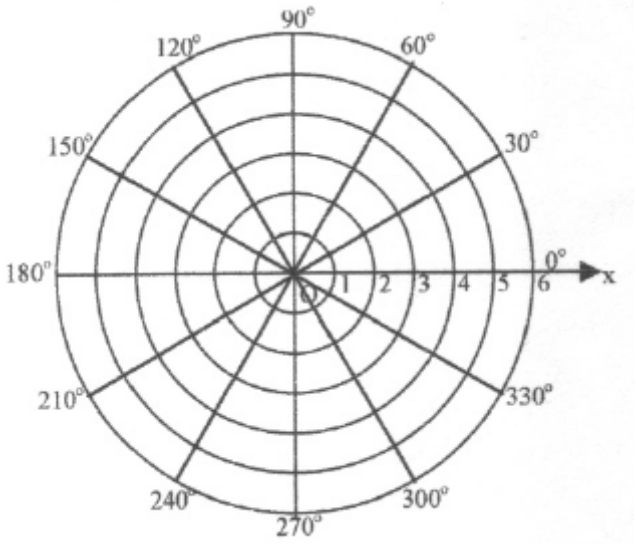
(d) D(_____, _____) $\xrightarrow{360^o}$ _____



G7.4 Polar Coordinates

A. Introduction to Polar Coordinates

The polar coordinate system is another kind of coordinate system which has wide applications in real life. For example, airports use polar coordinates to identify the precise positions of planes on the runway relative to the control tower.

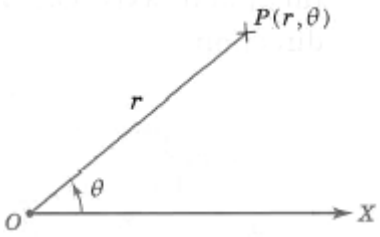


In the polar coordinate system, there is a fixed point of reference O, called the pole (極點), relative to which distances and directions are measured. There is a single horizontal axis OX, called the polar axis (極軸). The entire plane is called the polar coordinate plane (極坐標平面).

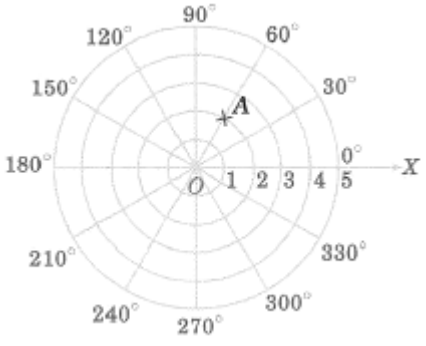
Suppose P is any point in the plane. The distance OP is called the radius vector (極徑), while the angle measured anti-clockwise from the polar axis to OP is called the polar angle (極角). Thus,

if the radius vector = r, the polar angle = θ,

then the polar coordinates pf P are (r, θ).

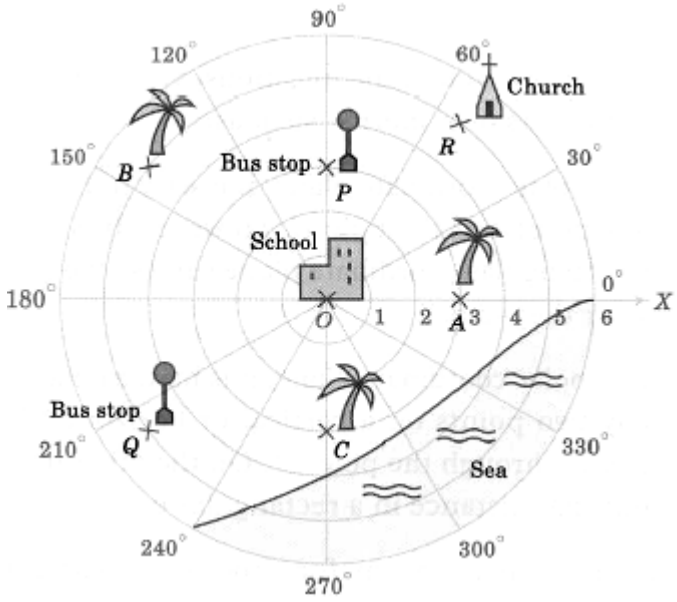


In the figure, the polar coordinates of A are $(2, 60^{o})$.



Checkpoint 20

Pora has produced a map of the school's neighbourhood using polar coordinate system.

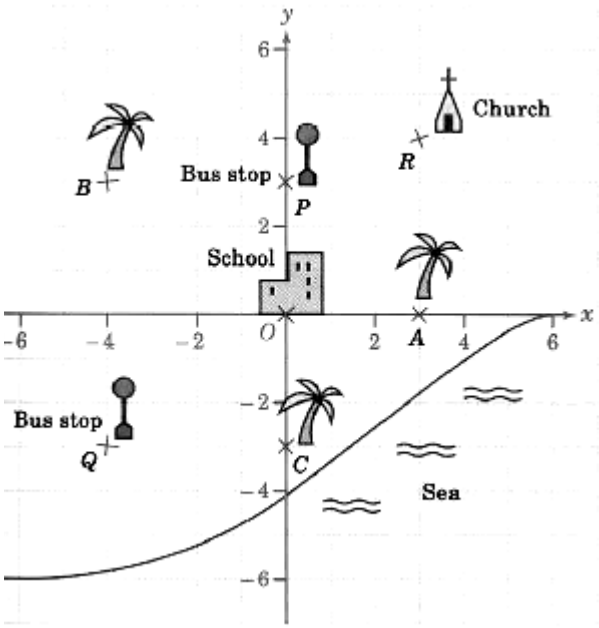


Write down the coordinates of the following points.

Point	O	A	B	C	P	Q	R
Coordinates							

B. Comparison between Rectangular and Polar Coordinates

Victor has also produced a map of the school's neighbourhood using rectangular coordinate system.



He has also written down the coordinates of the points.

Point	O	A	B	C	P	Q	R
Coordinates	(0, 0)	(3, 0)	(–4, 3)	(–3, 0)	(0, 3)	(–4, –3)	(3, 4)

Checkpoint 21

Referring to the maps produces by Pora and Victor again, answer the following questions.

(a) (i) According to Victor's map, what is the distance of tree A from the school O? Is the answer the same using Pora's map?

_____

(ii) According to Victor's map, what is the distance of bus stop P from school O? Is the answer the same using Pora's map?

_____

(b) (i) Find the distance between bus stop Q and tree B from Victor's map. _____

(ii) Find the distance between bus stop Q and tree C from Victor's map. _____

(iii) Is it easier to find the answers to (a) if we use Pora's map? _____

(iv) What is special about line segments BQ and QC? _____

(c) (i) Find the distance between church R and school O from Pora's map. _____

(ii) Find the distance between tree B and school O from Pora's map. _____

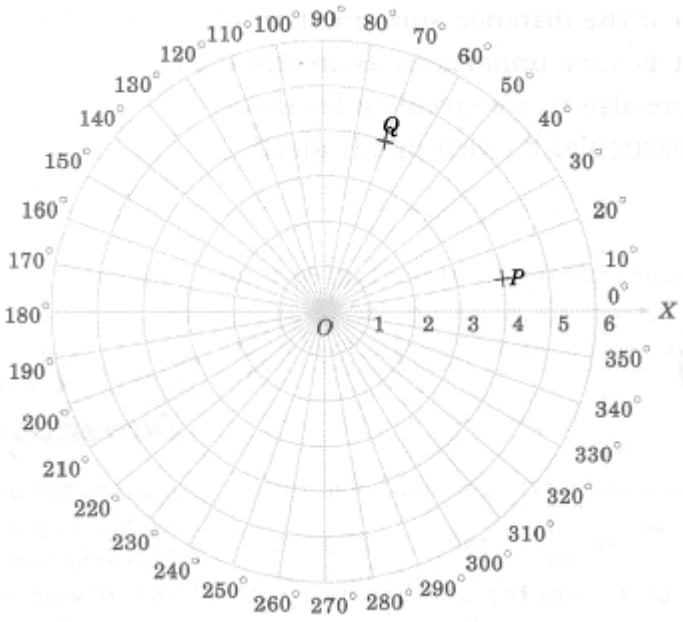
(iii) Is it easier to find the answers to (a) by using Victor's map? _____

From the above practice, we can see that:

(1) In a polar coordinate plane, it is easy to find the distance between any point and the pole O. However, it is not easy to do so in a rectangular coordinate plane (unless the point lies on either the *x*- or *y*-axis).

(2) In a polar coordinate plane, it is hard to find the distance between any two points on a vertical or horizontal line (unless the line passes through the pole O). On the other hand, it is easy to work out that distance in a rectangular coordinate plane.

Checkpoint 22



(a) Write down the polar coordinates of the points P and Q in the above polar coordinate plane.

(b) Prove that ΔOPQ is an equilateral triangle.

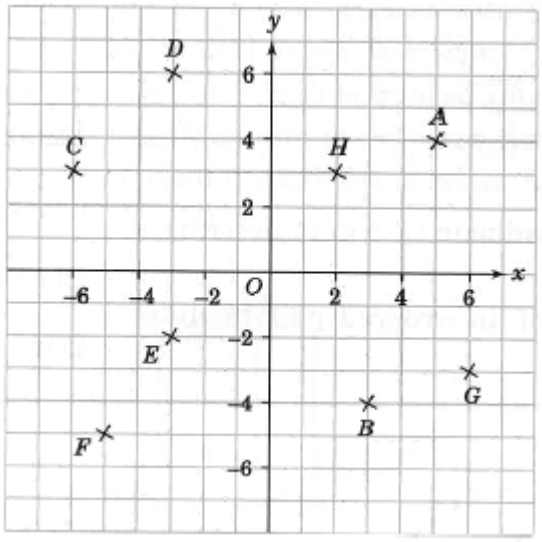
(c) (i) Plot the points R(4, 130^{o}), S(4, 190^{o}), T(4, 250^{o}) and U(4, 310^{o}), and join the points P, Q, R, S, T, U, P in order. Name the polygon formed.

(ii) Do the vertices of PQRSTU lie on a circle? Explain.

Exercise G7
Introduction to Coordinates

G7.1

1. Write down the coordinates of each point in the following figure.



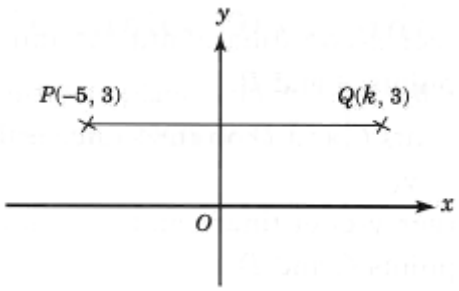
2. Plot the following points in the rectangular coordinate plane on a graph paper.
A(1, 3), B(–4, 3), C(6, –2), D(–5, –4), E(7, 7), F(2, –6), G(–4, –7) and H(–8, 5)

3. In each of the following
   (i) plot the given points in the rectangular coordinate plane provided;
   (ii) join the points in the given order;
   (iii) state the kind of figure that is formed in (ii).
   (a) Given: A(4, 0), B(–6, –5), C(–10, 10)
   Join: A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ A
   (b) Given: A(–8, 4), B(2, 7), C(2, –3), D(–8, –1)
   Join: A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ D $\rightarrow$ A

G7.2

4. In each of the following, find the distance between the two given points A and B.
   (a) A(4, 1), B(1, 1)
   (b) A(–7, 1), B(–7, 3)
   (c) A(–6, 4), B(6, 4)
   (d) A(–2, 5), B(–2, –3)

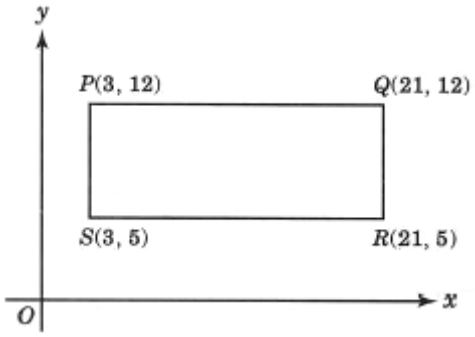
5. Given that the distance between the points P(–5, 3) and Q(k, 3) in the figure is 11 units, find the value of k.



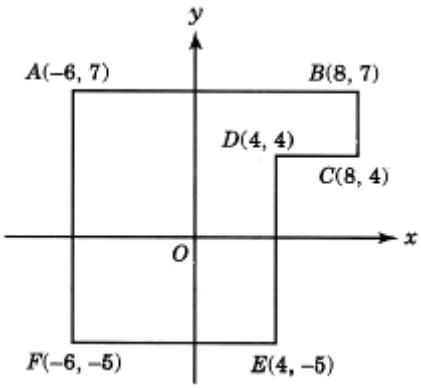
6. If the distance between the two points M($2a$ + 1, 1) and N($4a$ + 3, 1) is 6 units, find the two possible values of a.

7. Find the area of each of the following polygons.

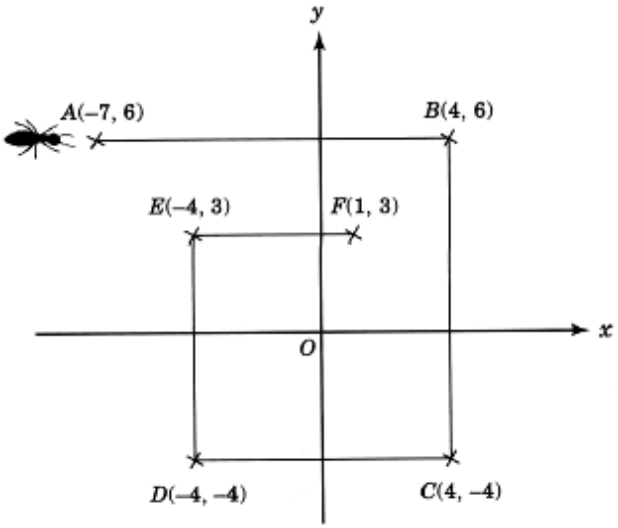
(a)



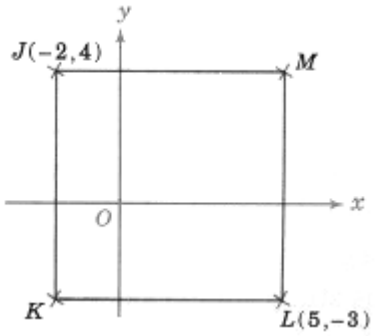
(b)



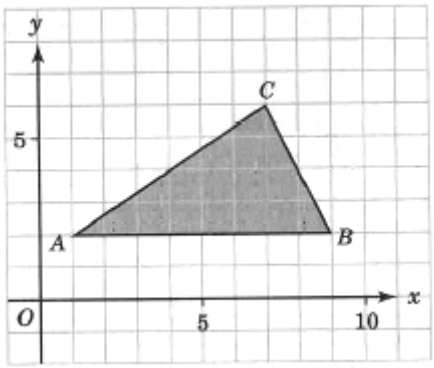
8. In the figure, an ant started from the point A. It walked along AB, BC, CD, DE, EF and arrived at the point F finally. Find the total distance traveled by the ant.



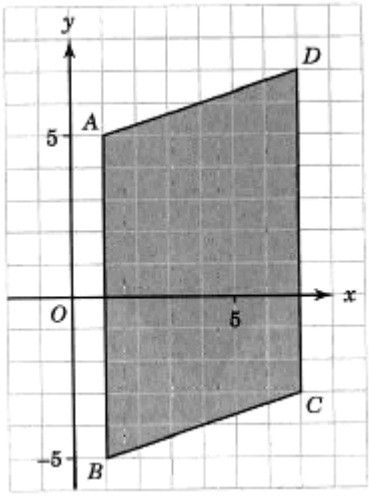
9. The figure shows a square JKLM whose sides are either horizontal or vertical. Find its perimeter and area.



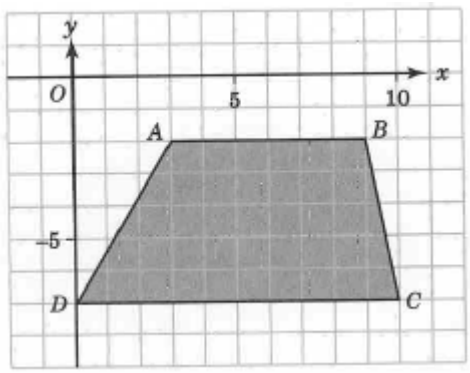
10. In the figure, let AB be the base of the triangle. Find
    (a) the length of the base;
    (b) the length of the height;
    (c) the area of the triangle.



11. In the figure, let AB be the base of the parallelogram. Find the area of the parallelogram.

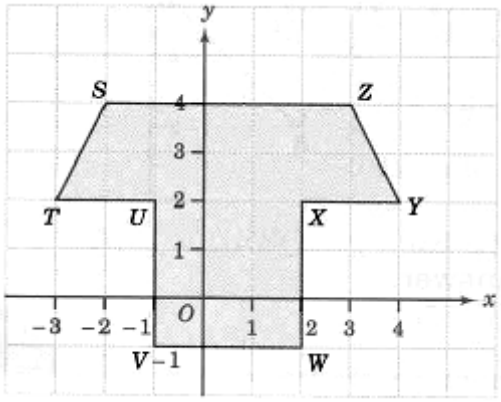


12. The figure shows a trapezium ABCD. Find its area.

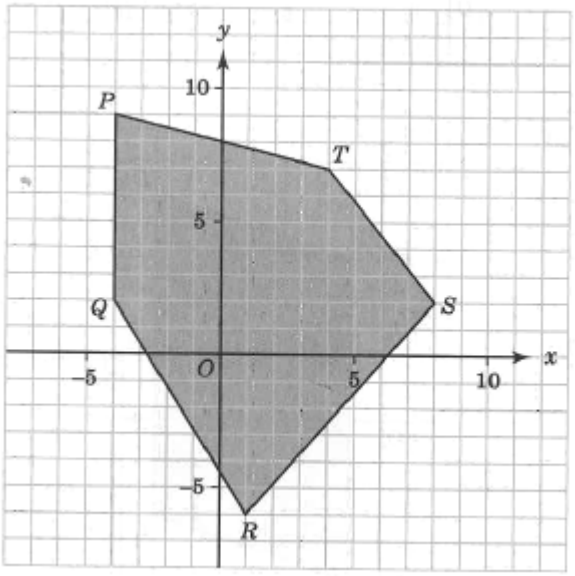


13. Find the area of each of the following figures.

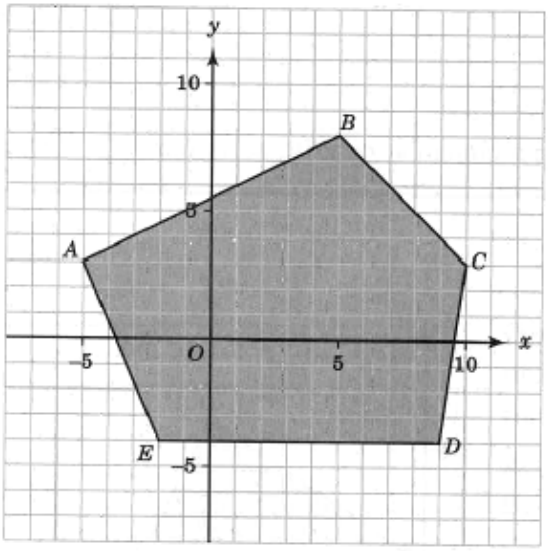
(a)



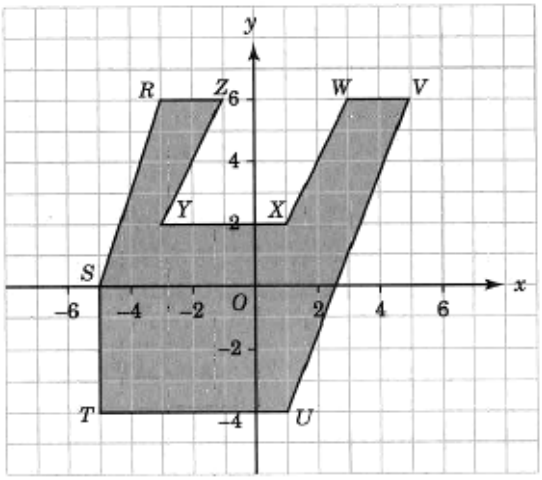
(b)



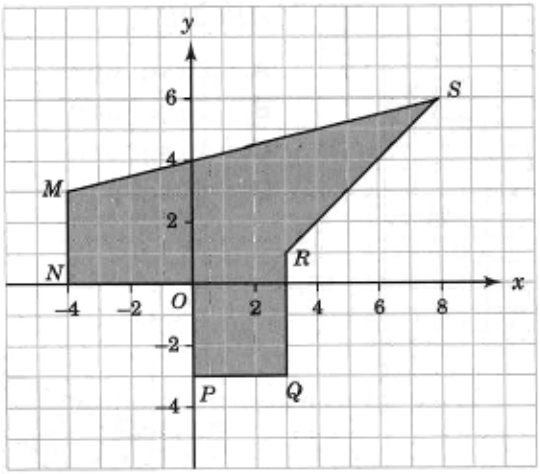
13. (c)


y
10
B
5
A
C
x
−5
O
5
10
E
−5
D

(d)


y
R
Z
6
W
V
4
Y
X
2
S
x
−6
−4
−2
O
2
4
6
−2
T
−4
U

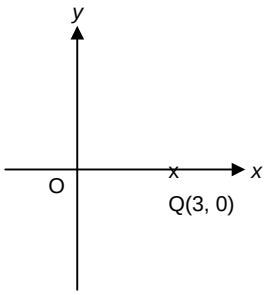
(e)


y
6
S
4
M
2
R
N
x
−4
−2
O
2
4
6
8
−2
P
Q
−4

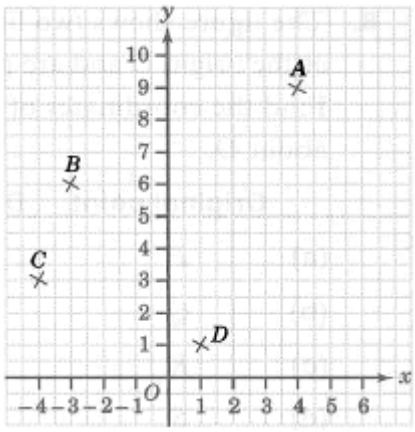
G7.3

14. (a) If the point Q(3, 0) is translated 6 units upward to the point M, what are the coordinates of M?

    (b) When the point N is translated 10 units to the left, it also reaches the point M. What are the coordinates of N?

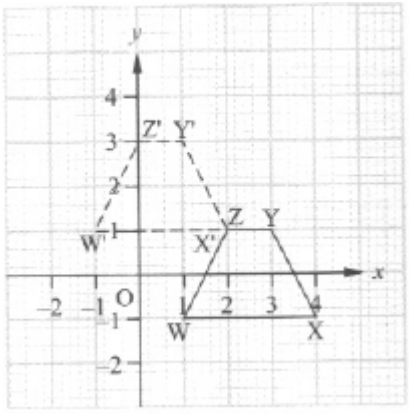


15. If the points A(4, 9), B(–3, 6), C(–4, 3) and D(1, 1) are reflected in the *y*-axis to the points A', B', C' and D' respectively, find the coordinates of these points.



16. Plot the points A(–5, –1), B(–3, –1), C(–3, –3) and D(–5, –3) on a graph paper and join A, B, C, D, A in order. Draw the images of the figure obtained after the translation $(x, y) \rightarrow (x + 1, y - 4)$.

17. The figure shows a trapezium WXYZ and its image W'X'Y'Z' after translation.

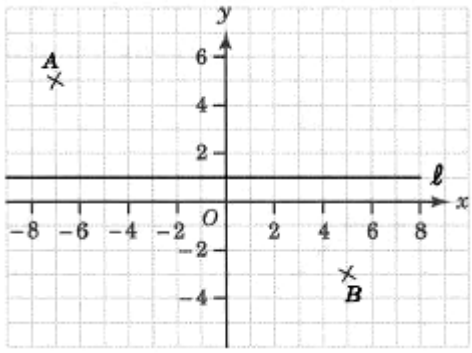


    (a) Write down the change in coordinates of each vertex after translation.
    [*Example: W(1, –1) → W'(___, ___)*]

    (b) Express the translation in terms of coordinates.

18. If the points C(–6, –2), D(2, 1), E(4, –4) and F(–3, 1) are reflected in the *x*-axis to the points C', D', E', F' and F' respectively, find the coordinates of these points.

19. In the figure, l is a line parallel to the *x*-axis and it passes through (0, 1).



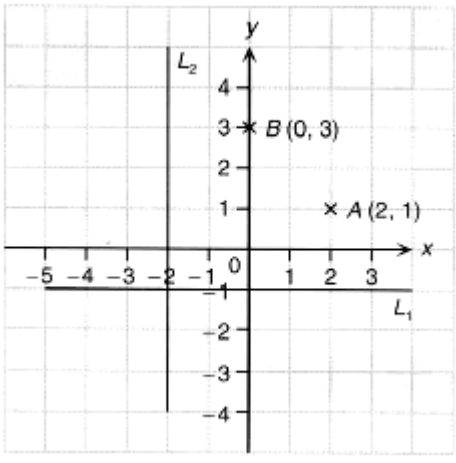
(a) Suppose C is the reflection of the point A(–7, 5) in the line l. Plot this point in the figure together with its coordinates.

(b) Suppose B(5, –3) is the reflection of a point D in the line l. Plot the point D in the figure together with its coordinates.

(c) Join A, C, B, D, A in order. Hence find the perimeter of ACBD.

20. If l is a line parallel to the *x*-axis and it passes through (0, 1), and R and Q(0, 1.5) are the reflections of P(4, –2) and S in the line l respectively, find the coordinates of R and S.

21. In the figure, A and B are two points whose coordinates are (2, 1) and (0, 3) respectively. The line L_1 parallel to the *x*-axis passes through (0, –1) and L_2 parallel to the *y*-axis passes through (–2, 0).



(a) If A_1 is the reflection of A in the line L_1, find the coordinates of A_1.

(b) If B_1 is the reflection of B in the line L_2, find the coordinates of B_1.

(c) Join A, B, B_1, A_1, A in order. What quadrilateral is ABB_1A_1?

22. Given 4 points: P(–5, 6), Q(–6, –5), R(5, –6), S(6, 5), In each of the following, write down the coordinates of the image after rotation.

(a) P is rotated clockwise about O through 360^o.

(b) Q is rotated anti-clockwise about O through 180^o.

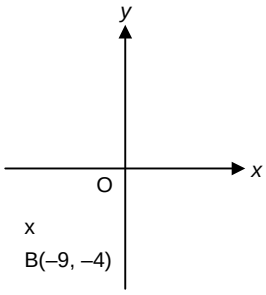
(c) R is rotated clockwise about O through 270^o.

(d) S is rotated anti-clockwise about O through 90^o.

23. The point B in the figure is obtained when a point A is rotated clockwise about O through 270^{o}.

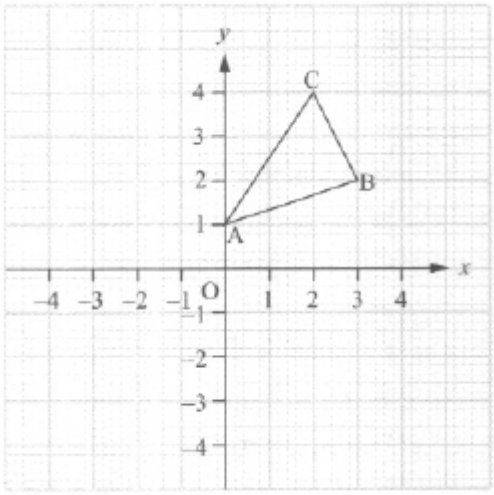
(a) What are the coordinates of A?

(b) If A is rotated about O through 180^{o} to the point C, then what are the coordinates of C?



24. Given that A(6, n + 1) is the image by rotating B(m – 3, 3) clockwise by 90^{o}. Find the coordinates of A and B.

25. (a) In the figure, draw the 3 images of ΔABC after rotating anti-clockwise about the origin by angle of 90^{o} 3 times. Label the images of A as A', A'' and A''' respectively, and label the images of B and C similarly.



(b) Write down a transformation in terms of coordinates that does the **same** thing as in (a), but clockwise direction instead of anti-clockwise direction. (i.e. Using this transformation, the images ΔA'B'C' and ΔA''B''C'', etc. will appear at the same location as in (a).)

G7.4

26. (a) In the polar coordinate plane, plot the points A(5, 60^{o}), B(3, 150^{o}) and C(4, 330^{o}).

(b) Join AB, BC and CA.

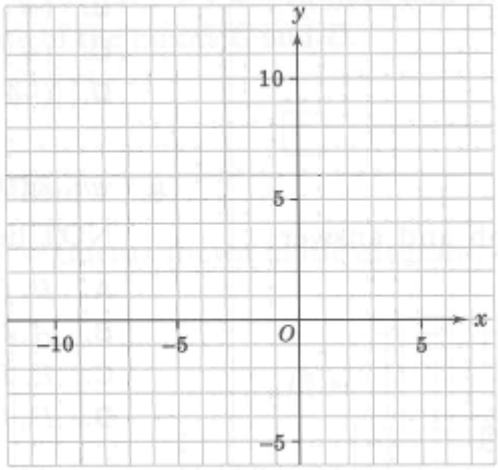
(c) Find the area of ΔABC.

[*Hint: Find the length of BC and ∠AOC.*]

27. (a) In the polar coordinate plane, plot the points A(4, 0^{o}), B(6, 90^{o}), C(3, 180^{o}) and D(5, 270^{o}).

(b) Join AB, BC, CD and DA. Find the area of quadrilateral ABCD.

Figures for Questions 3(a) and 3(b)



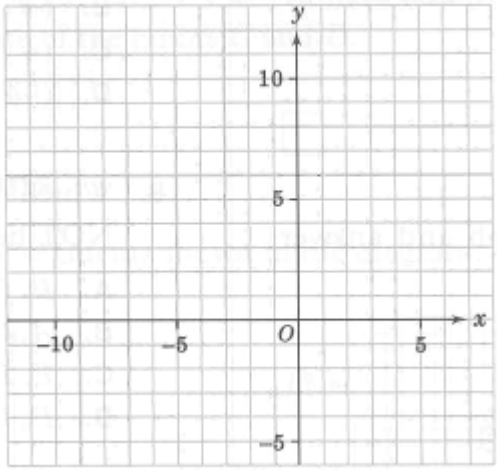


Figure for Question 13(a)

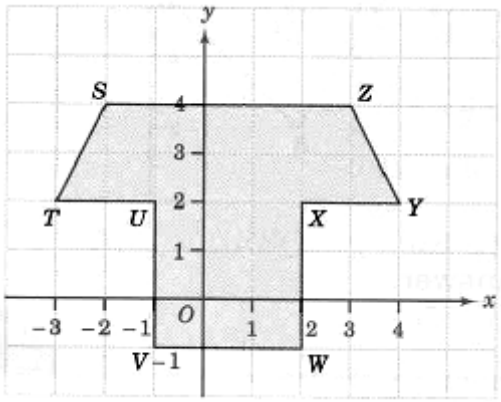


Figure for Question 13(b)

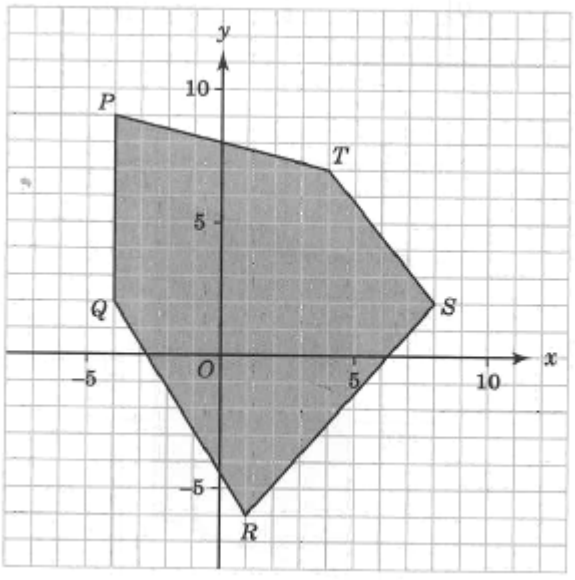


Figure for Question 13(c)

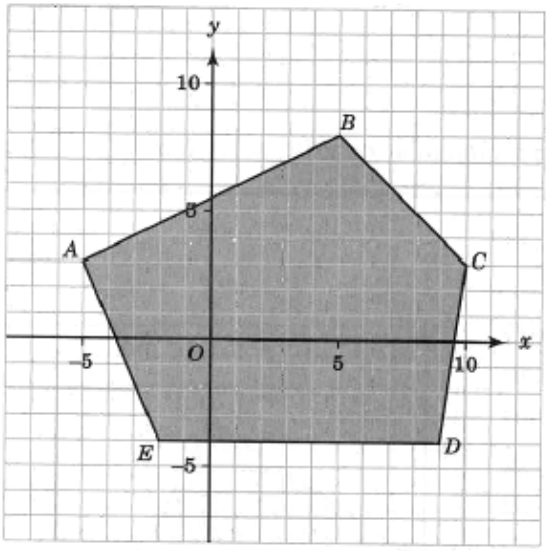


Figure for Question 13(d)

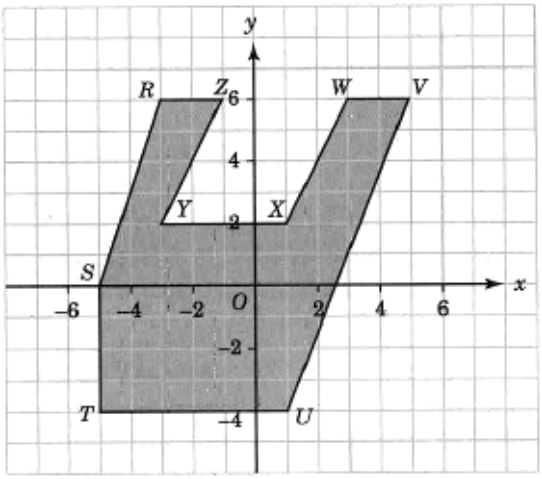


Figure for Question 13(e)

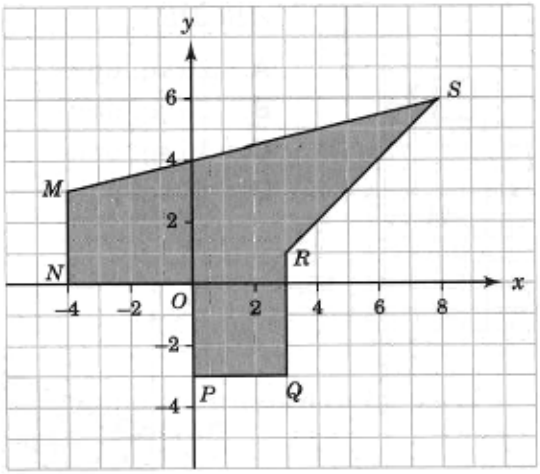


Figure for Question 17

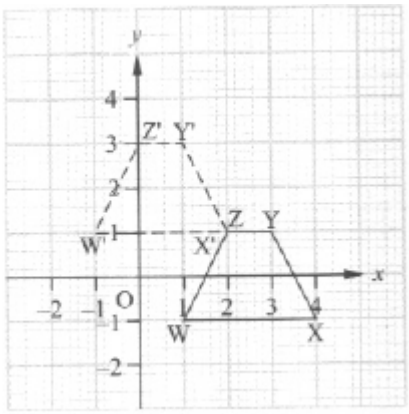


Figure for Question 19

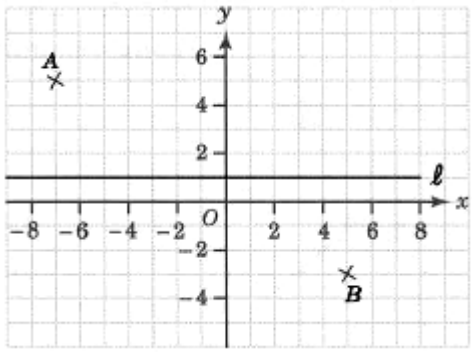
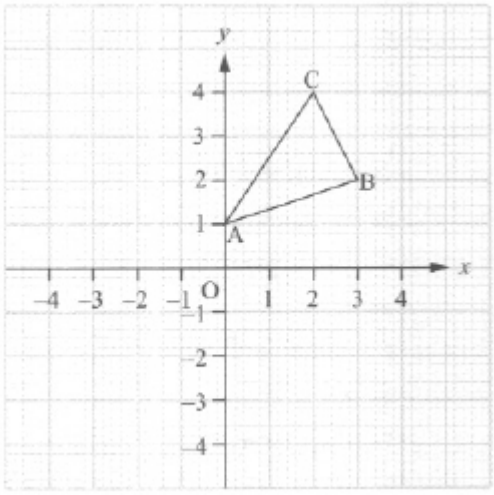
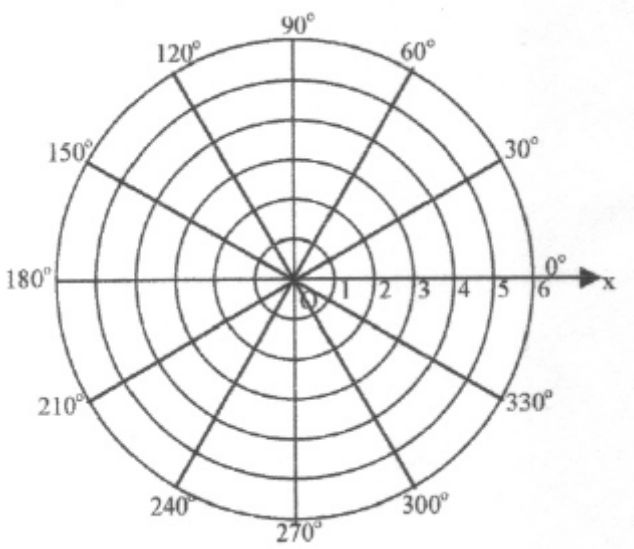


Figure for Question 23



Figures for Question 26



Figures for Question 27