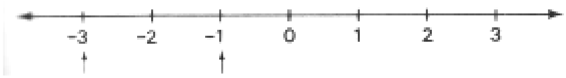
A9 Linear Inequalities in One Unknown

A9.1 Inequalities

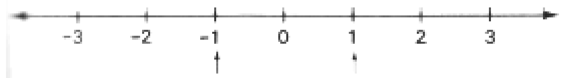
For a given number on the number line, it is greater than any number lying on its left and less than any number lying on its right. We use the symbol ‘>’ to represent ‘is greater than’ and the symbol ‘<’ to represent ‘is less than’.

e.g. (1) –3 lies on the left of –1,



hence –1 is greater than –3, i.e. –1 > –3.

(2) 1 lies on the right of –1,



hence –1 is less than 1, i.e. –1 < 1.

It is obvious that for any two numbers x and y, one of the following statements must be true:

(1) $x > y$,

(2) $x < y$,

(3) $x = y$.

This is known as the **law of trichotomy**.

A. Meaning of Inequality

The expression $x = y$ is called an equation while the expression $x > y$, $x < y$ are each called an inequality.

In fact, an inequality is a mathematical sentence in which two expressions are connected by a symbol of inequality. Some common used symbols of inequality are: $\neq$, >, <, $\geq$ and $\leq$.

Note: (1) ‘$\geq$’ means ‘is greater than or equal to’.

e.g. $x \geq 8$ means the variable x is greater than or equal to 8.

(2) ‘$\leq$’ means ‘is less than or equal to’.

e.g. $x \leq 5$ means the variable x is less than or equal to 5.

B. *Solutions of Inequalities*

For an inequality in one variable x, all values of x that can satisfy the inequality are called the **solutions** of the inequality.

e.g. (1) Some of the solutions of the inequality $x > -2$ are -1, 0, $-\frac{1}{2}$, etc.

(2) Some of the solutions of the inequality $x < 5$ are 4, 3, -2, etc.

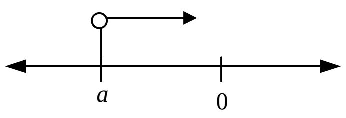
(3) Some of the solutions of the inequality $x \geq -8$ are -8, -7, 1, etc.

(4) Some of the solutions of the inequality $x \leq 5$ are 5, 0.2, -3, etc.

C. *Graphical Representation of Solutions of Inequalities*

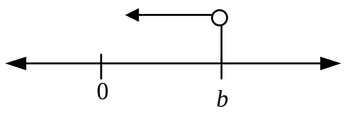
The solutions of an inequality can be represented by means of the number line. The following are the graphical representations of some inequalities on the number line.

(1) $x > a$ (assume a is negative)

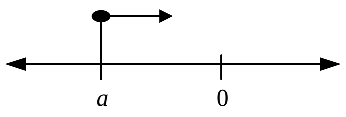


Note: The symbol (○) means that $x = a$ is NOT included as a solution to the inequality.

(2) $x < b$ (assume b is positive)

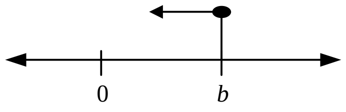


(3) $x \geq a$



Note: The symbol (●) means that $x = a$ is included as a solution to the inequality.

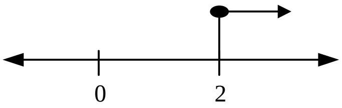
(4) $x \leq b$



Checkpoint 1

In each of the following, represent the solutions of the given inequality graphically: (The first one has been done for you as an example)

(a) $x \geq 2$



(b) $x \geq -2$

(c) $x \leq 5$

(d) $x < -3$

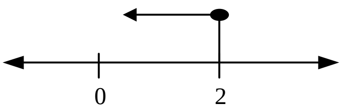
(e) $x \leq \frac{2}{3}$

(f) $x > -\frac{1}{4}$

Checkpoint 2

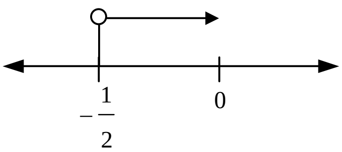
Write an inequality in the form $x > a$, $x < a$, $x \geq a$, $x \leq a$ for each of the following diagrams.

(a)



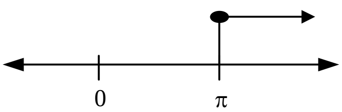
Inequality: ________________

(b)



Inequality: ________________

(c)



Inequality: ________________

D. Fundamental Properties of Inequalities

For any 3 numbers, *a*, *b*, and *c*, we have the following fundamental properties:

(1) If $a > b$ and $b > c$,
then $a > c$
and we may write $a > b > c$.
(Transitive Property)

(2) If $a > b$
then $a + c > b + c$
(Additive Property)

(3) If $a > b$,
then $ac > bc$ when $c > 0$,
or $ac < bc$ when $c < 0$.
(Multiplicative Property)

Note: When multiplying and dividing an inequality by a negative number, the inequality sign is reversed.

(4) If $a \neq 0$, then $a^2 > 0$

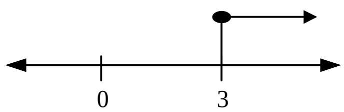
(5) If $a > b$, *a* and *b* are not equal to zero and are of the same sign, then $\frac{1}{a} < \frac{1}{b}$

Example 1

Find all the values of *x* such that $8x - 5 \geq x + 16$ and represent these values on a number line.

Solution

$$8x - 5 \geq x + 16$$
$$8x - x - 5 \geq 16$$
$$7x \geq 16 + 5$$
$$7x \geq 21$$
$$x \geq 3$$

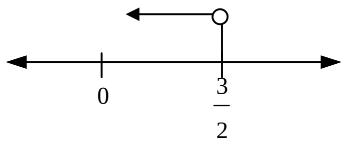


Example 2

Solve the inequality $\frac{4x-1}{2} < \frac{7x+2}{5}$ and represent the solutions graphically.

Solution

$$\frac{4x-1}{2} < \frac{7x+2}{5}$$
$$5(4x-1) < 2(7x+2)$$
$$20x-5 < 14x+4$$
$$20x-14x < 4+5$$
$$6x < 9$$
$$x < \frac{9}{6}$$
$$x < \frac{3}{2}$$



Checkpoint 3

Solve $\frac{-1}{4}x \leq 3$ and represent the solutions graphically.

Checkpoint 4

Solve the following inequalities and represent the solutions graphically:

(a) $3x - 1 \geq 2x$

(b) $5x + 2 \leq 4x + 1$

(c) $2(x + 3) > 3(2 - x)$

Checkpoint 5

Solve the following inequalities and represent the solutions graphically:

(a) $\frac{1}{4}x - \frac{3}{4} < \frac{1}{3}x$

(b) $\frac{3}{5}x - 1 \geq x - \frac{3}{5}$

Checkpoint 6

Solve the following inequalities and represent the solutions graphically:

(a) $\frac{1}{3}(x+2) > \frac{1}{2} + \frac{1}{4}(x-1)$

(b) $\frac{x-1}{2} - \frac{2x-3}{4} \geq \frac{2x+1}{3} + \frac{2x+5}{6}$

Checkpoint 7

The sum of two consecutive even integers is less than 15. What so you know about the smaller number?

Checkpoint 8

A piece of writing paper weighs 3 g and an envelope weighs 5 g. It costs \$1.50 to send a letter weighing 20 g or less to a certain country X. Anna has a \$1.50 stamp and she wants to write to her friend in country X, find the maximum number of pages she can write.

Exercise A9
Linear Inequalities in One Unknown

A9.1

1. Solve the following inequalities and represent the solutions graphically.

(a) $x+3>5$

(b) $2<\frac{1}{4}x$

(c) $\frac{-1}{4}x>\frac{1}{5}$

(d) $0>-4x$

(e) $6x-2\le 5x$

(f) $2x-5\ge 5$

(g) $25-5x<0$

(h) $4x\le -5x+3$

(i) $-2x\le 3x+5$

(j) $4x-2\le 2x-5$

(k) $5x-2\ge 19-2x$

(l) $-2(x+3)<3(2-x)$

(m) $-2(x-3)\ge -3(x+2)$

(n) $-x+\frac{3}{5}\le -\frac{2}{5}x$

(o) $\frac{x-1}{6}\ge\frac{3x-10}{4}$

(p) $\frac{1}{2}(x+1)\le -\frac{1}{3}(x+2)$

(q) $\frac{1}{2}(2-x)>\frac{1}{4}(3-x)+\frac{1}{4}$

(r) $\frac{x-2}{4}-\frac{2}{3}\le\frac{x-4}{6}$

(s) $\frac{x+1}{2}+\frac{3x-1}{4}\ge\frac{x+1}{4}+1$

(t) $1-\frac{2x-5}{4}\le\frac{x+3}{2}+\frac{2(x+1)}{5}$

2. The sum of two negative even numbers is greater than 21. What do you know about the value of the greater even number?

3. A factory produces 4 times as many radios as televisions in a month. If the number of radios produced must exceed the number of televisions produced by at least 3600 per month in order to make a profit, find the minimum number of televisions the factory should produce.