A5 Formulae

A5.1 Literal Equation

In the equation 3x + 5 = 7, we can find one variable and three numerals. If we replace the numerals by letters, we have a literal equation.

For example, the equation 3x + 5 = 7 can be written as

$$ax + b = c$$

The equation $ax + b = c$ is a literal equation.
To solve the above literal equation is to express the variable x in terms of the letters a, b and c.

(1) Solve equations involving like terms.

Simple equation	Literal equation
$2x+7x=27$	$ax+bx=c$
$x(2+7)=27$	$x(a+b)=c$
$x=\frac{27}{9}$	$x=\frac{c}{a+b}$ ← Assume $a+b\neq 0$
$x=3$	

(2) Solve equations involving brackets.

Simple equation	Literal equation
$3(x-2)-x=6$	$a(x-b)-x=c$
$3x-6-x=6$	$ax-ab-x=c$
$3x-x=6+6$	$ax-x=ab+c$
$2x=12$	$x(a-1)=ab+c$
$x=\frac{12}{2}$	$x=\frac{ab+c}{a-1}$ ← Assume $a-1\neq 0$
$x=6$	

(3) Solve equations involving fractions.

Simple equation	Literal equation
$\frac{x+2}{5}=1$	$\frac{x+a}{b}=c$ ← Assume $b\neq 0$
$x+2=5$	$x+a=bc$
$x=5-2$	$x=bc-a$
$x=3$	

Example 1

Solve the literal equation $a(x-b)=c(x+d)+e$ for x.

Solution

$$a(x-b)=c(x+d)+e$$
$$ax-ab=cx+cd+e$$
$$ax-cx=cd+e+ab$$
$$x(a-c)=ab+cd+e$$
$$x=\frac{ab+cd+e}{a-c}$$

Assume $a-c\neq 0$

Example 2

Solve the literal equation $\frac{x}{a}-\frac{x}{b}=c$, $a\neq 0, b\neq 0$ for x.

Solution

$$\frac{x}{a}-\frac{x}{b}=c$$
$$\frac{bx-ax}{ab}=c$$
$$\frac{x(b-a)}{ab}=c$$
$$x(b-a)=abc$$
$$x=\frac{abc}{b-a}$$

Assume $b-a\neq 0$

Example 3

Solve the literal equation $\frac{x}{a}=b(x-c)$, $a\neq 0$, for x.

Solution

$$\frac{x}{a}=b(x-c)$$
$$x=ab(x-c)$$
$$x=abx-abc$$
$$abc=abx-x$$
$$x(ab-1)=abc$$
$$x=\frac{abc}{ab-1}$$

Assume $ab-1\neq 0$

Checkpoint 1

Solve the following literal equations for x. Suppose that a, b, x represent non-zero numbers.

(a) $ab(x-c)+ac=c+xd$

(b) $ax-\frac{c}{b}=c$

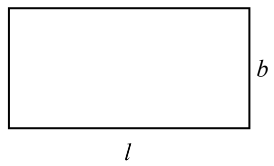
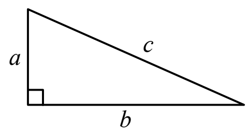
(c) $\frac{x}{a}+c=\frac{a}{b}(x-c)$

(d) $\frac{bx+c}{x}=a+d$

A5.2 Formulae and Substitution

We learnt that a formula is a rule or relation between two or more quantities and these quantities are usually represented by letters.

Some examples of formulae:

Area of rectangle:	$A = l \times b$	
Perimeter of rectangle:	$P = 2(l + b)$	
Speed:	$S = \frac{D}{T}$	S: speed D: distance T: time
Pythagoras' Theorem:	$c^2 = a^2 + b^2$	

In the formula, the value of a certain letter or variable can be obtained by the method of substitution if the values of the other letters are known.

Example 4

Find the value of P from the formula $D = \frac{T+2}{P}$, $P \neq 0$, when $D = 5$ and $T = 3$.

Solution

$$D = \frac{T+2}{P}$$

$$5 = \frac{3+2}{P}$$

$$5P = 5$$

$$P = 1$$

Checkpoint 2

In each of the following, a formula and the values of some of the variables are given. Find the value of the unknown.

(a) $s = ut + \frac{1}{2}ft^2$ If $u = 16$, $t = 2$ and $f = 10$, find s.

(b) $C = \frac{nE}{R + nr}$ If $C = 3.5$, $E = 17$, $R = 6$ and $r = 4$, find n.

(c) $d = t\left(1 - \frac{1}{u}\right)$ If $d = 3$ and $t = 12$, find u.

A5.3 Change of Subject of a Formula

In the formula $A = lb$, A is called the subject of the formula. For $l \neq 0$, this formula can be written as

$$b = \frac{A}{l}$$

Now, b is isolated on the left side of the equal sign and it becomes the subject of the formula.

Example 5

Make x the subject of the formula $y = 3x + 4$.

Solution

$$y = 3x + 4$$
$$3x = y - 4$$
$$x = \frac{y-4}{3}$$

Example 6

Change the subject of the formula $A = \frac{1}{2}(a+b)h$ to a.

Solution

$$A = \frac{1}{2}(a+b)h$$
$$2A = (a+b)h$$
$$2A = ha + hb$$
$$ha = 2A - hb$$
$$a = \frac{2A - hb}{h}$$

Assume $h \neq 0$

Example 7

Change the subject of the formula $T = 2rt - 4rs$ to r.

Solution

$$T = 2rt - 4rs$$
$$T = r(2t - 4s)$$
$$r = \frac{T}{2t - 4s}$$

Checkpoint 3

In each of the following, make the letter in bracket the subject of the formula.

(a) $A = \pi r^2 + 2\pi rh$ [h]

(b) $t(R - 2r) = 3R$ [R]

(c) $s = ut + \frac{1}{2}ft^2$ [f]

(d) $\frac{Ma - Mb}{c} = a$, $c \neq 0$ [M]

Example 8

Change the subject of the formula $y = \dfrac{a}{1-ax}$, $1-ax \neq 0$, to a.

Solution

$$y = \frac{a}{1-ax}$$
$$y(1-ax) = a$$
$$y - yax = a$$
$$a + axy = y$$
$$a(1+xy) = y$$
$$a = \frac{y}{1+xy}$$

Assume $1 + xy \neq 0$

Example 9

Change the subject of the formula $h = \dfrac{2nk}{1+(n+1)k}$, $1+(n+1)k \neq 0$, to

(a) n.
(b) k.

Solution

(a)
$$h = \frac{2nk}{1+(n+1)k}$$
$$h[1+(n+1)k] = 2nk$$
$$h(1+nk+k) = 2nk$$
$$h + nhk + hk = 2nk$$
$$h + hk = 2nk - nhk$$
$$n(2k - hk) = h + hk$$
$$n = \frac{h+hk}{2k-hk}$$

Assume $2k - hk \neq 0$

(b)
$$h = \frac{2nk}{1+(n+1)k}$$
$$h[1+(n+1)k] = 2nk$$
$$h + hk(n+1) = 2nk$$
$$2nk - hk(n+1) = h$$
$$k[2n - h(n+1)] = h$$
$$k = \frac{h}{2n-h(n+1)}$$

Assume $2n - h(n + 1) \neq 0$

Example 10

Change the subject of the formula $A = \dfrac{PI}{\sqrt{2R} + 1}$ to R.

Solution

$$\begin{aligned}
A &= \frac{PI}{\sqrt{2R} + 1} \\
A(\sqrt{2R} + 1) &= PI \\
\sqrt{2R} + 1 &= \frac{PI}{A} \\
\sqrt{2R} &= \frac{PI}{A} - 1 \\
2R &= \left(\frac{PI}{A} - 1 \right)^2 \\
R &= \frac{1}{2}\left(\frac{PI}{A} - 1 \right)^2
\end{aligned}$$

Checkpoint 4

Change the subject of the formula $m = \dfrac{1 - mnp}{np}$, $n, p \neq 0$, to n.

Checkpoint 5

In each of the following, make the letter in bracket the subject of the formula.

(a) $a + c = \dfrac{a - bc}{b}$, $b \neq 0$ [b]

(b) $mk = \dfrac{(m+1)n + k}{2n - k}$, $2n - k \neq 0$ [n]

(c) $\sqrt{2u} = vw$ [u]

(d) $\dfrac{a}{b + \sqrt{c}} = \sqrt{b} + a$, $b + \sqrt{c} \neq 0$ [c]

Example 11

Change the subject of the formula $b^2 = \frac{m}{n^2}$, $n \neq 0$ to *n*.

Solution

$$b^2 = \frac{m}{n^2}$$

$$n^2 = \frac{m}{b^2}$$

$$n = \pm\sqrt{\frac{m}{b^2}}$$

$$= \pm\frac{\sqrt{m}}{b}$$

Example 12

Change the subject of the formula $m + 2 = \frac{mn^2}{m + n^2}$, $m + n^2 \neq 0$, to *n*.

Solution

$$m + 2 = \frac{mn^2}{m + n^2}$$

$$(m + 2)(m + n^2) = mn^2$$

$$m^2 + 2m + mn^2 + 2n^2 = mn^2$$

$$2n^2 = -m^2 - 2m$$

$$n = \pm\sqrt{\frac{-m^2 - 2m}{2}}$$

Checkpoint 6

Change the subject of the formula $ak^2 - b = ck^2$ to *k*.

A5.4 Applications of Formulae

Example 13

Celsius (C) scale and Fahrenheit (F) scale are used to measure temperature. The relationship between F and C is given by the formula

$$F = \frac{9}{5}C + 32\,.$$

(a) Express C in terms of F.

(b) Find the reading on Celsius scale if the reading on Fahrenheit scale is

(i) –4; (ii) 32.

(c) At what temperature will the reading on Celsius scale the same as the reading on Fahrenheit scale?

Solution

(a)

$$F = \frac{9}{5}C + 32$$

$$\frac{9}{5}C = F - 32$$

$$C = \frac{5}{9}(F - 32)$$

(b) (i)

$$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}(-4 - 32)$$

$$= -20$$

(ii)

$$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}(32 - 32)$$

$$= 0$$

(c)

$$C = \frac{5}{9}(C - 32)$$

$$C = \frac{5}{9}C - \frac{5}{9} \times 32$$

$$C - \frac{5}{9}C = -\frac{160}{9}$$

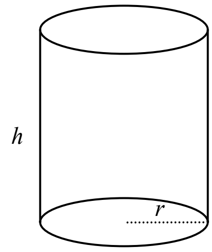
$$\frac{4}{9}C = -\frac{160}{9}$$

$$C = -\frac{160}{9} \times \frac{9}{4} = -40$$

Example 14

The volume (V) of a cylinder with base radius (r) and height (h) is given by the formula

$$V = \pi r^2 h .$$



(a) Make r the subject of the formula.

(b) Hence find r if $V = 88\text{ cm}^3$, $h = 7$ and $\pi = \dfrac{22}{7}$.

Solution

(a)
$$V = \pi r^2 h$$
$$r^2 = \frac{V}{\pi h}$$
$$r = \sqrt{\frac{V}{\pi h}} \quad \text{or} \quad -\sqrt{\frac{V}{\pi h}} \quad \text{(rejected)}$$

(b)
$$r = \sqrt{\frac{V}{\pi h}}$$
$$= \sqrt{\frac{88}{\frac{22}{7} \times 7}}$$
$$= \sqrt{4}$$
$$= 2\text{ cm}$$

Checkpoint 7

The measure of an interior angle (θ) of a rectangular polygon with n sides is given by the formula

$$\theta = \frac{180°(n-2)}{n} .$$

(a) Express n in terms of θ.

(b) If the measure of an interior angle of a regular polygon is 156°, find the number of sides of the polygon.

Exercise A5
Formulae

A5.1

1. Solve the following equations for x. All letters represent non-zero numbers.

(a) $3x - b = d$

(b) $4mx = 8n$

(c) $ax + b + c = 0$

(d) $4(x - 3t) = 7s$

(e) $5(x + 2b) = 3(x - 2c)$

(f) $mx - nx = pq$

(g) $ax + 9 = 3x + 14$

(h) $rx + h = sx - k$

(i) $3px = 2q(r - 5x)$

(j) $m(x - a) = n(x - b)$

(k) $\frac{2x - r}{s} = 3t$

(l) $\frac{x}{m} + \frac{x}{n} = \frac{1}{p}$

(m) $\frac{3ax + 2b}{c} = 4d$

(n) $\frac{5px - 6q}{3r} + x = 0$

A5.2

2. Given that $A = \frac{1}{2}bh$, find A when $b = 9$ and $h = 8$.

3. Given that $S = \frac{1}{2}n(n+1)$, find S when $n = 100$.

4. Given that $A = \pi r^2$, find A when $\pi = \frac{22}{7}$ and $r = 14$.

5. Given that $s = ut - \frac{1}{2}gt^2$, find s when $u = 40$, $t = 5$ and $g = 10$.

6. Given that $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, find f when $u = 15$ and $v = 30$.

7. Given that $F = \frac{9}{5}C + 32$, find C if $F = 50$.

8. Given that $V = blh$, find h if $V = 120$, $b = 4$ and $l = 10$.

9. Given that $A = P(1 + rt)$, find t if $A = 1240$, $P = 1000$ and $r = 0.06$.

10. Given that $g = \sqrt{ab}$, find b if $a = 9$ and $g = 12$.

11. Given that $y = \dfrac{1-3x}{4-x}$, find x if $y = 10$.

12. Given that $T = 2\pi\sqrt{\dfrac{l}{g}}$, find l if $T = 10\pi$ and $g = 10$.

A5.3

13. In each of the following, make the letter in the brackets the subject of the formula.

(a)	$V = \frac{1}{3}Ah$	$[h]$	(b)	$D = \frac{M}{V}$	$[V]$
(c)	$P = \frac{Fd}{t}$	$[F]$	(d)	$P = 2(l+b)$	$[b]$
(e)	$C = \frac{5}{9}(F-32)$	$[F]$	(f)	$3u + 7v = 10V$	$[v]$
(g)	$v = u + at$	$[t]$	(h)	$H = ms(T-t)$	$[T]$
(i)	$y = \frac{3+x}{3-x}$	$[x]$	(j)	$t = \frac{ms}{m+n}$	$[m]$
(k)	$F = \frac{m(v-u)}{t}$	$[v]$	(l)	$y = 5 + \frac{2-x}{x+1}$	$[x]$
(m)	$W = IR^2$	$[R]$	(n)	$F = G\frac{mn}{d^2}$	$[d]$
(o)	$v^2 = u^2 + 2as$	$[u]$	(p)	$w = \sqrt{\frac{m}{g}}$	$[m]$
(q)	$T = 4\sqrt{m}$	$[m]$	(r)	$r = \frac{1}{2}\sqrt{\frac{5s}{t}}$	$[t]$
(s)	$D = \sqrt{b^2 - 4ac}$	$[b]$	(t)	$x = a + \sqrt{b^2 + c^2}$	$[c]$
(u)	$a = \sqrt{\frac{1}{2}a + 5b}$	$[b]$	(v)	$A = \pi r^2 + r\sqrt{h^2 + r^2}$	$[h]$
(w)	$u = \sqrt{\frac{mv^2}{R}}$	$[v]$	(x)	$A = b + \sqrt{B^2 - C^2}$	$[B]$

A5.4

14. Celsius (C) scale and Fahrenheit (F) scale are used to measure temperature. The relationship between F and C is given by the formula

$$C = \frac{5}{9}(F - 32).$$

(a) Express F in terms of C.

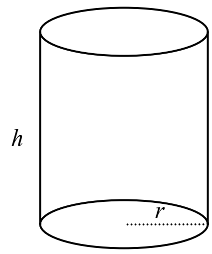
(b) Find the reading on Fahrenheit scale if the reading on Celsius scale is

(i) –5; (ii) 0; (iii) 5.

15. The total surface area (A) of a cylinder is given by the formula

$$A = 2\pi r^2 + 2\pi rh$$

where h is the height and r is the base radius of the cylinder.



(a) Make h the subject of the formula.

(b) Find the value of h if A = 550 cm^3, r = 5 cm and $\pi = \frac{22}{7}$.

16. The sum of the squares of first n positive integers is $S = \frac{n(n+1)(2n+1)}{6}$.

$$\text{(i.e. } 1^2 + 2^2 + 3^2 + ... + n^2 = S = \frac{n(n+1)(2n+1)}{6}\text{)}$$

(a) Find the sum of the squares of the first 50 positive integers.

(b) Find the sum of the squares of the positive integers from 51 to 100 inclusive (包括在內).