2005-CE

A MATH

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2005

ADDITIONAL MATHEMATICS

3:00 pm – 5:30 pm (2½ hours)

This paper must be answered in English

1. Answer **ALL** questions in Section A and any **FOUR** questions in Section B.
2. Write your name in the answer book provided. **For Sections A, there is no need to start each question on a fresh page.**
3. All workings must be clearly shown.
4. Unless otherwise specified, numerical answers must be exact.
5. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
6. The diagram in the paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$ $2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

* *

Section A (62 marks)

Answer **ALL** questions in this section.

1. Solve $4|x| = |x^2 + 3|$.

(3 marks)

2. Find $\int \sin x \cos^2 x \, dx$.

(4 marks)

3. Let A be a variable point on the curve $xy = -2$. B is the point (1, –2) and P is the mid-point of AB. Find the equation of the locus of P.

(4 marks)

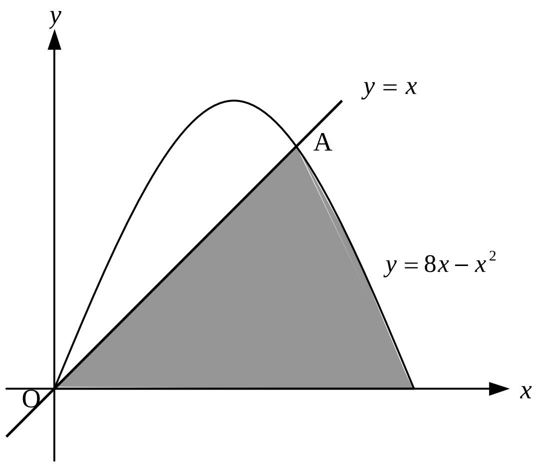
4. In the expansion of $(1-2x+x^2)^n$, the coefficient of x^2 is 153.

(a) Find the value(s) of *n*.

(b) Find the coefficient of *x*.

(5 marks)

5.



In the figure, the shaded region is bounded by the curve $y = 8x - x^2$, $y = x$ and the *x*-axis.

(a) Write down the coordinates of A.

(b) Find the area of the shaded region.

(5 marks)

6. (a) Find $\frac{d}{dx}\cos^3 x$.

(b) Hence find $\int \sin^3 x \, dx$.

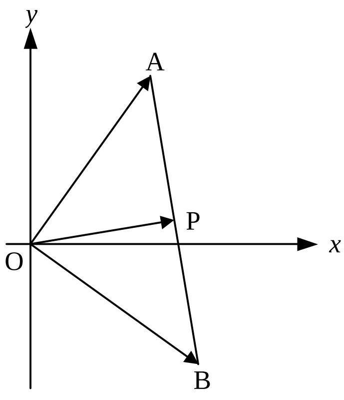
(5 marks)

7. (a) Show that $\cos 2x + \cos 4x + \cos 6x = \cos 4x(1 + 2\cos 2x)$.

(b) Hence show that $\frac{\sin 3x \cos 4x}{\cos 2x + \cos 4x + \cos 6x} = \sin x$.

(5 marks)

8. In the figure, let $\overrightarrow{OA} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j}$. P is a point on AB such that $AP : PB = t : 1 - t$.



(a) Express $\overrightarrow{OP}$ in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(b) If OP bisects $\angle AOB$, find the value of t.

(5 marks)

9. (a) Show that the equation of the tangent to the curve $x=2t-1$, $y=t^2+1$ at (1, 2) is $L: x-y+1=0$.

(b) Find the equation(s) of the line(s) parallel to L whose distance from L is 5 units.

(6 marks)

10. Given that α and β are the roots of the equation $x^2-2tx-(t^2+4)=0$, where t is a real constant.

(a) Find $\alpha+\beta$ and $\alpha\beta$ in terms of t.

(b) If $|\alpha|=\beta$, find t and α.

(6 marks)

11. (a) Prove, by mathematical induction, that

$$1\times 2+2\times 3+3\times 4+...+n(n+1)=\frac{n(n+1)(n+2)}{3}$$

for all positive integers n.

(b) Using the result of (a) and the identity

$$1+2+3+...+n=\frac{n(n+1)}{2},$$

find $1\times 100+2\times 99+3\times 98+...+100\times 1$.

(7 marks)

12. (a) A(5, 2), B(3, 3) are two vertices of the square ABCD. Find two possible equations of AC.

(b) A student says that

‘the two possible equations of AC in (a) are tangent to a circle *C* centred at the origin.’

Determine whether the student is correct or not.

(7 marks)

Section B (48 marks)

Answer any **FOUR** questions in this section.

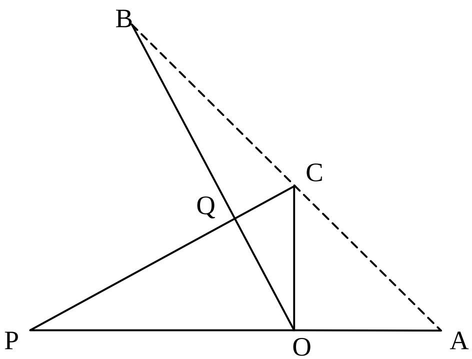
Each question carries 12 marks.

13. (a) O, A, B and C are 4 points on a plane such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = m\mathbf{a} + n\mathbf{b}$.

(i) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in terms of *m*, *n*, **a** and **b**.

(ii) Show that if A, B, C are collinear then $m + n = 1$.

(5 marks)



(b) In the figure, POA is a straight line and OP = 2 OA. Q is a point on OB such that $\text{OQ} = k\text{OB}$, where $0 < k < 1$ and PQ is produced to C such that $\text{PQ} = (1-k)\text{PC}$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(i) Show that $\overrightarrow{OC} = \dfrac{2k}{1-k}\mathbf{a} + \dfrac{k}{1-k}\mathbf{b}$.

(ii) Given that C lies on the straight line AB.

(1) Find the value of *k*.

(2) If OC is perpendicular to OA and $\angle AOB = 120°$, find $\dfrac{\text{OB}}{\text{OA}}$.

(7 marks)

14. The slope of the tangent *T* at any point (*x*, *y*) to the curve *C* is given by $\dfrac{1}{2\sqrt{x-1}}$. The *x*-intercept of *C* is 1.

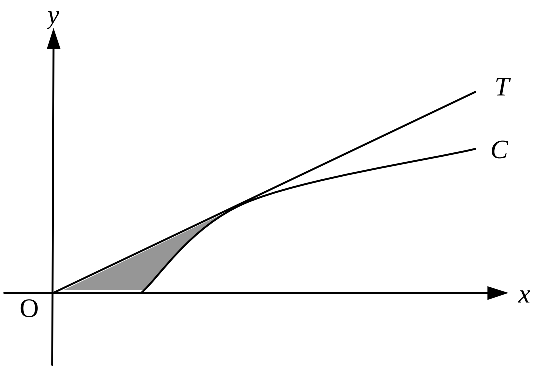
(a) Find the equation of *C*.

(4 marks)

(b) Find the equation of *T* parallel to the line $x-2y+5=0$.

Show that *T* passes through the origin.

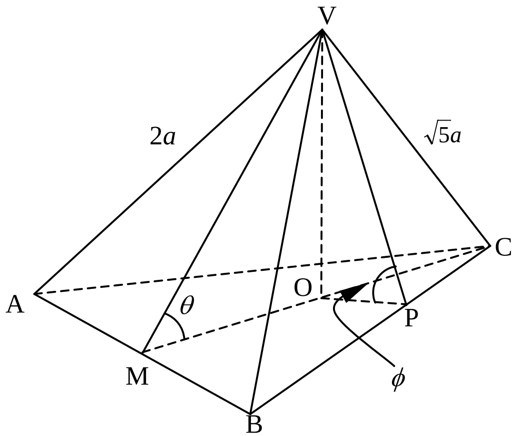
(4 marks)



(c) Find the volume generated by revolving the area of the shaded portion bounded by the tangent *T*, the curve *C* and the *x*-axis about the *x*-axis.

(4 marks)

15. The figure shows a tetrahedron such that VA = VB = AB = $2a$, $VC = \sqrt{5}a$ and $CA = CB = 3a$. O is the foot of the perpendicular from V to the base ABC. M is the mid-point of AB. P is a point on BC such that BP = ra where $0 \le r \le 3$. $\angle VMC = \theta$ and $\angle VPO = \phi$.



(a) Express CM^2 and VM^2 in terms of a.
Hence find the value of $\cos\theta$ in surd form.

(5 marks)

(b) Using the results of (a), find $\sin\theta$ and VO.
(*Give the answers in surd form.*)

(2 marks)

(c) Express VP^2 in terms of a and r.

Hence show that $\sin\phi = \sqrt{\dfrac{45}{8(3r^2 - 8r + 12)}}$.

(5 marks)

16. α, β are the roots of the equation $x^2 - 2mx + (m-1) = 0$, where m is a real constant.

(a) Show that α, β are real and distinct.

(3 marks)

(b) Find $(\alpha - \beta)^2$ in terms of m.

(2 marks)

(c) The graph $y = x^2 - 2mx + (m-1)$ cuts the x-axis at A and B. A circle C is drawn with AB as diameter.

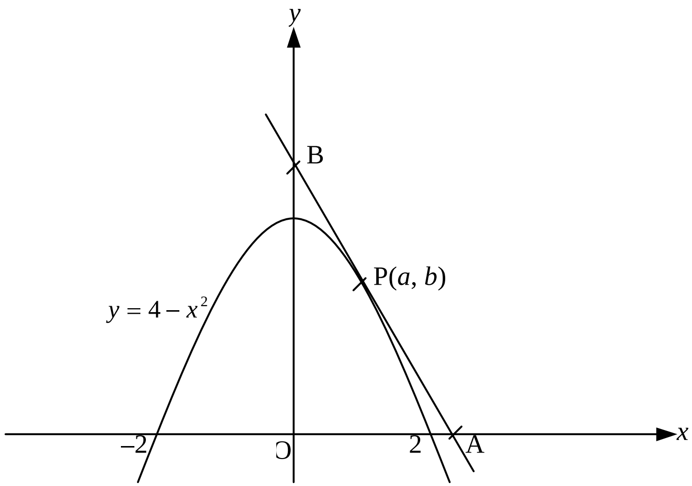
(i) Using the result of (b), show that the equation the circle C is $x^2 + y^2 - 2mx + m - 1 = 0$.

(ii) When $m = 1$ and $m = 4$, two circles C_1 and C_2 are formed.

A circle C_3 passes through the point (2, 1) and the points of intersection of C_1 and C_2. Find the equation of C_3.

(7 marks)

17. In the figure, P(a, b) is a point moving on the parabola $y = 4 - x^2$ in the first quadrant. The tangent at P cuts the axes at A and B.



(a) Express the coordinates of A and B in terms of a.

Hence show that the area of ΔAOB is $S = \dfrac{a^3}{4} + 2a + \dfrac{4}{a}$.

(5 marks)

(b) When $b = \dfrac{8}{3}$, B is moving down at a rate of $\dfrac{3}{4}$ unit per second.

(i) Find the rate at which point A is moving out.

(ii) A student says that the area of ΔAOB is decreasing at this moment. Determine whether the student is correct or not.

(7 marks)

END OF PAPER