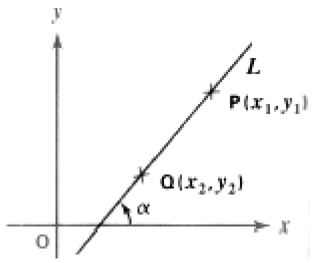
Chapter 9 Straight Lines

9.0 Review

9.0.1 Distance Between Two Points

The distance between two points P(x_1, y_1) and Q(x_2, y_2) can be found using the distance formula

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



9.0.2 Slope and Inclination of a Straight Line

The slope, m, of the straight line L joining the points P(x_1, y_1) and Q(x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let α be the angle that L makes with the positive x-axis, measured anti-clockwise, then α is called the inclination of L and $m = \tan\alpha$, where $0° < \alpha \le 180°$.

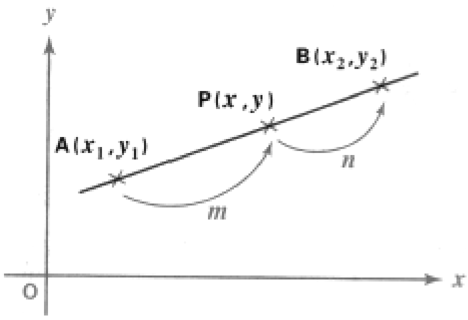
9.0.3 Parallel and Perpendicular Lines

Let m_1 and m_2 be the slopes of the straight lines L_1 and L_2, respectively.
If L_1 // L_2, then $m_1 = m_2$. Conversely, if $m_1 = m_2$, then L_1 // L_2.
If $L_1 \perp L_2$, then $m_1m_2 = -1$. Conversely, if $m_1m_2 = -1$, then $L_1 \perp L_2$.

9.0.4 Point of Division

The coordinates (x, y) of a point P which divides a straight line joining the points A(x_1, y_1) and B(x_2, y_2) internally in the ratio $m : n$ are given by the section formula

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}.$$



In particular, when P(x, y) is the mid-point of AB, then we have the mid-point formula

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}.$$

9.0.5 Different Forms of Equations of a Straight Line

(1) Point-slope Form

The equation of a line with slope m and passing through a point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)\,.$$

(2) Slope-intercept Form

The equation of a line with slope m and y-intercept c is given by

$$y = mx + c\,.$$

(3) Two-point Form

The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}\,.$$

(4) Intercept Form

The equation of a line with x-intercept a and y-intercept b is given by

$$\frac{x}{a} + \frac{y}{b} = 1\,.$$

(5) General Form

All forms of equation of a straight line can be transformed into the general form

$$Ax + By + C = 0\,.$$

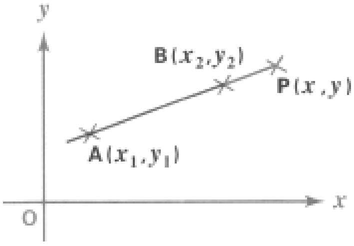
In this form, we have

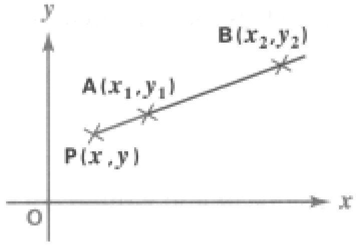
$$\text{slope} = -\frac{A}{B} \qquad \text{for } B \neq 0$$
$$x - \text{intercept} = -\frac{C}{A} \qquad \text{for } A \neq 0$$
$$y - \text{intercept} = -\frac{C}{B} \qquad \text{for } A \neq 0$$

9.1 External Point of Division

The coordinates (x, y) of a point P which divides a straight line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m : n$ are given by

$$x = \frac{mx_2 - nx_1}{m - n},\ \ y = \frac{my_2 - ny_1}{m - n}.$$

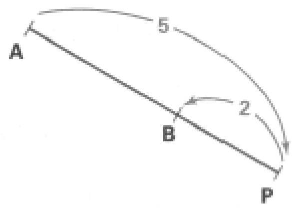




If a negative value for either m or n in the ratio $m : n$ is used, then we can apply the section formula $x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$ directly.

Example 9.1

Find the coordinates of the point P(x ,y) which divides the line joining A(–3, 5) and B(6, –8) externally in the ratio 5 : 2.



Solution

$$x = \frac{5(6) - 2(-3)}{5-2} = 12$$

$$y = \frac{5(-8) - 2(5)}{5-2} = -16\frac{2}{3}$$

$$\therefore \quad (x, y) = \left(12, -16\frac{2}{3}\right)$$

Example 9.2

Given two points M(5, 4) and N(11, 7). P is a point of division of MN. Find the coordinates of P if

(a) $\text{MP} : \text{PN} = 1 : -4$

(b) $\text{MP} : \text{PN} = -\frac{3}{2} : 1$

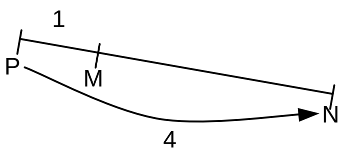
Solution

Let the coordinates of P be (x, y).

(a)
$$x = \frac{1(11) + (-4)(5)}{1 + (-4)} = 3$$

$$y = \frac{1(7) + (-4)(4)}{1 + (-4)} = 3$$

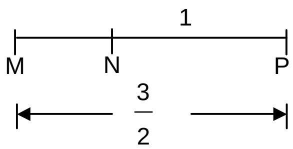
$\therefore$ The coordinates of P are (3, 3).



(b)
$$x = \frac{-\frac{3}{2}(11) + 1(5)}{-\frac{3}{2} + 1} = 23$$

$$y = \frac{-\frac{3}{2}(7) + 1(4)}{-\frac{3}{2} + 1} = 13$$

$\therefore$ The coordinates of P are (23, 13).



Checkpoint 9.1

Given two points A(–1, 3) and B(3, –5). P is an external point of division of AB. Find the coordinates of P if

(a) P divides AB in the ratio 1 : 5.

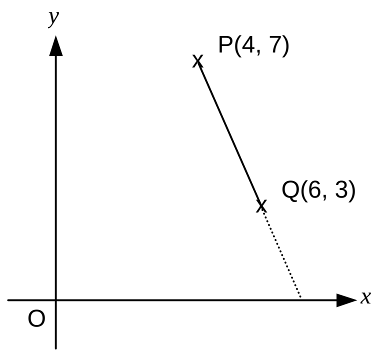
(b) $\text{AP} : \text{PB} = 7 : -3$.

Example 9.3

In the figure, the line PQ is produced to meet the x-axis at R.

(a) Find the ratio in which R divides PQ externally.

(b) Hence find the coordinates of R.



Solution

Let the required ratio be m : n and the coordinates of R be (x, 0).

(a)
$$0 = \frac{m(3) - n(7)}{m - n}$$
$$0 = \frac{3m - 7n}{m - n}$$
$$3m - 7n = 0$$
$$\frac{m}{n} = \frac{7}{3}$$

$\therefore$ R divides PQ externally in the ratio 7 : 3.

(b)
$$x = \frac{7(6) - 3(4)}{7 - 3} = \frac{15}{2}$$

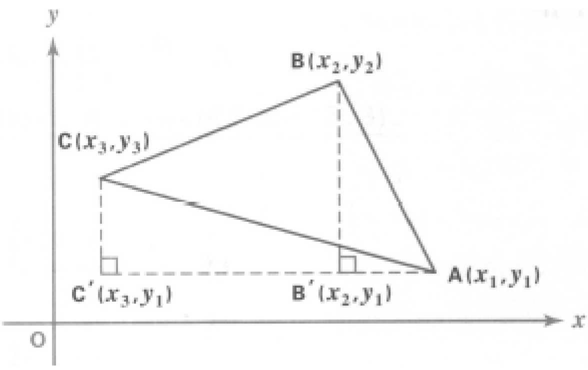
$\therefore$ The coordinates of R are $\left(\frac{15}{2}, 0\right)$.

Checkpoint 9.2

If A(1, 3), B(k, 0) and C(–6, –4) are three collinear points, find the ratio AC : BC, where C is an external point of division of AB. Hence find the value of k.

9.2 Areas of Rectilinear Figures

9.2.1 Areas of Triangles



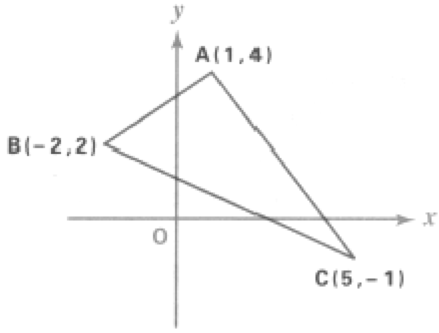
$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{vmatrix}$$

$$= \frac{1}{2}[x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3]$$

where (x_1, y_1), (x_2, y_2), (x_3, y_3) are the coordinates of the vertices of the triangle taken in ***anti-clockwise*** direction.

Example 9.4

Find the area of ΔABC in the figure.



Solution

$$\text{Area of }\Delta\text{ABC} = \frac{1}{2}\left|\begin{array}{cc} 1 & 4 \\ -2 & 2 \\ 5 & -1 \\ 1 & 4 \end{array}\right|$$

$$= \frac{1}{2}[(2+2+20-(-8+10-1)]$$

$$= \frac{23}{2}$$

What would be the result if we take the coordinates of A, B and C in an clockwise direction?

Example 9.5

Given the coordinates of the vertices of ΔABC are A(4, 2), B(–2, 5) and C(1, *y*). If the area of ΔABC is 22.5, find the possible values of *y*.

Solution

$$\text{Area of }\Delta\text{ABC} = \frac{1}{2}\left|\begin{array}{cc} 4 & 2 \\ 2 & 5 \\ 1 & y \\ 4 & 2 \end{array}\right|$$

$$= \frac{1}{2}[(20-2y+2)-(-4+5+4y)]$$

$$= \frac{1}{2}(21-6y)$$

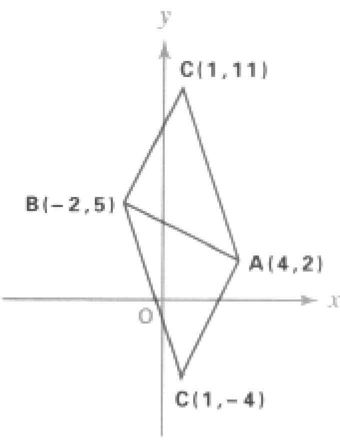
[Since the order of coordinates A, B and C is either clockwise or anti-clockwise, $\frac{1}{2}(21-6y)$ is either positive or negative.]

$\therefore$ Area of $\Delta\text{ABC} = \left|\frac{1}{2}(21-6y)\right|$

i.e. $22.5 = \frac{1}{2}(21-6y)$ or $22.5 = -\frac{1}{2}(21-6y)$

$21-6y = 45$ or $21-6y = -45$

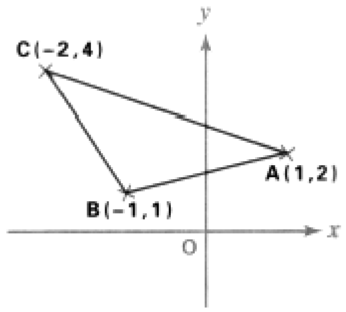
$y = -4$ or $y = 11$



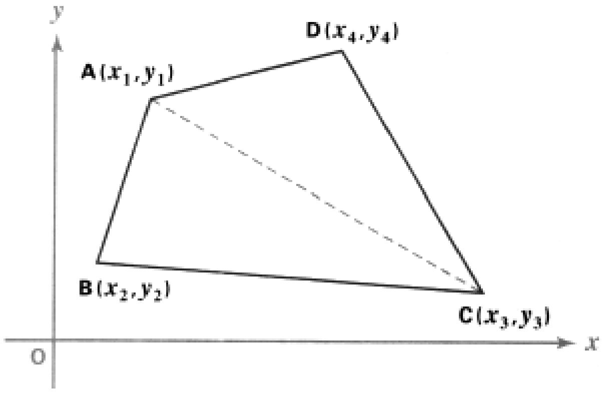
Checkpoint 9.3

(a) Find the area of ΔABC with vertices A(1, 2), B(–1, 1) and C(–2, 4).

(b) Hence evaluate the length (h) of the altitude from A to BC.



9.2.2 Areas of Polygons

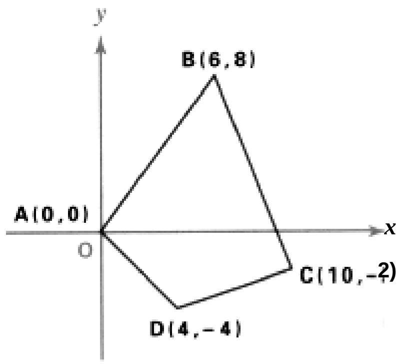


$$\text{Area of an } n\text{-sided polygon} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_n & y_n \\ x_1 & y_1 \end{vmatrix}$$

where (x_1, y_1), (x_2, y_2), …, (x_n, y_n) are the coordinates of the vertices of the polygon taken in anti-clockwise order.

Example 9.6

Given the quadrilateral whose vertices are A(0, 0), B(6, 8), C(10, –2), D(4, –4).



(a) Find the area of ABCD.

(b) Show that the area of the quadrilateral formed by joining the mid-points of adjacent sides of ABCD is equal to one-half the area of ABCD.

Solution

(a) Area of ABCD $= \frac{1}{2}\begin{vmatrix} 0 & 0 \\ 4 & -4 \\ 10 & -2 \\ 6 & 8 \\ 0 & 0 \end{vmatrix}$

$$= \frac{1}{2}[(0-8+80+0)-(0-40-12+0)]$$

$$= 62$$

(b) Let the mid-points of AB, BC, CD and DA be P, Q, R, S respectively. Then we have

Point	Coordinates
P	$\left(\frac{0+6}{2}, \frac{0+8}{2}\right) = (3, 4)$
Q	$\left(\frac{6+10}{2}, \frac{8-2}{2}\right) = (8, 3)$
R	$\left(\frac{10+4}{2}, \frac{-2-4}{2}\right) = (7, -3)$
S	$\left(\frac{4+0}{2}, \frac{-4+0}{2}\right) = (2, -2)$

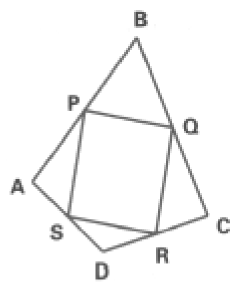
Area of PQRS $= \frac{1}{2}\begin{vmatrix} 3 & 4 \\ 2 & -2 \\ 7 & -3 \\ 8 & 3 \\ 3 & 4 \end{vmatrix}$

$$= \frac{1}{2}[(-6-6+21+32)-(8-14-24+9)]$$

$$= 31$$

$$= \frac{1}{2} \times \text{Area of ABCD}$$

∴ The area of the quadrilateral formed by joining the mid-points of adjacent sides of ABCD is equal to one-half the area of ABCD.



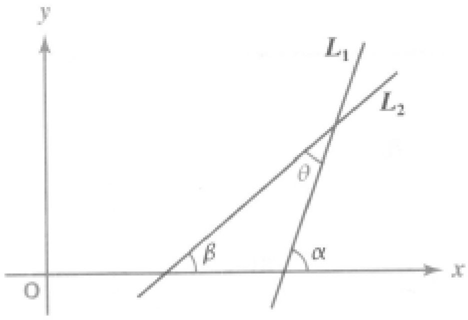
Checkpoint 9.4

P(2, 0), Q(4, 4), R(0, 7), S(–5, 0), T(x, y) and U(0, –3) are the vertices of a hexagon PQRSTU.

(a) Find the area of PQRSTU in terms of x and y.

(b) The area of PQRSTU is 58 square units. Find the coordinates of T if T lies on the line $y = 2x$.

9.3 Angle Between Two Straight Lines



Let θ be the acute angle between two straight lines with slopes m_1 and m_2, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

Example 9.7

Let θ be the acute angle between the two lines L_1: $y = -x$ and L_2: $y = 3x$.

(a) Find θ correct to the nearest degree.

(b) Find the equations of the lines which make an angle θ with L_1 and pass through the point $(0, -2)$.

Solution

(a) Slope of $L_1 = -1$

Slope of $L_2 = 3$

$$\tan\theta = \left|\frac{-1-3}{1+(-1)(3)}\right|$$

$$= 2$$

$$\theta = 63^\circ \quad \text{(corr. to the nearest degree)}$$

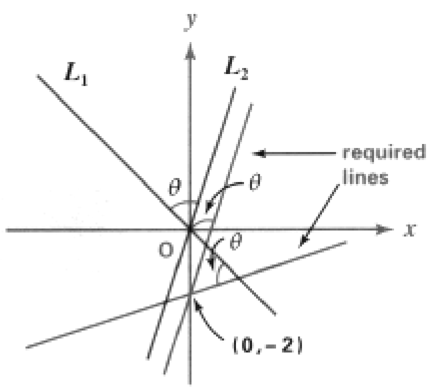
(b) Let the slope of the required line be m. Acute angle between the required line and $L_1 = \theta$.

$$\tan\theta = \left|\frac{-1-m}{1+(-1)m}\right|$$

$$2 = \left|\frac{-1-m}{1-m}\right|$$

$$\frac{-1-m}{1-m} = 2 \quad \text{or} \quad \frac{-1-m}{1-m} = -2$$

$$m = 3 \quad \text{or} \quad m = \frac{1}{3}$$



The equations of the required line are

$$y - (-2) = 3(x - 0) \text{ and } y - (-2) = \frac{1}{3}(x - 0)$$

i.e. $3x - y - 2 = 0$ and $x - 3y - 6 = 0$

Example 9.8

L_1 is a line passing through the point A(3, 7) and making an acute angle θ with the line L_1: $2x + y - 1 = 0$, where $\tan\theta = 2$. Find the two possible equations of L_1.

Solution

Let m be the slope of L_1.

Slope of $L = -\frac{2}{1} = -2$.

$\because$ L_1 makes an acute angle θ with L,

$$\tan\theta = \left|\frac{m-(-2)}{1+m(-2)}\right|$$

$$2 = \left|\frac{m+2}{1-2m}\right|$$

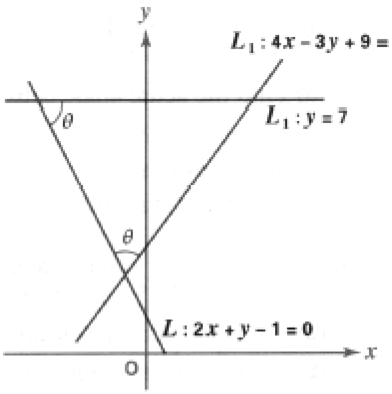
i.e. $\frac{m+2}{1-2m} = 2$ or $\frac{m+2}{1-2m} = -2$

$m = 0$ or $m = \frac{4}{3}$

When $m = 0$, the equation of L_1 is $y - 7 = 0\,(x - 3)$, i.e. $y = 7$.

When $m = \frac{4}{3}$, the equation of L_1 is $y - 7 = \frac{4}{3}(x-3)$, i.e. $4x - 3y + 9 = 0$.

$\therefore$ The two possible equations of L_1 are $y = 7$ and $4x - 3y + 9 = 0$.



Checkpoint 9.5

Find the acute angle θ between the lines L_1: $3x - y + 1 = 0$ and L_2: $2x + 7y = 0$.

Checkpoint 9.6

Given two straight lines L_1: $y = 2x + 3$ and L_2: $y = 6 - x$.

(a) Find the value of tan θ, where θ is the acute angle between L_1 and L_2.

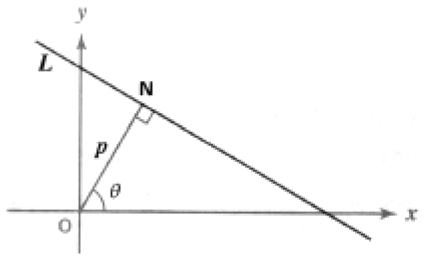
(b) Hence find the equation of L_3 which passes through (3, 1) and makes an angle of θ with L_2.

9.4 Equations of Straight Lines in the Normal Form

9.4.1 Normal Form

Let ON be the perpendicular (normal) from the origin O to the line *L*. If ON makes a positive angle θ with the positive *x*-axis and its length is *p*, then the equation of *L* can be expressed as

$$x\cos\theta + y\sin\theta - p = 0\,.$$



This is called the normal form of *L*.

9.4.2 Conversion of the General Form to the Normal Form

Given the equation of a straight line *L*. In the general form $Ax + By + C = 0$. To convert this equation to the normal form means to express it as

$$\frac{Ax + By + C}{\pm\sqrt{A^2 + B^2}} = 0$$

where the sign in front of $\sqrt{A^2 + B^2}$ is

(1) opposite to that of *C* if $C \neq 0$.

(2) the same as that of *B* if $C = 0$.

Example 9.9

Convert the following equations of straight lines to normal form.

(a) $5x - 12y + 18 = 0$

(b) $x - 4y = 0$

Solution

(a) [C is positive, so we add a negative sign to $\sqrt{A^2 + B^2}$.]

The normal form is $\dfrac{5x - 12y + 18}{-\sqrt{5^2 + (-12)^2}} = 0$

i.e. $-\dfrac{5}{13}x + \dfrac{12}{13}y - \dfrac{18}{13} = 0$.

(b) [C = 0, so the sign in front of $\sqrt{A^2 + B^2}$ must be the same as that of B. Since B = – 4, the sign in front of $\sqrt{A^2 + B^2}$ is negative.]

The normal form is $\dfrac{x - 4y}{-\sqrt{1^2 + (-4)^2}} = 0$

i.e. $-\dfrac{1}{\sqrt{17}}x + \dfrac{4}{\sqrt{17}}y = 0$.

Checkpoint 9.7

Convert the following equations to normal form.

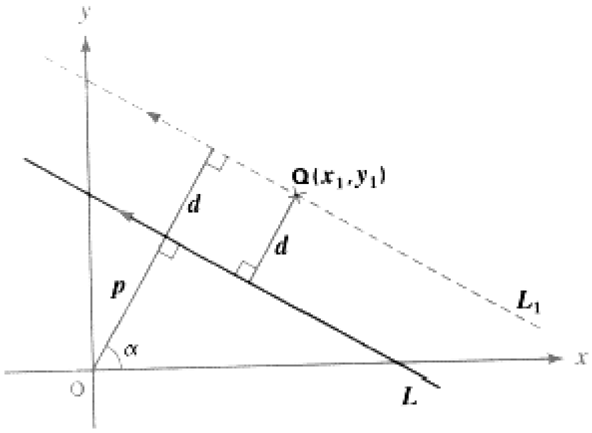
(a) $-6x + 8y + 3 = 0$ (b) $2y = 3x - 5$

9.5 Applications of the Normal Form

9.5.1 Distance from a Point to a Line

The distance d from the point (x_1, y_1) to the line $Ax + By + C = 0$ is given by

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|.$$



Example 9.10

Given a point P(2, –1) and the line L: $4x + y – 1 = 0$. Find the distance from P to L.

Solution

$$\text{The required distance} = \left| \frac{4(2) + (1) - 1}{\sqrt{4^2 + 1^2}} \right|$$

$$= \frac{6}{\sqrt{17}}$$

Example 9.11

Find two points on the line $y = x$ such that their distances from the line $2x – 3y + 5 = 0$ are both $\sqrt{13}$.

Solution

Let the required point be (a, a).

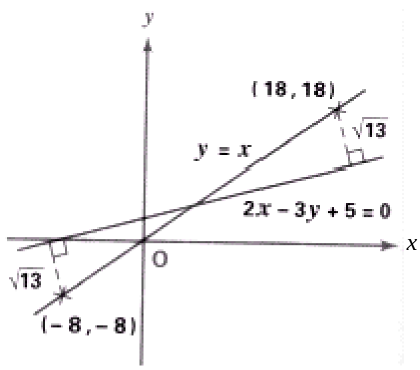
$$\left| \frac{2a - 3a + 5}{\sqrt{2^2 + (-3)^2}} \right| = \sqrt{13}$$

$$\left| \frac{5 - a}{\sqrt{13}} \right| = \sqrt{13}$$

$$|5 - a| = 13$$

$$5 - a = 13 \quad \text{or} \quad 5 - a = -13$$

$$a = -8 \quad \text{or} \quad a = 18$$



∴ The required points are (–8, –8) and (18, 18).

Checkpoint 9.8

(a) Find the distance from the point P(2, 0) to the line L: $3x - 4y = 0$.

(b) Find two points on the line passing through P with slope -1 such that the distance between the points and L is doubled the distance in (a).

Example 9.12

(a) Given two lines L_1: $x - y + 1 = 0$ and L_2: $7x + y - 3 = 0$, and the point P(x, y). Find, in terms of x and y, the distances from P to L_1 and L_2.

(b) Given that P lies on a line l and P is equidistant from L_1 and L_2, find the possible equations of l.

Solution

(a) The distance from P to $L_1 = \left|\dfrac{x-y+1}{\sqrt{1^2+(-1)^2}}\right|$

$$= \left|\frac{x-y+1}{\sqrt{2}}\right|$$

The distance from P to $L_2 = \left|\dfrac{7x+y-3}{\sqrt{7^2+1^2}}\right|$

$$= \left|\frac{7x+y-3}{\sqrt{50}}\right|$$

(b) Since P is equidistant from L_1 and L_2, we have

$$\left|\frac{x-y+1}{\sqrt{2}}\right| = \left|\frac{7x+y-3}{\sqrt{50}}\right|$$

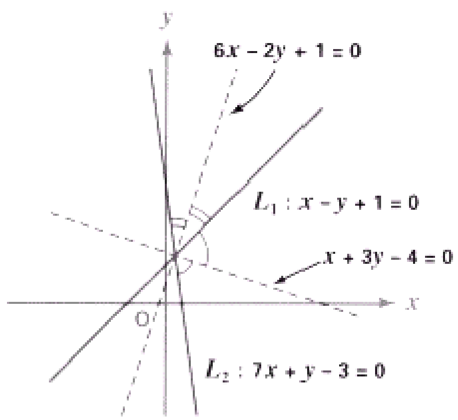
$$|x-y+1| = \frac{|7x+y-3|}{5}$$

$$x-y+1 = \pm\frac{7x+y-3}{5}$$

$5(x-y+1) = 7x+y-3$	or	$5(x-y+1) = -(7x+y-3)$
$5x-5y+5 = 7x+y-3$	or	$5x-5y+5 = -7x-y+3$
$-2x-6y+8=0$	or	$12x-4y+2=0$
$x+3y-4=0$	or	$6x-2y+1=0$

$\therefore$ The two possible equations are $x + 3y - 4 = 0$ and $6x - 2y + 1 = 0$.

Note: l is the angle bisector of the angle between L_1 and L_2. $x + 3y - 4 = 0$ is the angle bisector of the obtuse angle while $6x - 2y + 1 = 0$ is the angle bisector of the acute angle.



Checkpoint 9.9

Let L_1: $x + 7y - 23 = 0$ and L_2: $2x - 2y + 15 = 0$ be two given lines.

(a) Find the acute angle between L_1 and L_2 in degrees, correct to 2 decimal places.

(b) Find the equations of the two angle bisectors of L_1 and L_2.

(c) Show that the two angle bisectors found in (b) are perpendicular to each other.

9.5.2 Distance Between Two Parallel Lines

The distance d between two parallel lines $L_1 : Ax + By + C_1 = 0$ and $L_2 : Ax + By + C_2 = 0$ is given by

$$d = \left| \frac{C_2 - C_1}{\sqrt{A^2 + B^2}} \right| .$$

Example 9.13

Find the distance between the lines $L_1 : 3x + 4y - 2 = 0$ and $L_2 : 6x + 8y + 5 = 0$.

Solution

The equation of $L_1 : 3x + 4y - 2 = 0$ can be written as $6x + 8y - 4 = 0$.

$$\begin{aligned} \text{The required distance} &= \left| \frac{5 - (-4)}{\sqrt{6^2 + 8^2}} \right| \\ &= \frac{9}{10} \end{aligned}$$

Checkpoint 9.10

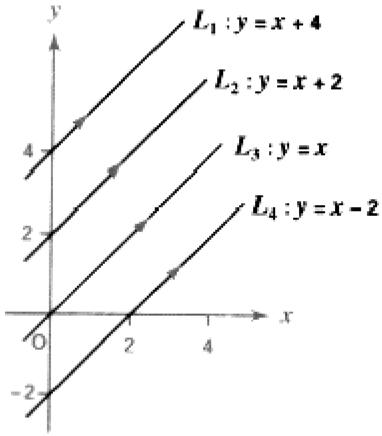
Find the distance between the lines $L_1 : 12x - 5y + 9 = 0$ and $L_2 : 24x - 10y + 21 = 0$.

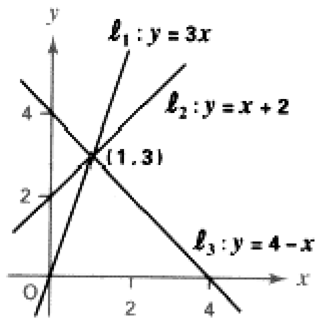
9.6 Family of Straight Lines

(1) The equation representing the family of straight lines with slope *m* is

$$y = mx + c$$

where *c* is any real number.





Example 9.14

(a) Write down the equation of the family of straight lines with slope –2.

(b) Find the equations of two straight lines with slope –2 which are $\sqrt{5}$ units from the point (0, 5).

Solution

(a) The required equation is $y = -2x + c$, where *c* is any real number.

(b) Rewrite $y = -2x + c$ as $2x + y - c = 0$.

The distance from (0, 5) to the line $2x + y - c = 0$ is $\sqrt{5}$, then we have

$$\left|\frac{2(0) + 5 - c}{\sqrt{2^2 + 1^2}}\right| = \sqrt{5}$$

$$\left|\frac{5 - c}{\sqrt{5}}\right| = \sqrt{5}$$

$$|5 - c| = 5$$

$$5 - c = 5 \quad \text{or} \quad 5 - c = -5$$

$$c = 0 \quad \text{or} \quad c = 10$$

$\therefore$ The required equations of straight lines are $2x + y = 0$ and $2x + y - 10 = 0$.

Checkpoint 9.11

(a) Write down the equation of the family of straight lines with slope 2.

(b) Find the equations of the straight lines with slope 2 which are $\sqrt{5}$ units from the point (1, 1).

(2) The equation representing the family of straight lines passing through the point (a, b) is

$$y-b=m(x-a)$$

where m is any real number.

Example 9.15

(a) Show that $7x - 2y + 20 = 0$ is a line in the family of straight lines represented by $y=m(x+2)+3$.

(b) Find the equations of two straight lines in the family of straight lines represented by $y=m(x+2)+3$ such that each line inclines at $45°$ to a line $L: 2x-y=0$.

Solution

(a) $y=m(x+2)+3$ represents the family of straight lines which passes through (–2, 3).

$\because$ $7(-2)-2(3)+20=0$

$\therefore$ (–2, 3) lies on $7x - 2y + 20 = 0$.

i.e. $7x - 2y + 20 = 0$ is a line in the given family.

(b) Slope of $L=\dfrac{-2}{-1}=2$

Since the angle between L and the line $y=m(x+2)+3$ is $45°$, we have

$$\tan 45° = \left|\frac{m-2}{1+2m}\right|$$

$$1 = \left|\frac{m-2}{1+2m}\right|$$

$$\frac{m-2}{1+2m}=1 \quad \text{or} \quad \frac{m-2}{1+2m}=-1$$

$$m-2=1 \quad \text{or} \quad m-2=-1-2m$$

$$m=-3 \quad \text{or} \quad m=\frac{1}{3}$$

$\therefore$ The required equations of straight lines are $y=-3(x+2)+3$ and $y=\dfrac{1}{3}(x+2)+3$,

i.e. $3x+y+3=0$ and $x-3y+11=0$.

Checkpoint 9.12

(a) Write down the equation of the family of straight lines other than the *y*-axis, which passes through the origin.

(b) Find the equation of the straight lines in the family in (a) which is 2 units from (2, 1).

(3) The equation representing the family of straight lines passing through the intersection of two lines $L_1 : A_1x + B_1y + C_1 = 0$ and $L_2 : A_2x + B_2y + C_2 = 0$ is

$$A_1x + B_1y + C_1 + k(A_2x + B_2y + C_2) = 0$$

where k is any real number.

Example 9.16

Given two straight lines

$$L_1 : x + y - 2 = 0 \text{ and } L_2 : 2x - y + 1 = 0 .$$

Find the equation of a straight line which passes through the point of intersection of L_1 and L_2, and the line

(a) has slope 3,

(b) passes through (4, 1).

Solution

Let the line be $L: x+y-2+k(2x-y+1)=0$,

i.e. $(1+2k)x+(1-k)y+k-2=0$.

(a) Slope of $L=-\frac{1+2k}{1-k}=3$

$\therefore$ $-1-2k=3-3k$

$k=4$

$\therefore$ The required equation is $x+y-2+4(2x-y+1)=0$,

i.e. $9x-3y+2=0$.

(b) Since L passes through (4, 1), we have

$4+1-2+k[2(4)-1+1]=0$

$3+8k=0$

$k=-\frac{3}{8}$

$\therefore$ The required equation is $x+y-2-\frac{3}{8}(2x-y+1)=0$,

i.e $2x+11y-19=0$.

Example 9.17

The line $(1-k)x+(2+k)y+3k+6=0$ passes through a fixed point P for any value of *k*. Find the coordinates of P.

Solution

Rewrite the equation $(1-k)x+(2+k)y+3k+6=0$ as $x+2y+6+k(-x+y+3)=0$.

Therefore the given equation represents a family of straight lines passing through the point of intersection of

$$x+2y+6=0 \quad ....(1)$$

$$-x+y+3=0 \quad ....(2)$$

$(1)+(2):$ $3y+9=0$

$y=-3$

Substituting $y = -3$ into (2), we have

$-x-3+3=0$

$x=0$

$\therefore$ The coordinates of P are (0, –3).

Checkpoint 9.13

Given two straight lines

$$L_1 : x + y - 2 = 0 \text{ and } L_2 : 2x - y + 1 = 0 .$$

(a) Write down the family of straight lines passing through the point of intersection of L_1 and L_2.

(b) Find the possible equations of the line in the family at unit distance from the origin.

Exercise 9 Straight Lines

9.1

1. Find the coordinates of the points which divide the line segment joining A(–3, 2) and B(11, 6) into three equal parts.

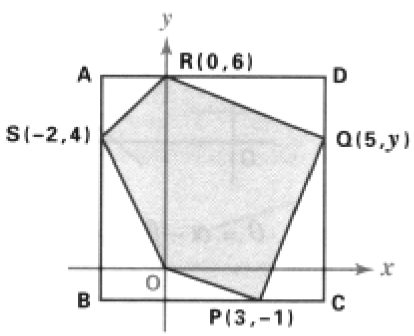
2. A and B are the points (2, 0) and (5, 3) respectively. P is a point on the line segment AB such that $\frac{\text{AP}}{\text{PB}} = k$.
   (a) Write down the coordinates of P in terms of k.
   (b) Hence find the ratio in which the line $4x + 3y - 25 = 0$ divides the line segment AB.

9.2

3. A triangle has vertices A(–1, 6), B(0, 2) and C(k, –5). Find the possible values of k if the area of ΔABC is 6 square units.

4. The lines $y = 2x$, $2x + y = 12$ and $y = 2$ enclose a triangular region in the coordinate plane. Find the coordinates of the vertices of the region and hence find the area of this region.

5. In the figure, ABCD is a rectangle whose sides are parallel to the x-axis or the y-axis. A pentagon OPQRS is inscribed inside ABCD.



   (a) Express the area of OPQRS in terms of y.
   (b) Hence, find the value of y such that the area of OPQRS is maximum. What is the maximum area?

9.3

6. Find the acute angle between the two lines $x-4y+7=0$ and $3x+5y+9=0$.

7. Given two straight lines $L_1: x-3y+5=0$ and $L_2: 6x-y+11=0$.
   (a) Find the acute angle between L_1 and L_2.
   (b) Given $L_3: 3x+y-7=0$.
      (i) Show that $L_1 \perp L_3$.
      (ii) Hence, find the acute angle between L_2 and L_3.

8. A line $L_1: px-y+6=0$, where p is a constant, makes an angle 45° with the line $L_2: 3x-7y+8=0$. Find the two values of p.

9. Find the equations of two lines through (0, 1), each making an angle θ with the line $y=4x$, where $\tan\theta=\frac{1}{3}$.

9.4, 9.5

10. Convert the equation of the straight line $8x+15y-34=0$ to the normal form, and find the values of θ in degree and p, correct to 2 decimal places.

11. Find the distance between each of the following pairs of parallel lines.
   (a) $L_1: 3x-4y+6=0$, $L_2: 9x-12y+7=0$
   (b) $L_1: x-3y=0$; $L_1: 3x-9y=8$.

12. Find the values of c if the distance from (1, 1) to the straight line $3x+y+c=0$ is $\sqrt{10}$ units.

13. Find two points on the straight line $x+y-1=0$ such that its distance to the line $x+3y-4=0$ is $\sqrt{13}$ units.

14. A point Q is called the reflection of the point P with respect to the straight line L if L is the perpendicular bisector of PQ.
   Find the coordinates of Q if Q is the point of reflection of the point (2, 9) with respect to the straight line $y=2x$.

15. Consider two parallel lines

$$L_1: \quad 3x + 4y - 7 = 0$$
$$L_2: \quad 3x + 4y + 8 = 0$$

(a) Find the distance between L_1 and L_2.

(b) A line L cuts L_1 and L_2 at P and Q respectively and $\text{PQ} = 3\sqrt{2}$.

(i) Find the acute angle between PQ and L_1.

(ii) Hence, find the slope(s) of the line L.

(c) If L passes through the point (2, 3), find the equation(s) of L.

9.6

16. (a) Write down the equation of the family of straight lines with slopes –4.

(b) Find the equation of the straight line L in the family in (a) such that the area of the triangle formed by L and the two positive coordinate axes is 8 square units.

17. (a) Write down the equation of the family of straight lines which passes through (1, 3).

(b) Find the equations of the two straight lines in the family in (a) which are inclined at $45°$ to the line $L_1: 3x - y + 4 = 0$.

18. Let $L_1: 2x + 4y - 3 = 0$ and $L_2: x - y + 5 = 0$ be two given straight lines.

(a) Write down the equation of the family of straight lines which passes through the point of intersection of L_1 and L_2.

(b) Find the equation of the straight line in the family in (a) which

(i) is parallel to the x-axis;

(ii) is perpendicular to $x = 2y$.

19. The equation $L: (2x - y + 8) + k(x - 4y + 7) = 0$ represents a family of straight lines which pass through a fixed point P for any value of k.

(a) Find the coordinates of P.

(b) Find the equation of the straight line in the family which is parallel to the line $x + 3y - 6 = 0$.

(c) Find the two lines in the family which incline at $45°$ to the line $y = 2x + 3$.