# *Spiking Neurons*

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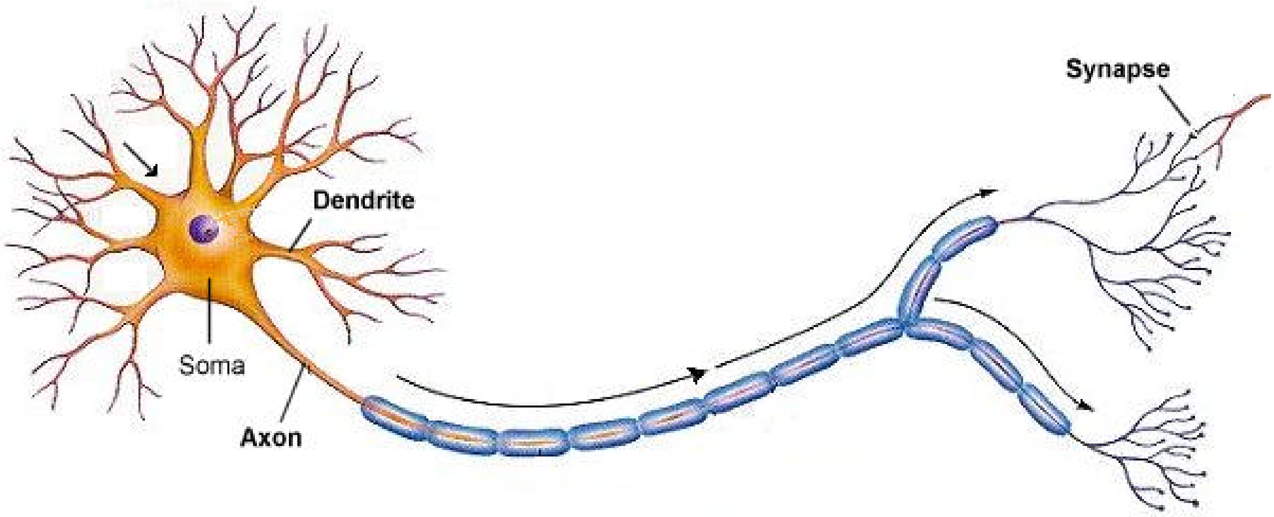
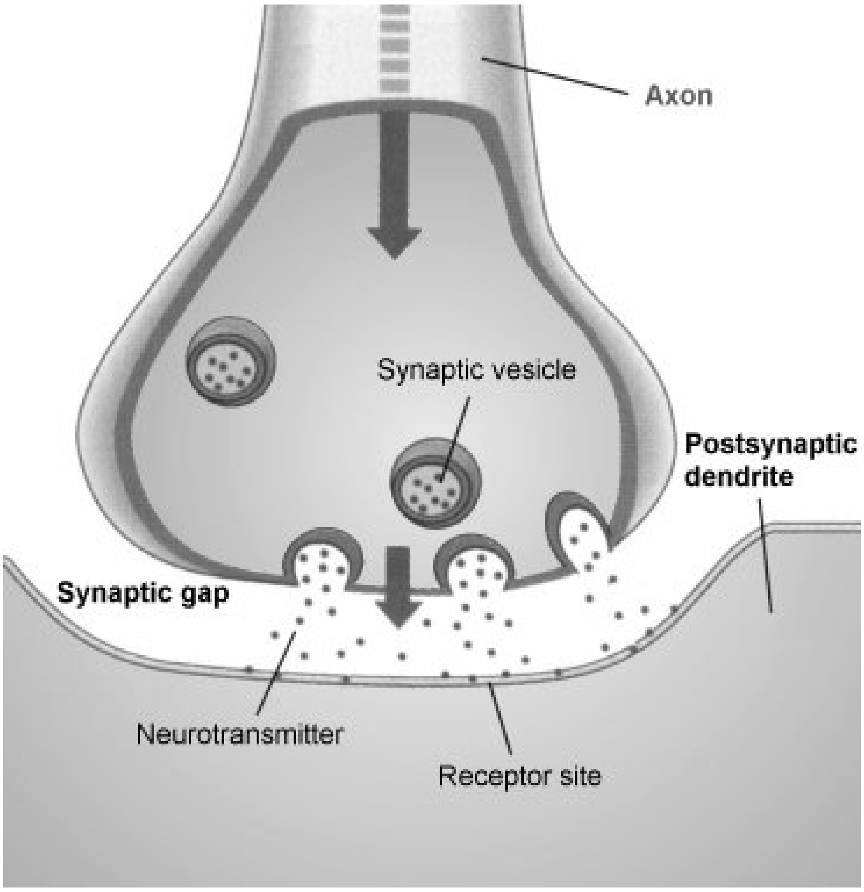
## ***Neuron model***

$$\boldsymbol{y} = f(\boldsymbol{x}, \boldsymbol{t})$$


Synapse
Dendrite
Soma
Axon




Axon
Synaptic vesicle
Postsynaptic dendrite
Synaptic gap
Neurotransmitter
Receptor site

Realism

Compartmental models

Hodking-Huxley

Integrate and fire

Spiking Neuron

Rate models



Realism / complexity

Compartmental models — *Spatial structure, ...*

Hodking-Huxley — *Ionic currents*

Integrate and fire — *R and C in parallel, $f(V)$*

Spiking Neuron — $f(\tau)$

Rate models — *Large temporal scale*

Realism

Compartmental models

Hodking-Huxley

Integrate and fire

*Differential equations*

Spiking Neuron — *Integral / sum on t*

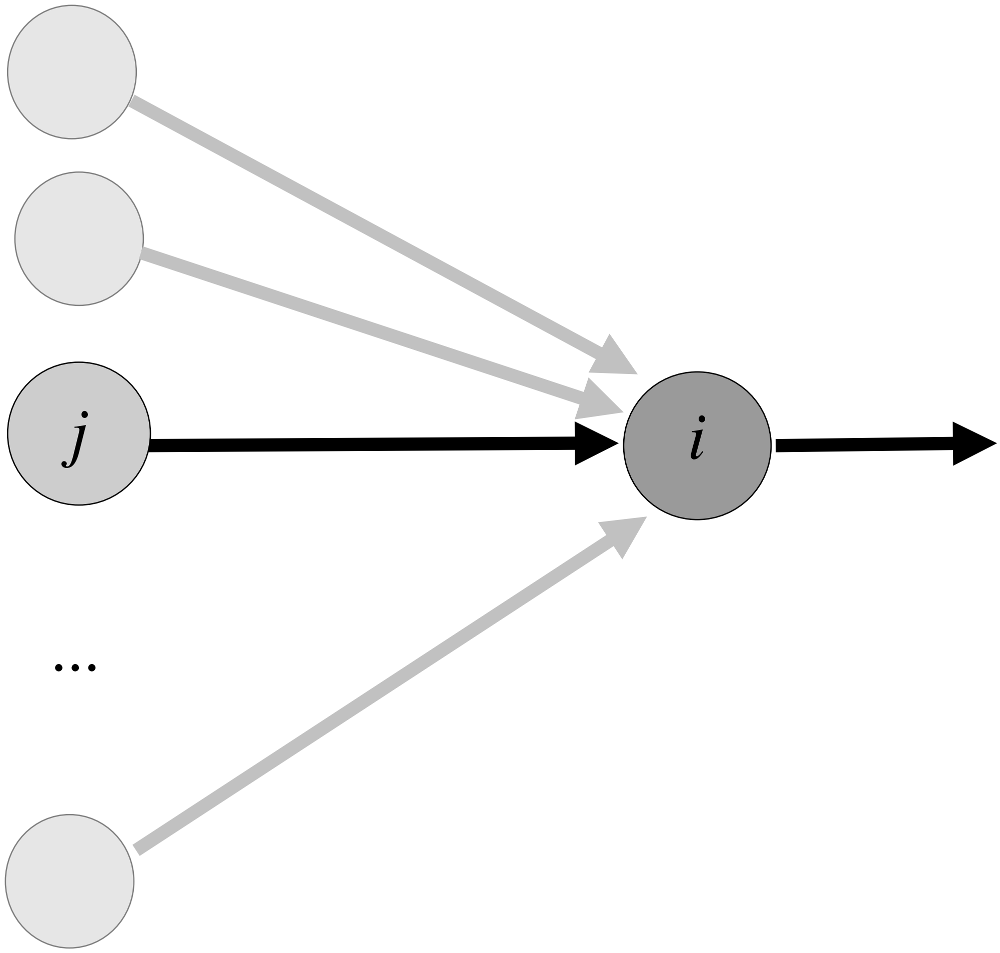
Rate models — *Algebraic equations*



# In the brain:

- A spike lasts for 1-2 ms.

- In general, spikes are very similar with each other, so the only information they carry is contained in their time stamp.






# Notation:

$t_i^{(f)}$ time when neuron *i* generated the spike *f*

$u_i(t)$ potential of neuron *i*

$w_{ij}$ synaptic weight between neurons *j* e *i*

$$\mathcal{F}_i = \left\{ t_i^{(1)}, ..., t_i^{(n)} \right\}$$

$$\eta_i\left(t - t_i^{(f)}\right) \qquad \epsilon_{ij}\left(t - t_i^{(f)}\right)$$

# Potential (1)

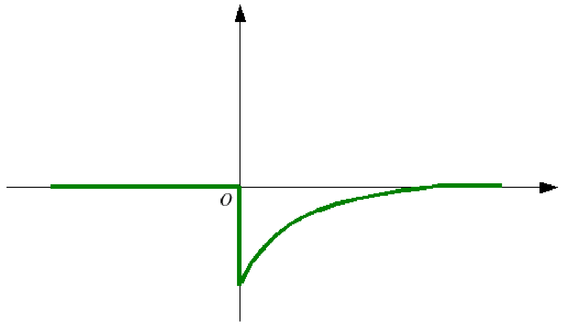
$$u_i(t) = \sum_{t_i^{(f)} \in \mathcal{F}_i} \eta_i \left(t - t_i^{(f)}\right) + \sum_{j \in \Gamma_i} \sum_{t_j^{(f)} \in \mathcal{F}_j} w_{ij} \epsilon_{ij} \left(t - t_j^{(f)}\right)$$



# Potential (2)

$$u_i(t) = \sum_{t_i^{(f)} \in \mathcal{F}_i} \eta_i \left(t - t_i^{(f)}\right) + \sum_{j \in \Gamma_i} \sum_{t_j^{(f)} \in \mathcal{F}_j} w_{ij} \epsilon_{ij} \left(t - t_j^{(f)}\right)$$

Refractory function

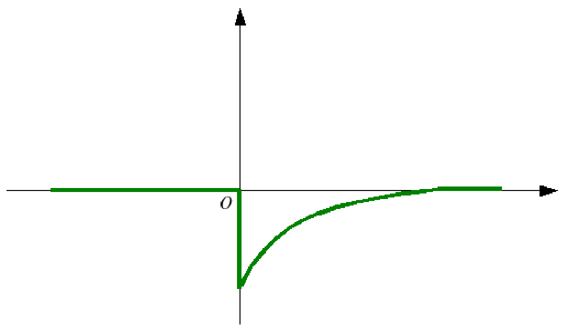


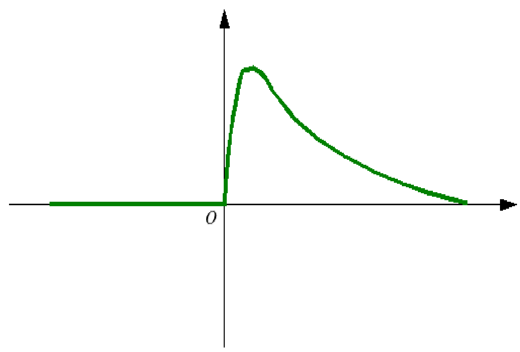
# Potential (3)

$$u_i(t) = \sum_{t_i^{(f)} \in \mathcal{F}_i} \eta_i \left(t - t_i^{(f)}\right) + \sum_{j \in \Gamma_i} \sum_{t_j^{(f)} \in \mathcal{F}_j} w_{ij} \epsilon_{ij} \left(t - t_j^{(f)}\right)$$

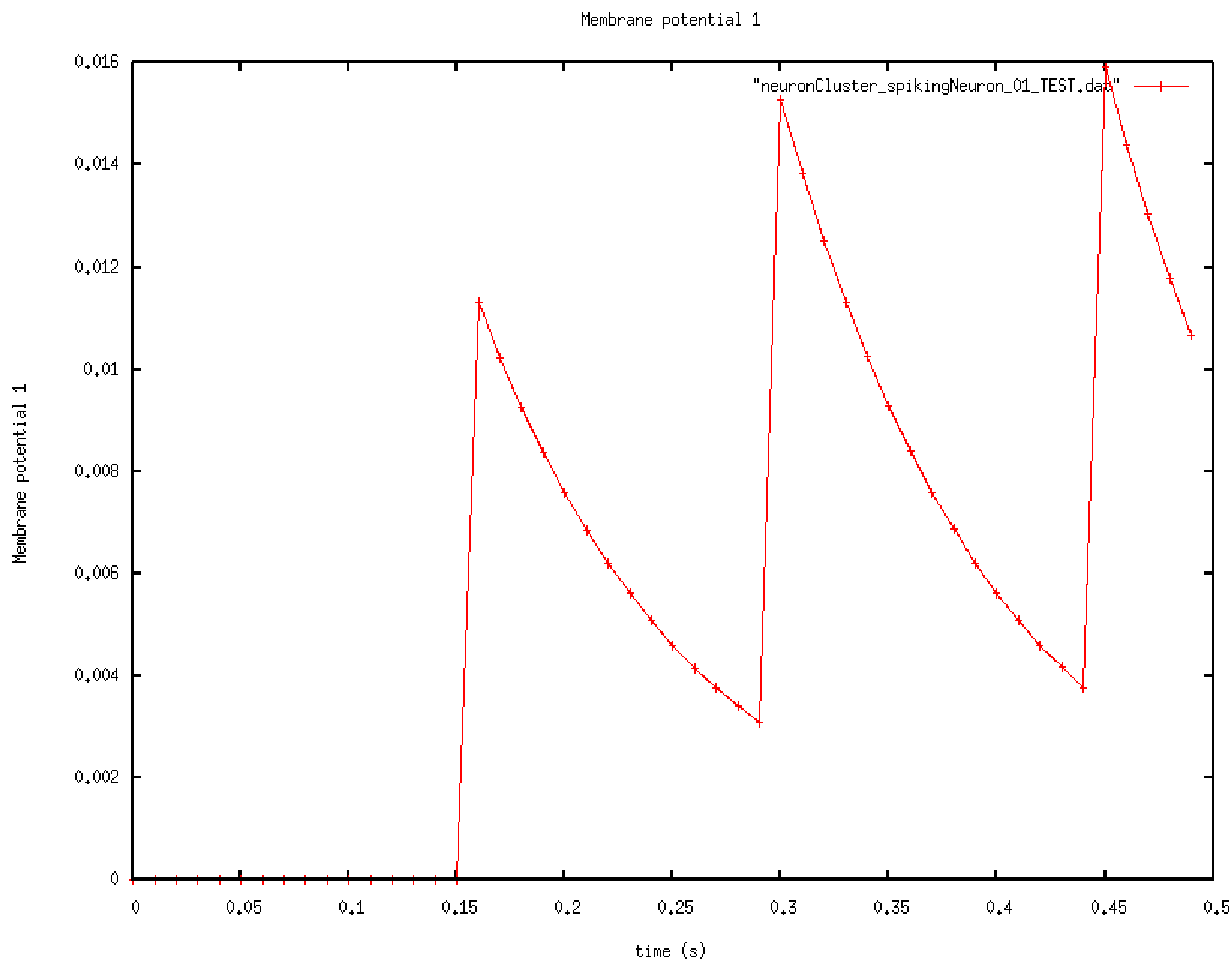
Refractory function

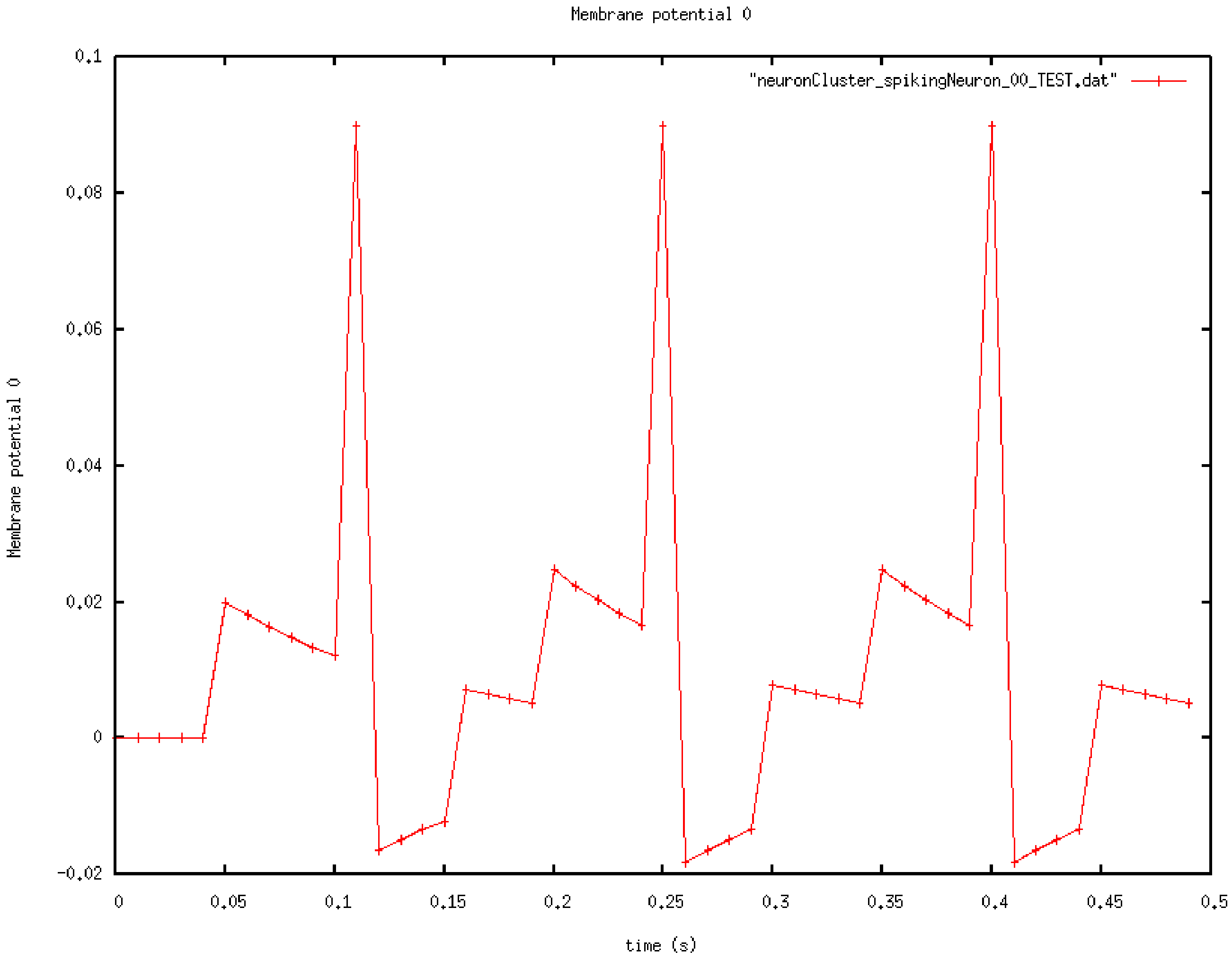
PSP



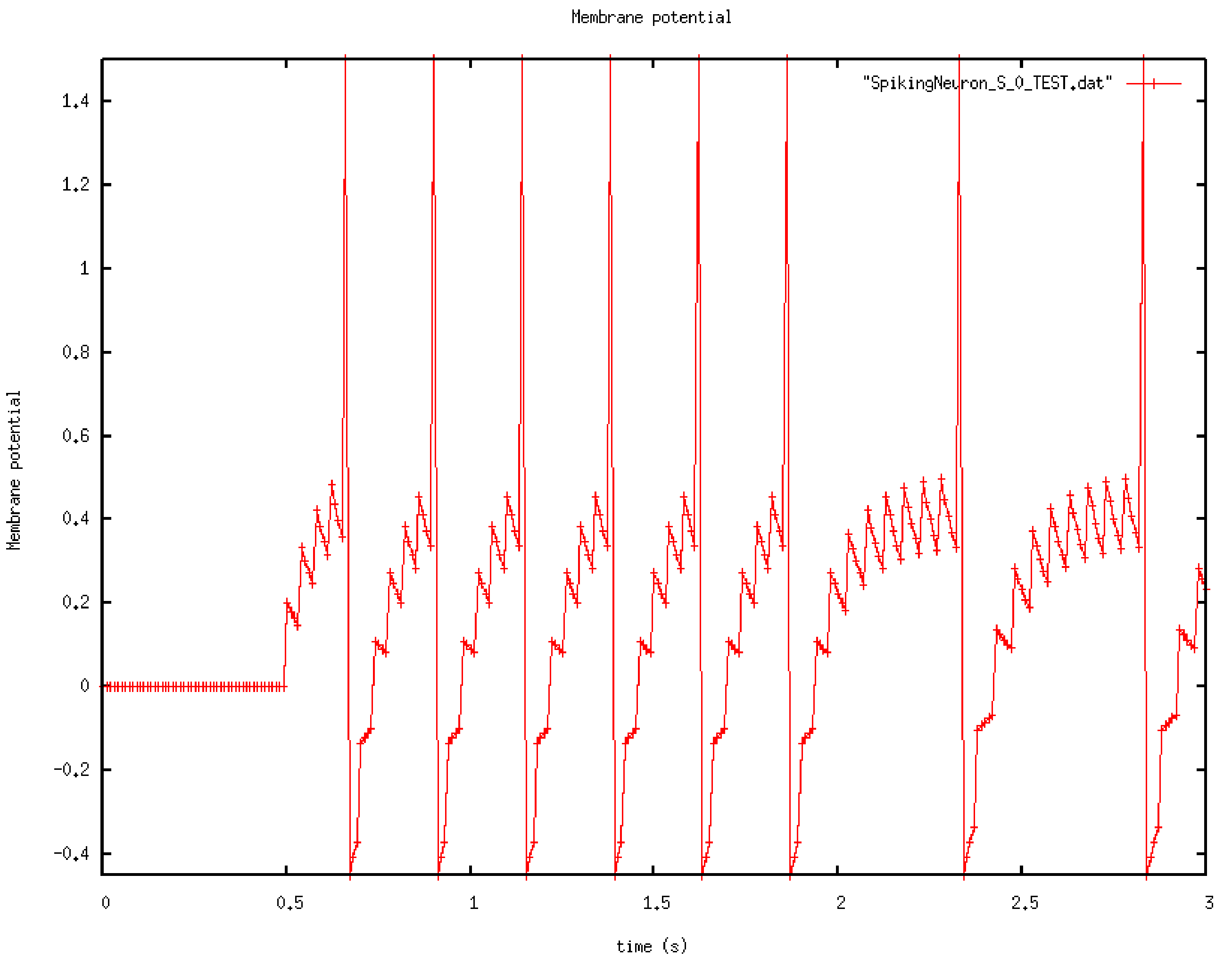



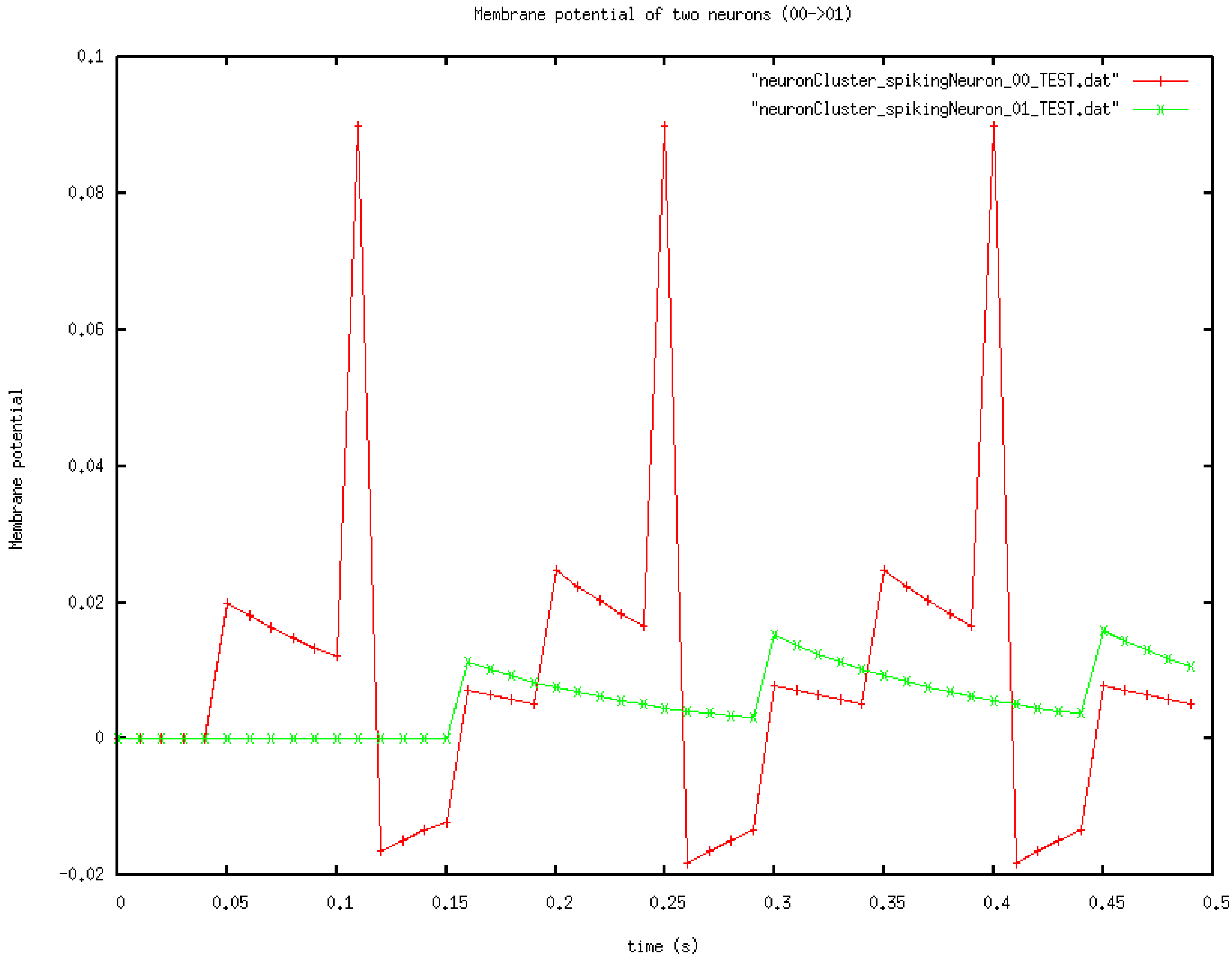



Membrane potential 0
"neuronCluster_spikingNeuron_00_TEST.dat"
Membrane potential 0
0.1
0.08
0.06
0.04
0.02
0
-0.02
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5
time (s)

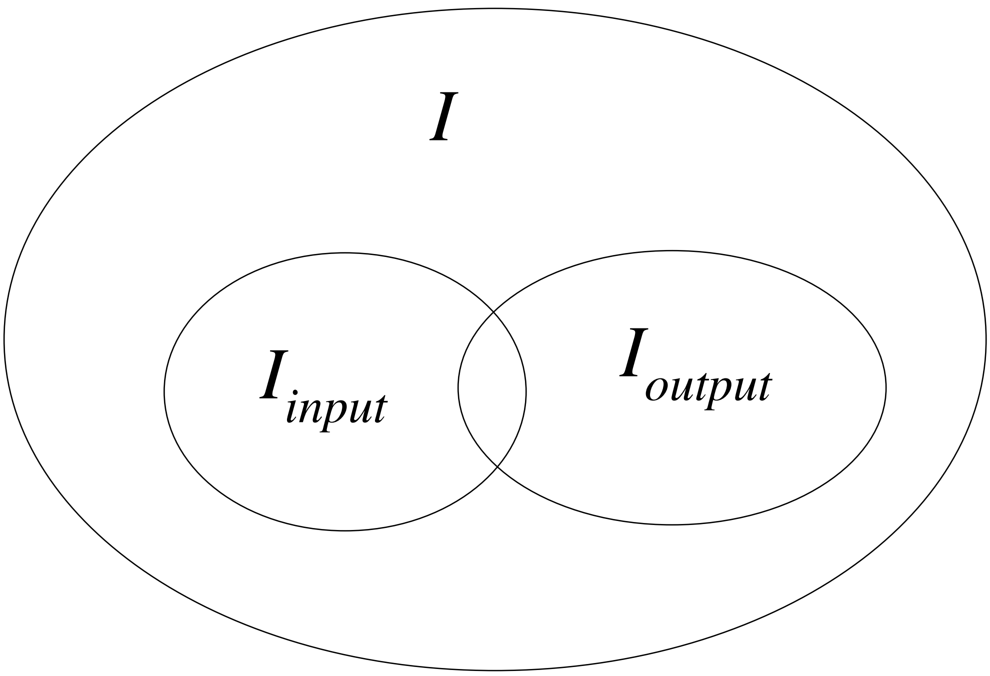



Membrane potential
"SpikingNeuron_S_0_TEST.dat"
Membrane potential
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0 0.5 1 1.5 2 2.5 3
time (s)






# Network of spiking neurons



# Network of spiking neurons

$$\langle \mathcal{F}_i \rangle_{i \in I_{input}} \longrightarrow \langle \mathcal{F}_i \rangle_{i \in I_{output}}$$



# Learning (1.1)

$\downarrow$

The following parameters are updated:

Synaptic weights: $w_{ij}$

Thresholds: $\vartheta \quad \vartheta_i$

Axonal transmission delays: $\Delta_{ij}$

# Learning (1.2)

The following parameters are updated:

Synaptic weights: $w_{ij}$

Thresholds: $\vartheta \quad \vartheta_i$

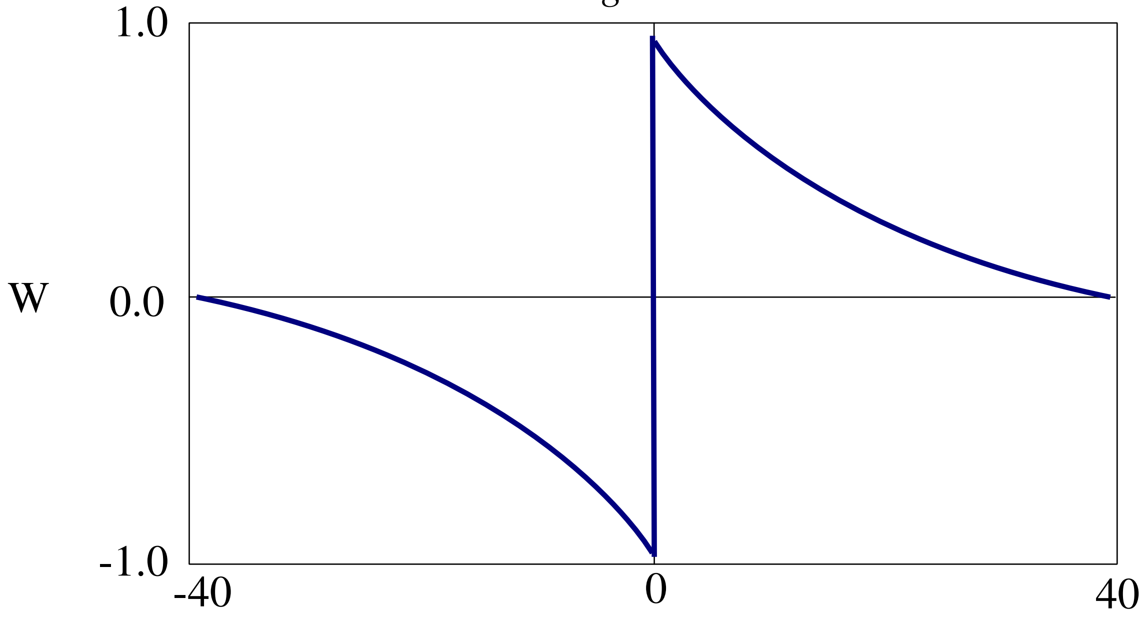
Axonal transmission delays: $\Delta_{ij}$

$$\left(t_i^{(f)} - t_j^{(f)}\right)$$



# Learning (1.3)

*Learning window W*




$$\left(t_i^{(f)} - t_j^{(f)}\right)$$

*ms*

# Learning (2)

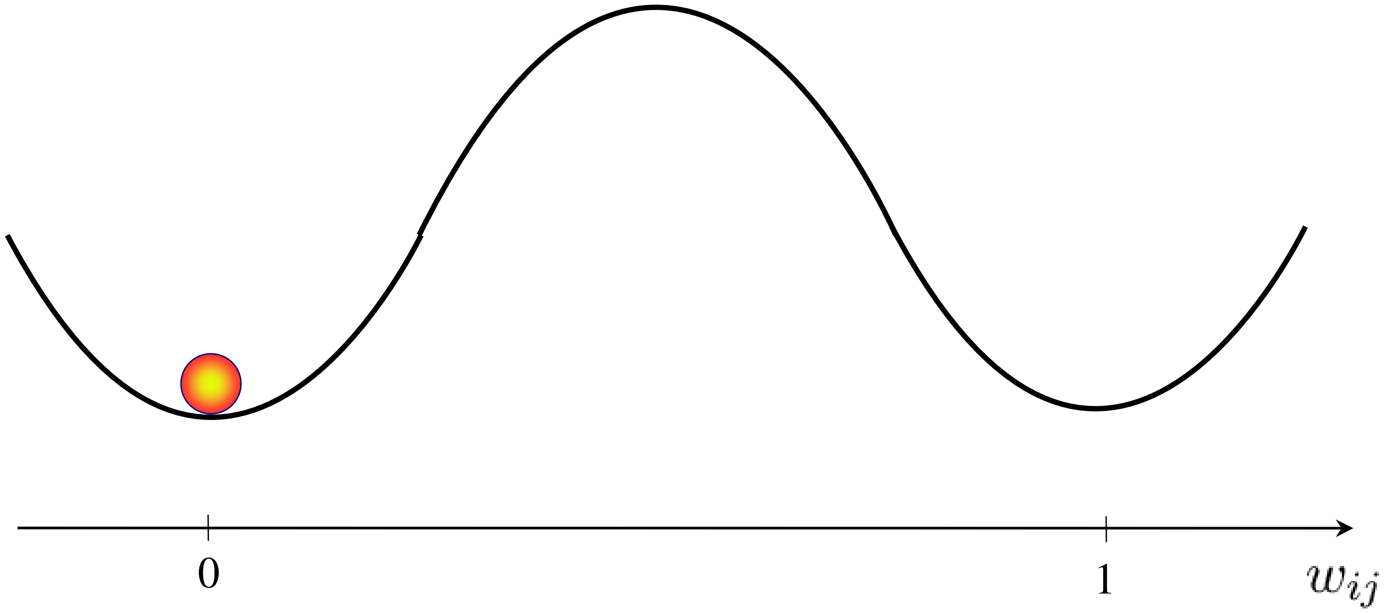
| Phenomenon | Duration | Locus of Induction |
|---|---|---|
| ***Short-term Enhancement*** | | |
| Paired-Pulse Facilitation (PPF) | 100 ms | Pre |
| Augmentation | 10 s | Pre |
| Post-tetanic potentiation | 1 min | Pre |
| ***Long-term Enhancement*** | | |
| Short-Term Potentiation (STP) | 15 min | Post |
| Long-Term Potentiation (LTP) | >30 min | Pre and Post |
| ***Depression*** | | |
| Paired-Pulse Depression (PPD) | 100 ms | Pre |
| Depletion | 10 s | Pre |
| Long-term Depression (LTD) | >30 min | Pre and Post |



# *Learning (3.1)*

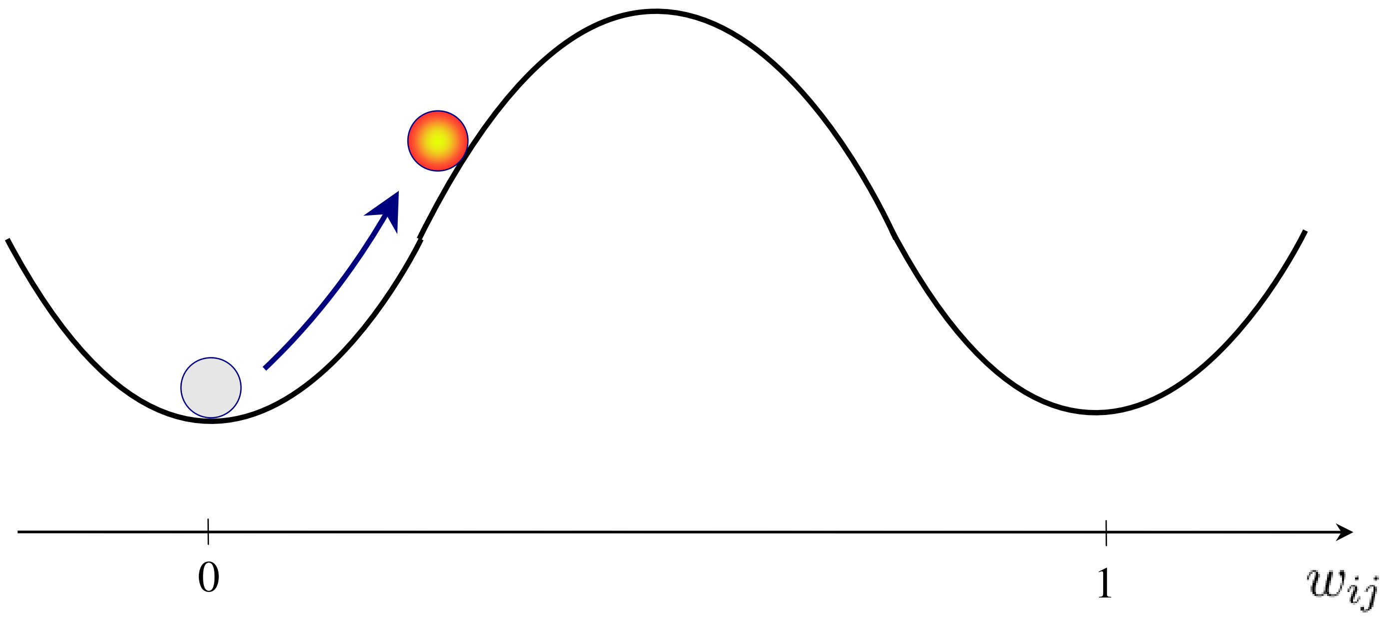
## ***Stability / Plasticity dilemma***

## *Learning (3.2)*

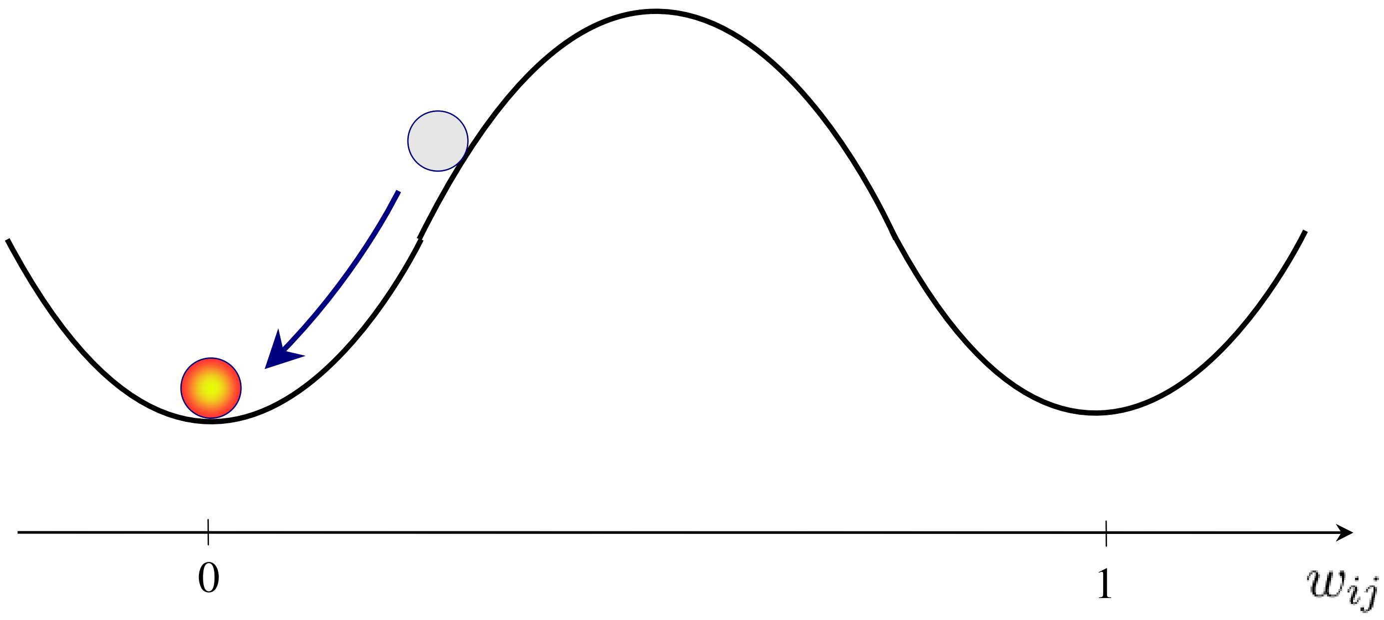





## *Learning (3.3)*

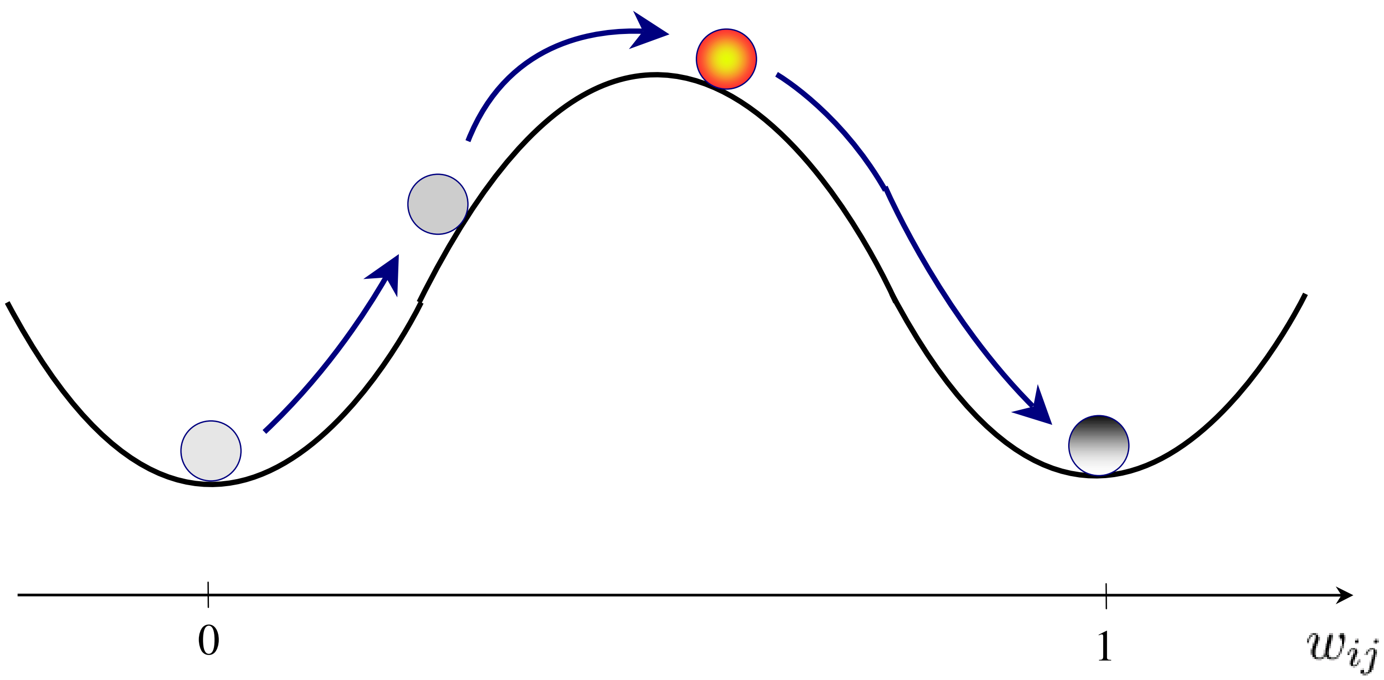


# *Learning (3.4)*






# *Learning (3.5)*



# Implementation: notes



Average number of spikes generated
by each neuron in a time interval

↓

Different approach in the representation of spike trains

Average number of spikes generated
by each neuron in a time interval: **low**

↓

## Spike event list

| |
|---|
| … |
| $t_3$ , $n_6$ |
| $t_2$ , $n_2$ |
| $t_1$ , $n_3$ |
| $t_0$ , $n_2$ |

time ↓



Average number of spikes generated
by each neuron in a time interval: **high**

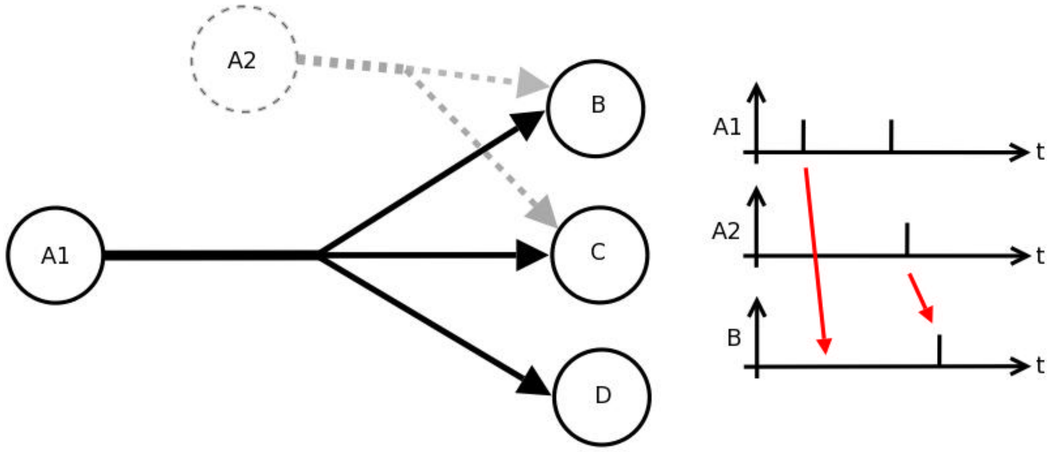
↓

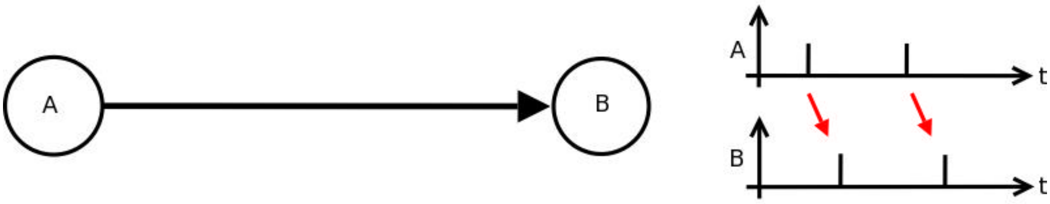
For each neuron:

| |
|---|
| … |
| 1 |
| 0 |
| 1 |
| 1 |
| 0 |
| 1 |
| 0 |
| 1 |

*time slot* {

time ↓






$$PSP \propto \left( w_{ji}, \frac{1}{N_t} \right)$$

*The end*