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Axion-dilation black holes with $SL(2, Z)$ symmetry through APT-FGP model

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Abstract. – Keeping in view the partial breaking of the supersymmetry of dyons in $N = 4$ supersymmetric theories, which is due to the presence of a central charge in the algebra, and the mass of the BPS dyon in supersymmetric Yang-Mills theories, we have analyzed the ratios of charges of supersymmetric black holes with 1/4 of unbroken $N = 4$ supersymmetry and demonstrated that in spontaneous breaking of the $N = 4$ global supersymmetry to $N = 2$, the parameters of electric and magnetic Fayet-Iliopoulos terms can be considered proportional to the electric and magnetic charges of the dyonic black holes.

Supersymmetric gauge theories of monopoles and dyons have been much explored [1–12] partly because of the phenomenological interest, and recent results [9–11] have emerged about their strong-coupling behaviour. One could have guessed that black holes of the $N = 2$ theory with one-half of supersymmetry unbroken may be somehow relevant to models with spontaneous breaking of the $N = 2$ supersymmetry to $N = 1$ [13]. The models of spontaneously broken $N = 2$ to $N = 1$ global supersymmetries and those of $N = 4$ to $N = 2$ lead to the possibility that the parameters of electric and magnetic Fayet-Iliopoulos terms can be considered proportional to the electric and magnetic charges of the dyonic black hole. The choice of a superpotential in such models will be related to the central charge of the graviphoton, *i.e.* to the black-hole mass as a function of moduli and conserved charges of the black hole.

Keeping in view the Prasad-Sommerfield limit [14] and Gauss's law, the expressions for the electric and magnetic charges of fields associated with dyons in the non-Abelian gauge theory [15] in $N = 4$ supersymmetric theories can be written as

$$Q_{\mathrm{E}} = \int \mathrm{d}^3x\, \partial_i(E_i\phi), \qquad Q_{\mathrm{M}} = \int \mathrm{d}^3x\, \partial_i(B_i\phi) = k. \tag{1}$$

This Q_{E} arises from the components of the six-dimensional momentum in extra dimensions [16] while the magnetic charge Q_{M} has a topological origin.

The presence of these central charges in the algebra implies that

$$M^2 \geq Q_{\mathrm{E}}^2 + Q_{\mathrm{M}}^2. \tag{2}$$

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The charges (1) can be given as

$$Q_{\mathrm{E}} = ve \qquad \text{and} \qquad Q_{\mathrm{M}} = vg. \tag{3}$$

Then (2) reduces to

$$M \geq v|q|, \tag{4}$$

which is saturated in the form of equation (Prasad-Sommerfield limit)

$$V(\phi) = 0, \qquad \text{but} \qquad v = \langle \phi \rangle \neq 0,$$

at classical level, for all states in the theory. As a result of the partial breaking of supersymmetry [17], one could have guessed that the black holes of the $N = 2$ and $N = 4$ theories with one-half of supersymmetry unbroken may be somehow relevant to models with spontaneous breaking of the $N = 2$ supersymmetry to $N = 1$ and that of $N = 4$ to $N = 2$. The central charge defining the mass of the dyons in rigid supersymmetric theory is defined [18] by the charge of the vector multiplet. It is defined by the sympletic section of the given $N = 2$ theory (X^Λ, F_Λ) and by the conserved charges (q_Λ, p^Λ) of the dyon and it is given by

$$\left(M_{\mathrm{YM}}^{\mathrm{dyon}}\right)^2 = \left|Z^{\mathrm{rigid}}(t, \bar{t}, q, p)\right|^2 = \left|X^\Lambda(t) q_\Lambda - F_\Lambda(t) p^\Lambda\right|^2. \tag{5}$$

This formula also gives the mass of the BPS dyons in supersymmetric Yang-Mills theory. In our case the central charge of the gravitational multiplet is the charge of the graviphoton, the supersymmetric partner of the graviton in theories with local supersymmetry. It is defined as [19]

$$\left(M_{\mathrm{bh}}^{\mathrm{dyon}}\right)^2 = \left|Z^{\mathrm{local}}(t, \bar{t}, q, p)\right|^2 = \left|e^{K(t,\bar{t})/2}\left[X^\Lambda(t) q_\Lambda - F_\Lambda(t) p^\Lambda\right]\right|^2, \tag{6}$$

where $K(t, \bar{t})$ is a Kahler potential,

$$K = -\,\mathrm{Im}\left(\overline{X}^\Lambda F_\Lambda - X^\Lambda \overline{F}_\Lambda\right). \tag{7}$$

In all cases of supersymmetric black holes with a non-vanishing area of the horizon and Bertotti-Robinson–type geometry near the black-hole horizon the following has been proved [20]. The extremum of the square of the graviphoton central charge [21] in the moduli space relates the values of moduli to the ratios of electric and magnetic charges. As a result, the value of the square of the central charge at the extremum in moduli defines the area of the black-hole horizon, which depends only on the conserved charges. The extremal value of this mass is also related to the size of the infinite throat of the Bertotti-Robinson geometry and is independent of the values of moduli far away from the horizon:

$$\left(\left|Z(t, \bar{t}(q,p))\right|^2\right)_{\bar{\sigma}|Z(t,\bar{t}(q,p))|/(\partial t)=0} = \frac{1}{\pi} S(q,p). \tag{8}$$

We will show in what follows that the idea of the supersymmetric attractors [20, 21], which explains that moduli are driven to a fixed point of attraction near the black-hole horizon, may be realized in effective theories of global supersymmetry in a standard way. For this to happen we need three conditions to be satisfied:

A) The potential depending on moduli $t, \bar{t}$ and on some parameters (E, M) has to be proportional to the square of the central charge, depending on the same moduli $t, \bar{t}$ and on the black-hole charge (q, p):

$$V\big(t, \bar{t}(E, M)\big) \approx \left|Z^{\mathrm{local}}\big(t, \bar{t}(q,p)\big)\right|^2 = \left|e^{K(t,\bar{t})/2}\left[X^\Lambda(t) q_\Lambda - F_\Lambda(t) p^\Lambda\right]\right|^2. \tag{9}$$

B) The parameters (E,M) in the potential have to be proportional to the black-hole charge:

$$(E,M) \approx (q,p). \tag{10}$$

C) The relevant supersymmetric black hole has to have a finite area of the horizon for the potential to have a stable minimum with regular values of moduli.

Spontaneous partial breaking of the $N=2$ supersymmetry to $N=1$ was discovered recently in the context of globally supersymmetric theories by Antoniadis, Partouche and Taylor [22]. The partial supersymmetry breaking was found by a suitable flat limit of local $N=2$ supergravity models by Ferrara, Girardello and Parrati [23]. These two theories are essentially equivalent. The APT-FGP potential has a stable minimum. The parameters (E,M) in the potential turn out to be the parameters of the electric and magnetic Fayet-Iliopoulos terms, and they will be related to the black-hole charge via the scale of supersymmetry breaking:

$$(E,M) = \Lambda^2(q,p). \tag{11}$$

We will start with the black-hole side first. From all known black holes it is the family of axion-dilation black holes with manifest $SL(2,Z)$ symmetry [24] that turns out to be relevant to the APT-FGP mechanism.

For analyzing the superpotential in the FTP-AGP model and black holes near the horizon, the APT model as well as the flat limit of the FGP model in terms of manifest $N=1$ supersymmetry consist of the usual form

$$-\frac{i}{4}\int \mathrm{d}^2\theta\lambda\overline{\omega} + \mathrm{c.c.} + \int \mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}K, \tag{12}$$

where $\overline{\omega}$ is the gauge field superfield and K is the *Kahler* potential. This action is supplemented by the FI term

$$\Lambda^2\sqrt{2}\xi D, \tag{13}$$

as well as by an unusual superpotential term

$$\Lambda^2\int \mathrm{d}^2\theta\overline{\omega} + \mathrm{c.c.} \tag{14}$$

Here

$$\overline{\omega} = eb + m\wp_b, \tag{15}$$

where b is a chiral superfield and $\wp_b$ is the derivative of the prepotential over b.

In terms of the manifest $N=2$ superfield the APT Lagrangian is

$$\frac{i}{4}\int \mathrm{d}^2\theta_1\mathrm{d}^2\theta_2\left[\wp(B) - \beta^D B\right] + \frac{1}{2}\left(\overline{E}\cdot\overline{Y} + \overline{M}\cdot\overline{Y}^D\right) + \mathrm{c.c.}, \tag{16}$$

where B is an unconstrained chiral $N=4$ multiplet, B^D is a constrained chiral $N=2$ multiplet playing the role of the Lagrangian multiplier. The constant vectors $\vec{E},\vec{M}$ ($\vec{M}$ being real) define the electrical and the magnetic Fayet-Iliopoulos term, since they are the coefficients in front of the auxiliary fields of the B, B^D multiplets, $\overline{Y}$, $\overline{Y}^D$. In the APT-FGP model the FI parameters that lead to spontaneous breaking of the $N=2$ supersymmetry to $N=1$ are chosen to be

$$\mathrm{Re}\,\vec{E} = \Lambda^2(0,e,\xi), \qquad \vec{M} = \Lambda^2(0,m,0). \tag{17}$$

Upon elimination of auxiliary fields the scalar potential is

$$V = \frac{|\,\mathrm{Re}\,\vec{E} + \lambda \vec{M}|^2}{\mathrm{Im}\,\lambda} + \cdots\,, \tag{18}$$

where the ellipsis denotes terms independent of moduli λ. At the moment, from the black-hole side we do not have information on the field-independent part of the potential and in what follows we will compare the APT-FGP field-dependent part of the potential with the black-hole central charge. A stable minimum of the potential for the scalar field in APT theory exists at

$$\begin{aligned}(\mathrm{Im}\,\lambda)_{\min} &= \left(e^{-2\phi}\right)_{\min} = \left|\frac{\xi}{m}\right|, \\ (\mathrm{Re}\,\lambda)_{\min} &= (a)_{\min} = -\frac{e}{m}.\end{aligned} \tag{19}$$

It is fairly easy to see that, if one took the three types of black-hole solution above and in each case chose only three non-vanishing charges, one would reproduce the relevant values of the scalars at the potential minimum from the fixed point of moduli in black holes. In particular, in the first case we may take a black hole with

$$\begin{aligned}&m_2 = 0, \qquad \left(e^{-2\phi}\right)_{\mathrm{fix}} = \left|\frac{m}{n_1}\right| = \left|\frac{\xi}{m}\right| = \left(e^{-2\phi}\right)_{\min}, \\ &(a)_{\mathrm{fix}} = \frac{n_1}{m_1} = -\frac{e}{m} = (a)_{\min}.\end{aligned} \tag{20}$$

In the second case with

$$\begin{aligned}&p^0 = 0, \qquad \left(e^{-2\phi}\right)_{\mathrm{fix}} = \left|\frac{q_0}{q_1}\right| = \left|\frac{\xi}{m}\right| = \left(e^{-2\phi}\right)_{\min}, \\ &(a)_{\mathrm{fix}} = \frac{p^1}{q_1} = -\frac{e}{m} = (a)_{\min}.\end{aligned} \tag{21}$$

In the third case with

$$\begin{aligned}&\hat{p}^1 = 0, \qquad \left(e^{-2\phi}\right)_{\mathrm{fix}} = \left|\frac{\hat{q}_1}{\hat{q}_1}\right| = \left|\frac{\xi}{m}\right| = \left(e^{-2\phi}\right)_{\min}, \\ &(a)_{\mathrm{fix}} = \frac{\hat{q}^0}{\hat{p}^1} = -\frac{e}{m} = (a)_{\min}.\end{aligned} \tag{22}$$

This gives the relation between the ratios of the parameters of the FI term leading to spontaneous breaking of the $N = 2$ supersymmetry to the $N = 1$ supersymmetry in the APT-FGP model and the ratios of charges of supersymmetric black holes with $1/4$ of unbroken $N = 4$ supersymmetry or $1/2$ of the $N = 2$ one, which confirms our argument given in the beginning of this paper.

Discussion. – The mass formula given by eq. (2), or eq. (4) in saturated form survives quantization by requiring that the supersymmetric charge algebra is saturated involving central charges which arise as surface terms when spontaneous symmetry breaking occurs. The unbroken supersymmetry pairs bosonic zero modes with fermionic zero modes in the $N = 4$ theory [25]. As such, the effective action governing the low-energy dynamics of the dyons of the $N = 4$ supersymmetry is given by the $N = 8$ supersymmetric quantum mechanics. In the $N = 4$ supersymmetric theory, the electrons and monopoles have the same quantum numbers and hence the electric-magnetic duality is possessed contrary to the $N = 2$ supersymmetric theory of dyons. In the model of spontaneous breaking of the $N = 4$ to $N = 2$ global supersymmetric theory the parameters of electric and magnetic Fayet-Iliopoulos terms have been considered proportional to the electric and magnetic charges of the dyonic black hole. The APT-FGP model clearly shows that the origin of the superpotential stabilizing the moduli is related to supersymmetric black holes with a non-vanishing area of the horizon. It is also important to stress that, whereas black holes required the concept of static geometry-breaking Lorentz invariance, the low-energy action with partially spontaneously broken supersymmetry describes a Lorentz-covariant theory that codifies the most important properties of extreme black holes with partially broken supersymmetry. Our observation opens various directions for further investigations. It may be possible to construct models with global $N = 4$ supersymmetry with the black-hole–type potential for breaking the supersymmetry spontaneously down to $N = 1$, since there exist black holes that break 3/4 of the $N = 4$ supersymmetry.

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REFERENCES

[1] Hoker E. D. and Vinet L., *Phys. Lett. B*, **137** (1984) 72; *Nucl. Phys. B*, **260** (1985) 79; *Lett. Math. Phys.*, **8** (1984) 439.
[2] Witten E. and Seiberg N., *Nucl. Phys. B*, **426** (1994) 19.
[3] Ferrari F. and Bilal A., *Nucl. Phys. B*, **469** (1996) 387; **480** (1996) 589.
[4] Terashima S. and Yang S. K., *Phys. Lett. B*, **391** (1997) 107.
[5] Joshi S. C. and Rajput B. S., *Ind. J. Pure Appl. Phys.*, **38** (2000) 437.
[6] Joshi S. C. and Rajput B. S., to be published in *Pramana*.
[7] Joshi S. C. and Rajput B. S., submitted to *Int. J. Theor. Phys.*
[8] Jackiw R. and Rebbi C., *Phys. Rev. D*, **13** (1976) 3398.
[9] Minikata H., *Phys. Rev. D*, **32** (1985) 2134.
[10] Rajput B. S. and Rana J. M. S., *Phys. Rev. D*, **43** (1991) 3550.
[11] Seiberg N., *Phys. Rev. D*, **49** (1994) 6857; **50** (1994) 1092.
[12] Seiberg N. and Witten E., *Nucl. Phys. B*, **431** (1994) 484.
[13] Joshi S. C. and Rajput B. S., submitted to *Nucl. Phys. B*.
[14] Prasad M. K. and Sommerfield C. M., *Phys. Rev. Lett.*, **35** (1975) 760.
[15] Rajput B. S., Bisht R. and Chandola H. C., *Nuovo Cimento A*, **104** (1991) 697.
[16] Gauntlett J. P., *Nucl. Phys. B*, **411** (1994) 443; **400** (1993) 103.
[17] Joshi S. C. and Rajput B. S., submitted to *Nucl. Phys. B*.
[18] Seiberg N. and Witten E., *Nucl. Phys. B*, **431** (1994) 484.
[19] Ceresole A., D'Auria R., Ferrara S. and Van Proeyen A., *Nucl. Phys. B*, **444** (1995) 92.
[20] Ferrara S. and Kallosh R., *Phys. Rev. D*, **54** (1996) 1514; 1525.
[21] Ferrara S., Kallosh R. and Strominger A., *Phys. Rev. D*, **52** (1995) 5412; Strominger A., *Macroscopic Entropy of $N = 2$ External Black Holes*, Report no. hep-th/9602111.
[22] Antoniadis I., Partouche H. and Taylor T. R., *Phys. Lett. B*, **372** (1996) 83; Antoniadis I., Partouche H. and Taylor T. R., *Dual $N = 2$ SUSY Breaking*, Report no. hep-th/9604062.
[23] Ferrara S., Girardello L. and Porrati M., *Phys. Lett. B*, **376** (1996) 275.
[24] Kallosh R. and Ortin T., *Phys. Rev. D*, **48** (1993) 742; Bergshoeff E., Kallosh R. and Ortin T., *Stationary Axion/Dilation Solutions and Supersymmetry*, Report no. hep-th/9605059.
[25] Blum J. D., *Phys. Lett. B*, **333** (1996) 4709.