

SOLUTIONS TO SELECTED EXERCISES

Section 7.2

Part I: Monadic Arguments

(1)	1.	$\forall xWx \ \& \ \forall yYy$	PR	$\therefore \exists x(Wx \vee Yx)$
	2.	$\forall yYy$	CE 1	
	3.	$\sim Wa$	PA-CP	
	4.	Ya	UI 3	
	5.	$\sim Wa \rightarrow Ya$	CP 3-4	
	6.	$Wa \vee Ya$	Impl 5	
	7.	$\exists x(Wx \vee Yx)$	EG 6	

(2)	1.	$\forall x(Ax \ \& \ Bx)$	PR	$\therefore \exists xAx \ \& \ \exists xBx$
	2.	$Aa \ \& \ Ba$	UI 1	
	3.	Aa	CE 2	
	4.	$\exists xAx$	EG 3	
	5.	Ba	CE 2	
	6.	$\exists xBx$	EG 5	
	7.	$\exists xAx \ \& \ \exists xBx$	CI 4,6	

(4) Using only what's available to us, I can't figure out how to derive the universally quantified statement. But we can fairly easily derive the existentially quantified statement.

1.	$\forall x(Ax \vee Bx)$	PR	
2.	$\forall x(Ax \rightarrow \sim Bx)$	PR	$\therefore \exists x(Ax \leftrightarrow \sim Bx)$
3.	$Aa \rightarrow \sim Ba$	UI 2	
4.	$Aa \vee Ba$	UI 1	
5.	$\sim Aa \rightarrow Ba$	Impl 4	
6.	$\sim Ba$	PA-CP	
7.	$\sim \sim Aa$	MT 5,6	
8.	Aa	DN 7	
9.	$\sim Ba \rightarrow Aa$	CP 6-8	
10.	$(Aa \rightarrow \sim Ba) \ \& \ (\sim Ba \rightarrow Aa)$	CI 3,9	
11.	$Aa \leftrightarrow \sim Ba$	Equiv 10	
12.	$\exists x(Ax \leftrightarrow \sim Bx)$	EG 11	

- (5)
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|----|---|--------|-----------------|
| 1. | $Pr \rightarrow Qr$ | PR | |
| 2. | $\forall x[\exists yRy \rightarrow Px]$ | PR | |
| 3. | Ra | PR | $\therefore Qr$ |
| 4. | $\exists yRy \rightarrow Pr$ | UI 2 | |
| 5. | $\exists yRy$ | EG 3 | |
| 6. | Pr | MP 4,5 | |
| 7. | Qr | MP 1,6 | |
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- (6)
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|-----|--|---------|--------------------------|
| 1. | $\forall x(Sx \rightarrow Tx)$ | PR | |
| 2. | $\exists x\sim Sx \rightarrow (\forall xTx \vee Rr)$ | PR | |
| 3. | $\sim Ta$ | PR | $\therefore \exists xRx$ |
| 4. | $Sa \rightarrow Ta$ | UI 1 | |
| 5. | $\sim Sa$ | MT 3,4 | |
| 6. | $\exists x\sim Sx$ | EG 5 | |
| 7. | $\forall xTx \vee Rr$ | MP 2,6 | |
| 8. | $\sim\forall xTx \rightarrow Rr$ | Impl 7 | |
| 9. | $\exists x\sim Tx \rightarrow Rr$ | Q 8 | |
| 10. | $\exists x\sim Tx$ | EG 3 | |
| 11. | Rr | MP 9,10 | |
| 12. | $\exists xRx$ | EG 11 | |
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- (8)
- | | | | |
|----|---|---------|--|
| 1. | $\exists x\sim Bx \rightarrow \exists x\sim Cx$ | PR | $\therefore \forall xCx \rightarrow \forall xBx$ |
| 2. | $\forall xCx$ | PA-CP | |
| 3. | $\sim\forall xBx$ | PA-RAA | |
| 4. | $\exists x\sim Bx$ | Q 3 | |
| 5. | $\exists x\sim Cx$ | MP 1,4 | |
| 6. | $\sim\exists x\sim Cx$ | Q 2 | |
| 7. | $\exists x\sim Cx \ \& \ \sim\exists x\sim Cx$ | CI 5,6 | |
| 8. | $\forall xBx$ | RAA 3-7 | |
| 9. | $\forall xCx \rightarrow \forall xBx$ | CP 2-8 | |
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- (9)
- | | | | |
|----|---------------------------------------|--------|---------------------------------------|
| 1. | $Aa \vee \sim\forall xBx$ | PR | $\therefore Aa \vee \exists x\sim Bx$ |
| 2. | $\sim Aa$ | PA-CP | |
| 3. | $\sim Aa \rightarrow \sim\forall xBx$ | Impl 1 | |
| 4. | $\sim\forall xBx$ | MP 2,3 | |

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|----|---|--------|
| 5. | $\exists x \sim Bx$ | Q 4 |
| 6. | $\sim Aa \rightarrow \exists x \sim Bx$ | CP 2-5 |
| 7. | $Aa \vee \exists x \sim Bx$ | Impl 6 |

(10) When the premise reads as it does in the book, I can't figure out how to do the proof. However, I'm OK when the premise reads as follows: ' $Aa \vee \sim \exists x Bx$ '. Here's the proof given that premise.

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|-----|---|---------|--|
| 1. | $Aa \vee \sim \exists x Bx$ | PR | $\therefore \exists y \exists x (Bx \rightarrow Ay)$ |
| 2. | $\sim Aa \rightarrow \sim \exists x Bx$ | Impl 1 | |
| 3. | Ba | PA-CP | |
| 4. | $\sim Aa$ | PA-RAA | |
| 5. | $\sim \exists x Bx$ | MP 2,4 | |
| 6. | $\forall x \sim Bx$ | Q 5 | |
| 7. | $\sim Ba$ | UI 6 | |
| 8. | $Ba \ \& \ \sim Ba$ | CI 3,7 | |
| 9. | Aa | RAA 4-8 | |
| 10. | $Ba \rightarrow Aa$ | CP 3-9 | |
| 11. | $\exists x (Bx \rightarrow Aa)$ | EG 10 | |
| 12. | $\exists y \exists x (Bx \rightarrow Ay)$ | EG 11 | |

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|------|----|---|--------|---------------------------|
| (11) | 1. | $\forall x \sim Bx \rightarrow \forall x \sim Ax$ | PR | |
| | 2. | $\exists x Ax$ | PR | $\therefore \exists x Bx$ |
| | 3. | $\sim \forall x \sim Ax$ | Q 2 | |
| | 4. | $\sim \forall x \sim Bx$ | MT 1,3 | |
| | 5. | $\exists x Bx$ | Q 4 | |

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|------|-----|--|--------|-----------------|
| (12) | 1. | $\forall x [(Ax \vee Bx) \rightarrow Cx]$ | PR | |
| | 2. | $\sim (Ca \vee Cb)$ | PR | |
| | 3. | $\forall x [\sim (Aa \vee Bb) \rightarrow Hx]$ | PR | $\therefore Hc$ |
| | 4. | $(Aa \vee Ba) \rightarrow Ca$ | UI 1 | |
| | 5. | $(Ab \vee Bb) \rightarrow Cb$ | UI 1 | |
| | 6. | $\sim (Aa \vee Bb) \rightarrow Hc$ | UI | |
| | 7. | $\sim Ca \ \& \ \sim Cb$ | DeM 2 | |
| | 8. | $\sim Ca$ | CE 6 | |
| | 9. | $\sim Cb$ | CE | |
| | 10. | $\sim (Aa \vee Ba)$ | MT 4,7 | |
| | 11. | $\sim Aa \ \& \ \sim Ba$ | DeM 10 | |
| | 12. | $\sim (Ab \vee Bb)$ | MT 5,9 | |
| | 13. | $\sim Ab \ \& \ \sim Bb$ | DeM 12 | |

- 14. $\sim Aa$ CE 11
- 15. $\sim Bb$ CE 13
- 16. $\sim Aa \ \& \ \sim Bb$ CI 14,15
- 17. $\sim(Aa \vee Bb)$ DeM 16
- 18. Hc MP 6,17

- (14)
- 1. $\forall x(Fx \rightarrow \forall yGy)$ PR
 - 2. $\sim \exists x \sim Fx$ PR
 - 3. $\forall x(Hx \vee \sim Gx)$ PR $\therefore \exists x Hx$
 - 4. $\forall x Fx$ Q 2
 - 5. $Fa \rightarrow \forall y Gy$ UI 1
 - 6. Fa UI 4
 - 7. $\forall y Gy$ MP 5,6
 - 8. Ga UI 7
 - 9. $Ha \vee \sim Ga$ UI 3
 - 10. $\sim Ha \rightarrow \sim Ga$ Impl 9
 - 11. $\sim \sim Ga$ DN 8
 - 12. $\sim \sim Ha$ MT 10,11
 - 13. Ha DN 12
 - 14. $\exists x Hx$ EG 13

- (15)
- 1. $\forall x \sim(Ax \vee Bx)$ PR
 - 2. $\sim(Aa \vee Bb) \rightarrow \forall z(Cz \rightarrow Bz)$ PR $\therefore \sim(Ca \vee Cb)$
 - 3. $\sim(Aa \vee Ba)$ UI 1
 - 4. $\sim Aa \ \& \ \sim Ba$ DeM 3
 - 5. $\sim Aa$ CE 4
 - 6. $\sim(Ab \vee Bb)$ UI 1
 - 7. $\sim Ab \ \& \ \sim Bb$ DeM 6
 - 8. $\sim Bb$ CE 7
 - 9. $\sim Aa \ \& \ \sim Bb$ CI 5,8
 - 10. $\sim(Aa \vee Bb)$ DeM 9
 - 11. $\forall z(Cz \rightarrow Bz)$ MP 2,10
 - 12. $Ca \rightarrow Ba$ UI 11
 - 13. $\sim Ba$ CE 4
 - 14. $\sim Ca$ MT 12,13
 - 15. $Cb \rightarrow Bb$ UI 11
 - 16. $\sim Cb$ MT 8,15
 - 17. $\sim Ca \ \& \ \sim Cb$ CI 14,16
 - 18. $\sim(Ca \vee Cb)$ DeM 17

Part II: Polyadic Arguments

- (16) 1. $\forall x \forall y Rxy$ PR $\therefore \exists y \exists x Rxy$
 2. $\forall y Ray$ UI 1
 3. Rab UI 2
 4. $\exists x Rxb$ EG 3
 5. $\exists y \exists x Rxy$ EG 4
- (17) 1. $\forall x \forall y (Axy \rightarrow \sim Ayx)$ PR $\therefore \exists x \exists y \sim Axy$
 2. $\forall y (Aay \rightarrow \sim Aya)$ UI 1
 3. $Aaa \rightarrow \sim Aaa$ UI 2
 4. Aaa PA-RAA
 5. $\sim Aaa$ MP 3,4
 6. $Aaa \& \sim Aaa$ CI 4,5
 7. $\sim Aaa$ RAA 4-6
 8. $\exists y \sim Aay$ EG 7
 9. $\exists x \exists y \sim Axy$ EG 8
- (18) 1. Ga PR
 2. $\forall y (\exists x Gx \rightarrow Hby)$ PR $\therefore \exists x Hxb$
 3. $\sim \exists x Hxb$ PA-RAA
 4. $\forall x \sim Hxb$ Q 3
 5. $\exists x Gx \rightarrow Hbb$ UI 2
 6. $\exists x Gx$ EG 1
 7. Hbb MP 5,6
 8. $\sim Hbb$ UI 4
 9. $Hbb \& \sim Hbb$ CI 7,8
 10. $\exists x Hxb$ RAA 3-9
- (19) 1. $\forall x \forall y \forall z [(Kxy \& Kyz) \rightarrow Kxz]$ PR
 2. $Kab \& Kbc$ PR $\therefore \exists x (Kax \& Kbx)$
 3. Kbc CE 2
 4. $\forall y \forall z [(Kay \& Kyz) \rightarrow Kaz]$ UI 1
 5. $\forall z [(Kab \& Kbz) \rightarrow Kaz]$ UI 4
 6. $(Kab \& Kbc) \rightarrow Kac$ UI 5
 7. Kac MP 2,6
 8. $Kac \& Kbc$ CI 3,7
 9. $\exists x (Kax \& Kbx)$ EG 8
- (20) 1. $\sim \exists x \exists y (Gxy \& Lxy)$ PR

2.	Gab	PR	$\therefore \sim Lab$
3.	$\forall x \sim \exists y (Gxy \ \& \ Lxy)$	Q 1	
4.	$\sim \exists y (Gay \ \& \ Lay)$	UI 3	
5.	$\forall y \sim (Gay \ \& \ Lay)$	Q 4	
6.	$\sim (Gab \ \& \ Lab)$	UI 5	
7.	$\sim Gab \vee \sim Lab$	DeM 6	
8.	$Gab \rightarrow \sim Lab$	Impl 7	

9.	Lab	PA-RAA
10.	$\sim \sim Lab$	DN 9
11.	$\sim Gab$	MT 8,10
12.	$Gab \ \& \ \sim Gab$	CI 2,11

13. $\sim Lab$ RAA 9-12

(22)	1.	$\forall x \forall y [Kxy \rightarrow \forall z (Lz \leftrightarrow \exists u Mu)]$	PR	
	2.	$Kab \ \& \ Ma$	PR	$\therefore \exists x Lx$
	3.	$\forall y [Kay \rightarrow \forall z (Lz \leftrightarrow \exists u Mu)]$	UI 1	
	4.	$Kab \rightarrow \forall z (Lz \leftrightarrow \exists u Mu)$	UI 3	
	5.	Kab	CE 2	
	6.	$\forall z (Lz \leftrightarrow \exists u Mu)$	MP 4,5	
	7.	$Lb \leftrightarrow \exists u Mu$	UI 6	
	8.	Ma	CE 2	
	9.	$\exists u Mu$	EG 8	
	10.	$(Lb \rightarrow \exists u Mu) \ \& \ (\exists u Mu \rightarrow Lb)$	Equiv 7	
	11.	$\exists u Mu \rightarrow Lb$	CE 10	
	12.	Lb	MP 9,11	
	13.	$\exists x Lx$	EG 12	

(23)	1.	$\forall x (Axa \vee Bbx)$	PR	$\therefore \exists x \exists y Axy \vee \exists u \exists z Buz$
	2.	$Aca \vee Bbc$	UI 1	
	3.	$\sim Aca \rightarrow Bbc$	Impl 2	
	4.	$\forall x \sim \exists y Axy \ \& \ \forall u \sim \exists z Buz$	PA-RAA	
	5.	$\forall x \sim \exists y Axy$	CE 4	
	6.	$\forall u \sim \exists z Buz$	CE 4	
	7.	$\sim \exists y Axy$	UI 5	
	8.	$\sim \exists z Buz$	UI 6	
	9.	$\forall y \sim Axy$	Q 7	
	10.	$\forall z \sim Bbz$	Q 8	
	11.	$\sim Aca$	UI 9	
	12.	$\sim Bbc$	UI 10	
	13.	$\sim \sim Aca$	MT 3,12	
	14.	Aca	DN 13	
	15.	$Aca \ \& \ \sim Aca$	CI 11,14	

16. $\sim(\forall x\sim\exists yAxy \ \& \ \forall u\sim\exists zBuz)$ RAA 4-15
 17. $\sim\forall x\sim\exists yAxy \vee \sim\forall u\sim\exists zBuz$ DeM 16
 18. $\exists x\exists yAxy \vee \sim\forall u\sim\exists zBuz$ Q 17
 19. $\exists x\exists yAxy \vee \exists u\exists zBuz$ Q 18

(24) 1. $\sim\exists x\exists y(Axy \vee \sim Axy) \vee \forall x(\exists yBxy \vee (Cx \ \& \ \sim Cx))$ PR $\therefore \exists yBay$

2. $\sim\exists yBay$ PA-RAA

3. $\sim\sim\exists x\exists y(Axy \vee \sim Axy) \rightarrow \forall x(\exists yBxy \vee (Cx \ \& \ \sim Cx))$ Impl 1

4. $\exists x\exists y(Axy \vee \sim Axy) \rightarrow \forall x(\exists yBxy \vee (Cx \ \& \ \sim Cx))$ DN 3

5. $\sim\exists x\exists y(Axy \vee \sim Axy)$ PA-RAA

6. $\forall x\sim\exists y(Axy \vee \sim Axy)$ Q 5

7. $\sim\exists y(Aay \vee \sim Aay)$ UI 6

8. $\forall y\sim(Aay \vee \sim Aay)$ Q 7

9. $\sim(Aab \vee \sim Aab)$ UI 8

10. $\sim Aab \ \& \ \sim\sim Aab$ DeM 9

11. $\exists x\exists y(Axy \vee \sim Axy)$ RAA 5-10

12. $\forall x(\exists yBxy \vee (Cx \ \& \ \sim Cx))$ MP 4,11

13. $\exists yBay \vee (Ca \ \& \ \sim Ca)$ UI 12

14. $\sim\exists yBay \rightarrow (Ca \ \& \ \sim Ca)$ Impl 13

15. $Ca \ \& \ \sim Ca$ MP 2,14

16. $\sim\sim\exists yBay$ RAA 2-15

17. $\exists yBay$ DN 16

(25) 1. $\forall x\forall y\forall z(Axy \rightarrow Bxz)$ PR
 2. $\forall x\forall y\forall z(Bxy \rightarrow Cxz)$ PR $\therefore Aab \rightarrow \exists x\exists yCxy$

3. $\forall y\forall z(Aay \rightarrow Baz)$ UI 1

4. $\forall z(Aab \rightarrow Baz)$ UI 3

5. $Aab \rightarrow Bac$ UI 4

6. Aab PA-CP

7. Bac MP 5,6

8. $\forall y\forall z(Bay \rightarrow Caz)$ UI 2

9. $\forall z(Bac \rightarrow Caz)$ UI 8

10. $Bac \rightarrow Cab$ UI 9

11. Cab MP 7,10

12. $\exists yCay$ EG 11

13. $\exists x\exists yCxy$ EG 12

14. $Aab \rightarrow \exists x\exists yCxy$ CP 6-13

Section 7.3

Part I: Monadic Arguments

(1)	1.	$\forall x(Ax \rightarrow Bx)$	PR	
	2.	$\forall xAx$	PR	$\therefore \forall xBx$
	3.	$Aa \rightarrow Ba$	UI 1	
	4.	Aa	UI 2	
	5.	Ba	MP 3,4	
	6.	$\forall xBx$	UG 5	
(2)	1.	$\forall x(Bx \rightarrow Ax)$	PR	
	2.	$\exists x(Bx \ \& \ \sim Cx)$	PR	$\therefore \exists x(Ax \ \& \ \sim Cx)$
	3.	$Ba \ \& \ \sim Ca$	PA-EI 2	
	4.	$Ba \rightarrow Aa$	UI 1	
	5.	Ba	CE 3	
	6.	Aa	MP 4,5	
	7.	$\sim Ca$	CE 3	
	8.	$Aa \ \& \ \sim Ca$	CI 6,7	
	9.	$\exists x(Ax \ \& \ \sim Cx)$	EG 8	
	10.	$\exists x(Ax \ \& \ \sim Cx)$	EI 3-9	
(4)	1.	$\exists x(Ax \ \& \ Bx)$	PR	
	2.	$\exists x(Bx \ \& \ Cx)$	PR	$\therefore \forall xAx \rightarrow \exists x(Ax \ \& \ Cx)$
Cx)	3.	$\forall xAx$	PA-CP	
	4.	$Ba \ \& \ Ca$	PA-EI 2	
	5.	Ca	CE 4	
	6.	Aa	UI 3	
	7.	$Aa \ \& \ Ca$	CI 5,6	
	8.	$\exists x(Ax \ \& \ Cx)$	EG 7	
	9.	$\exists x(Ax \ \& \ Cx)$	EI 4-8	
	10.	$\forall xAx \rightarrow \exists x(Ax \ \& \ Cx)$	CP 3-9	

(5)	1.	$\forall x(Ax \vee Bx)$	PR	
	2.	$\forall x(Ax \rightarrow Cx)$	PR	
	3.	$\sim \exists xCx$	PR	$\therefore \forall xBx$
	4.	$\forall x \sim Cx$	Q 3	
	5.	$\sim \forall xBx$	PA-RAA	
	6.	$\exists x \sim Bx$	Q 5	
	7.	$\sim Ba$	PA-EI 6	
	8.	$\sim Ca$	UI 4	
	9.	$Aa \rightarrow Ca$	UI 2	
	10.	$\sim Aa$	MT 8,9	
	11.	$Aa \vee Ba$	UI 1	
	12.	$\sim Aa \rightarrow Ba$	Impl 11	
	13.	Ba	MP 10,12	
	14.	$P \ \& \ \sim P$	IR 7,13	
	15.	$P \ \& \ \sim P$	EI 7-14	
	16.	$\forall xBx$	RAA 5-15	
(6)	1.	$\forall x(S \rightarrow Bx)$	PR	$\therefore S \rightarrow \forall xBx$
	2.	S	PA-CP	
	3.	$S \rightarrow Ba$	UI 1	
	4.	Ba	MP 2,3	
	5.	$\forall xBx$	UG 4	
	6.	$S \rightarrow \forall xBx$	CP 2-5	
(7)	1.	$\exists x(Ax \rightarrow S)$	PR	$\therefore \forall xAx \rightarrow S$
	2.	$\forall xAx$	PA-CP	
	3.	$Aa \rightarrow S$	PA-EI 1	
	4.	Aa	UI 2	
	5.	S	MP 3,4	
	6.	S	EI 3-5	
	7.	$\forall xAx \rightarrow S$	CP 2-6	
(8)	1.	$\forall x(Ax \rightarrow S)$	PR	$\therefore \exists xAx \rightarrow S$
	2.	$\exists xAx$	PA-CP	

	3.	Aa	PA-EI 2	
	4.	$Aa \rightarrow S$	UI 1	
	5.	S	MP 3,4	
	6.	S	EI 3-5	
	7.	$\exists xAx \rightarrow S$	CP 2-6	
(9)	1.	$\exists x(S \rightarrow Ax)$	PR	$\therefore S \rightarrow \exists xAx$
	2.	S	PA-CP	
	3.	$S \rightarrow Aa$	PA-EI 1	
	4.	Aa	MP 2,3	
	5.	$\exists xAx$	EG 4	
	6.	$\exists xAx$	EI 3-5	
	7.	$S \rightarrow \exists xAx$	CP 2-6	
(10)	1.	$\forall xAx \ \& \ \forall xBx$	PR	$\therefore \forall x(Ax \ \& \ Bx)$
	2.	$\forall xAx$	CE 1	
	3.	$\forall xBx$	CE 1	
	4.	Aa	UI 2	
	5.	Ba	UI 3	
	6.	$Aa \ \& \ Ba$	CI 4,5	
	7.	$\forall x(Ax \ \& \ Bx)$	UG 6	
(11)	1.	$\exists xFx \vee \exists xGx$	PR	$\therefore \exists x(Fx \vee Gx)$
	2.	$\sim \exists x(Fx \vee Gx)$	PA-RAA	
	3.	$\forall x \sim (Fx \vee Gx)$	Q 2	
	4.	$\sim (Fa \vee Ga)$	UI 3	
	5.	$\sim Fa \ \& \ \sim Ga$	DeM 4	
	6.	$\sim Fa$	CE 5	
	7.	$\sim Ga$	CE 5	
	8.	$\sim \exists xFx \rightarrow \exists xGx$	Impl 1	
	9.	$\forall x \sim Fx \rightarrow \exists xGx$	Q 8	
	10.	$\forall x \sim Fx$	UG 6	
	11.	$\exists xGx$	MP 9,10	
	12.	$\forall x \sim Gx$	UG 7	
	13.	$\sim \exists xGx$	Q 12	
	14.	$\exists xGx \ \& \ \sim \exists xGx$	CI 11,13	

	15.	$\exists x(Fx \vee Gx)$	RAA 2-14	
(12)	1.	$\forall x(Ax \rightarrow Bx)$	PR	
	2.	$\forall x(Bx \rightarrow Cx)$	PR	$\therefore \forall x(Ax \rightarrow Cx)$
	3.	Aa	PA-CP	
	4.	$Aa \rightarrow Ba$	UI 1	
	5.	Ba	MP 3,4	
	6.	$Ba \rightarrow Ca$	UI 2	
	7.	Ca	MP 5,6	
	8.	$Aa \rightarrow Ca$	CP 3-7	
	9.	$\forall x(Ax \rightarrow Cx)$	UG 8	
(14)	1.	$\forall x\exists y(Fx \& Gy)$	PR	$\therefore \exists y\forall x(Fx \& Gy)$
	2.	$\sim\exists y\forall x(Fx \& Gy)$	PA-RAA	
	3.	$\forall y\sim\forall x(Fx \& Gy)$	Q 2	
	4.	$\sim\forall x(Fx \& Ga)$	UI 3	
	5.	$\exists x\sim(Fx \& Ga)$	Q 4	
	6.	$\sim(Fb \& Ga)$	PA-EI 5	
	7.	$\exists y(Fb \& Gy)$	UI 1	
	8.	Fb & Gc	PA-EI 7	
	9.	$\sim Fb \vee \sim Ga$	DeM 6	
	10.	$Fb \rightarrow \sim Ga$	Impl 9	
	11.	Fb	CE 8	
	12.	$\sim Ga$	MP 10,11	
	13.	$\forall x\sim Gx$	UG 12	
	14.	$\sim\exists xGx$	Q 13	
	15.	Gc	CE 8	
	16.	$\exists xGx$	EG 15	
	17.	P & $\sim P$	IR 14,16	
	18.	P & $\sim P$	EI 8-17	
	19.	P & $\sim P$	EI 6-18	
	20.	$\exists y\forall x(Fx \& Gy)$	RAA 2-19	
(15)	1.	$\forall x\forall y(Fx \rightarrow Gy)$	PR	$\therefore \exists xFx \rightarrow \exists yGy$
	2.	$\exists xFx$	PA-CP	

	3.	Fa	PA-EI 2	
	4.	$\forall y(Fa \rightarrow Gy)$	UI 1	
	5.	$Fa \rightarrow Gb$	UI 4	
	6.	Gb	MP 3,5	
	7.	$\exists yGy$	EG 6	
	8.	$\exists yGy$	EI 3-7	
	9.	$\exists xFx \rightarrow \exists yGy$	CP 2-8	
(16)	1.	$\forall x\forall y(Fx \rightarrow Gy)$	PR	$\therefore \forall y(\exists xFx \rightarrow Gy)$
	2.	$\exists xFx$	PA-CP	
	3.	Fa	PA-EI 2	
	4.	$\forall y(Fa \rightarrow Gy)$	UI 1	
	5.	$Fa \rightarrow Gb$	UI 4	
	6.	Gb	MP 3,5	
	7.	Gb	EI 3-6	
	8.	$\exists xFx \rightarrow Gb$	CP 2-7	
	9.	$\forall y(\exists xFx \rightarrow Gy)$	UG 8	
(17)	1.	$\forall x[(Ax \ \& \ \forall yGy) \rightarrow \forall zRz]$	PR	$\therefore \exists y[(\exists xAx \ \& \ Gy) \rightarrow \forall zRz]$
	2.	$\sim\exists y[(\exists xAx \ \& \ Gy) \rightarrow \forall zRz]$	PA-RAA	
	3.	$\forall y\sim[(\exists xAx \ \& \ Gy) \rightarrow \forall zRz]$	Q 2	
	4.	$\sim[(\exists xAx \ \& \ Ga) \rightarrow \forall zRz]$	UI 3	
	5.	$\sim[\sim(\exists xAx \ \& \ Ga) \vee \forall zRz]$	Impl 4	
	6.	$(\exists xAx \ \& \ Ga) \ \& \ \sim\forall zRz$	DeM 5	
	7.	$\exists xAx \ \& \ Ga$	CE 6	
	8.	$\sim\forall zRz$	CE 6	
	9.	$\exists xAx$	CE 7	
	10.	Ga	CE 7	
	11.	$\forall yGy$	UG 10	
	12.	Ab	PA-EI 9	
	13.	$Ab \ \& \ \forall yGy$	CI 11,12	
	14.	$(Ab \ \& \ \forall yGy) \rightarrow \forall zRz$	UI 1	
	15.	$\forall zRz$	MP 13,14	
	16.	$\forall zRz$	EI 12-15	

17. $\forall zRz \ \& \ \sim\forall zRz$ CI 8,16
18. $\exists y[(\exists xAx \ \& \ Gy) \rightarrow \forall zRz]$ RAA 2-17

Part II: Polyadic Arguments

- (18) 1. $\forall x[Px \leftrightarrow \exists y(Fxy \vee Mxy)]$ PR
2. $\exists xFax$ PR $\therefore Pa$
3. $\sim Pa$ PA-RAA
3. Fab PA-EI 2
4. $Pa \leftrightarrow \exists y(Fay \vee May)$ UI 1
5. $[Pa \rightarrow \exists y(Fay \vee May)] \ \& \ [\exists y(Fay \vee May) \rightarrow Pa]$ Equiv 4
6. $\exists y(Fay \vee May) \rightarrow Pa$ CE 5
7. $\sim\exists y(Fay \vee May)$ MT 3,6
8. $\forall y\sim(Fay \vee May)$ Q 7
9. $\sim(Fab \vee Mab)$ UI 8
10. $\sim Fab \ \& \ \sim Mab$ DeM 9
11. $\sim Fab$ CE 10
12. $P \ \& \ \sim P$ IR 3,11
13. $P \ \& \ \sim P$ EI 3-12
14. Pa RAA 3-13
- (19) 1. $\forall x\forall y(Axy \rightarrow Axy) \rightarrow \forall x\forall yBxy$ PR $\therefore \exists x\exists yBxy$
2. $\sim\forall x\forall y(Axy \rightarrow Axy)$ PA-RAA
3. $\exists x\sim\forall y(Axy \rightarrow Axy)$ Q 2
4. $\sim\forall y(Aay \rightarrow Aay)$ PA-EI 3
5. $\exists y\sim(Aay \rightarrow Aay)$ Q 4
6. $\sim(Aab \rightarrow Aab)$ PA-EI 5
7. $\sim(\sim Aab \vee Aab)$ Impl 6
8. $\sim\sim Aab \ \& \ \sim Aab$ DeM 7
9. $P \ \& \ \sim P$ IR 8
10. $P \ \& \ \sim P$ EI 6-9
11. $P \ \& \ \sim P$ EI 4-10

	12.	$\forall x\forall y(Axy \rightarrow Axy)$		RAA 2-11
	13.	$\forall x\forall yBxy$		MP 1,12
	14.	$\forall yBay$		UI 13
	15.	Bab		UI 14
	16.	$\exists yBay$		EG 15
	17.	$\exists x\exists yBxy$		EG 16
(20)	1.	$\forall x\forall y(Axy \rightarrow Bxy)$		PR
	2.	$\exists x\exists y\sim Bxy$		PR $\therefore \exists x\exists y\sim Axy$
	3.	$\exists y\sim Bay$		PA-EI 2
	4.	$\sim Bab$		PA-EI 3
	5.	$\forall y(Aay \rightarrow Bay)$		UI 1
	6.	$Aab \rightarrow Bab$		UI 5
	7.	$\sim Aab$		MT 4,6
	8.	$\exists y\sim Aay$		EG 7
	9.	$\exists x\exists y\sim Axy$		EG 8
	10.	$\exists x\exists y\sim Axy$		EI 4-9
	11.	$\exists x\exists y\sim Axy$		EI 3-10
(21)	1.	$\forall x\forall y(Axy \rightarrow Bxy)$		PR
	2.	$\forall x\forall y\sim Bxy$		PR $\therefore \sim\exists x\exists yBxy$
	3.	$\exists x\exists yBxy$		PA-RAA
	4.	$\exists yBay$		PA-EI 3
	5.	Bab		PA-EI 4
	6.	$\forall y\sim Bay$		UI 2
	7.	$\sim Bab$		UI 6
	8.	$P \ \& \ \sim P$		IR 5,7
	9.	$P \ \& \ \sim P$		EI 5-8
	10.	$P \ \& \ \sim P$		EI 4-9
	11.	$\sim\exists x\exists yBxy$		RAA 3-10
(22)	1.	Ba		PR

2.	$\sim\forall x(Fxa \rightarrow \exists yBy) \vee \forall xFbx$	PR	$\therefore Ba \ \& \ Fba$
3.	$\exists x\sim(Fxa \rightarrow \exists yby) \vee \forall xFbx$	Q 2	
4.	$\sim\exists x\sim(Fxa \rightarrow \exists yby) \rightarrow \forall xFbx$	Impl 3	
5.	$\forall x(Fxa \rightarrow \exists yby) \rightarrow \forall xFbx$	Q 4	
6.	Fca	PA-CP	
7.	$\exists yBy$	EG 1	
8.	$Fca \rightarrow \exists yBy$	CP 6-7	
9.	$\forall x(Fxa \rightarrow \exists yBy)$	UG 8	
10.	$\forall xFbx$	MP 5,9	
11.	Fba	UI 10	
12.	Ba & Fba	CI 1,11	

(23)	1.	$\exists x\exists y\exists z(\sim Pxy \vee Qyz)$	PR	$\therefore \exists y(\exists x\sim Pxy \vee \exists zQyz)$
	2.	$\exists y\exists z(\sim Pay \vee Qyz)$	PA-EI 1	
	3.	$\exists z(\sim Pab \vee Qbz)$	PA-EI 2	
	4.	$\sim Pab \vee Qbc$	PA-EI 3	
	5.	$\forall xPxd$	PA-CP	
	6.	Pad	UI 5	
	7.	$Pad \rightarrow Qdc$	Impl 4	
	8.	Qdc	MP 5,7	
	9.	$\exists zQdz$	EG 8	
	10.	$\forall xPxd \rightarrow \exists zQdz$	CP 5-9	
	11.	$\forall xPxd \rightarrow \exists zQdz$	EI 4-10	
	12.	$\forall xPxd \rightarrow \exists zQdz$	EI 3-11	
	13.	$\forall xPxd \rightarrow \exists zQdz$	EI 2-12	
	14.	$\sim\exists x\sim Pxd \rightarrow \exists zQdz$	Q13	
	15.	$\exists x\sim Pxd \vee \exists zQdz$	Impl 14	
	16.	$\exists y(\exists x\sim Pxy \vee \exists zQyz)$	EG 15	

(24)	1.	$\exists x\exists y(Axy \ \& \ \sim Ayx)$	PR	
	2.	$\forall x\forall y[(Axy \ \& \ \sim Ayx) \rightarrow \exists zBxyz]$	PR	
	3.	$\forall x\forall y\forall z(Cxy \leftrightarrow \sim Bxyz)$	PR	$\therefore \sim\forall x\forall yCxy$

4.	$\forall x\forall yCxy$	PA-RAA
5.	$\exists y(Aay \& \sim Aya)$	PA-EI 1
6.	$Aab \& \sim Aba$	PA-EI 5
7.	$\forall y[(Aay \& \sim Aya) \rightarrow \exists zBayz]$	UI 2
8.	$(Aab \& \sim Aba) \rightarrow \exists zBabz$	UI 7
9.	$\exists zBabz$	MP 6,8
10.	$Babc$	PA-EI 9
11.	$\forall y\forall z(Cay \leftrightarrow \sim Bayz)$	UI 3
12.	$\forall z(Cab \leftrightarrow \sim Babz)$	UI 11
13.	$Cab \leftrightarrow \sim Babc$	UI 12
14.	$[Cab \rightarrow \sim Babc] \& [\sim Babc \rightarrow Cab]$	Equiv 13
15.	$Cab \rightarrow \sim Babc$	CE 14
16.	$\sim Cab$	MT 10,15
17.	$\exists y\sim Cay$	EG 16
18.	$\sim \forall yCay$	Q 17
19.	$\exists x\sim \forall yCxy$	EG 18
20.	$\sim \forall x\forall yCxy$	Q 19
21.	$\forall x\forall yCxy \& \sim \forall x\forall yCxy$	CI 4,20
22.	$\forall x\forall yCxy \& \sim \forall x\forall yCxy$	EI 10-21
23.	$\forall x\forall yCxy \& \sim \forall x\forall yCxy$	EI 6-22
24.	$\forall x\forall yCxy \& \sim \forall x\forall yCxy$	EI 5-23
25.	$\sim \forall x\forall yCxy$	RAA 4-24
(26)	1. $\forall x\forall y\forall z[(Ax \& (Pyx \& Czx)) \rightarrow (Vx \leftrightarrow \sim \exists u(Iu \& (Tyu \& \sim Tzu)))]$	PR
	2. $Aa \& \exists u[Iu \& (Tbu \& \sim Tcu)]$	PR
	$\therefore (Pba \& Cca) \rightarrow \sim Va$	
	3. $Pba \& Cca$	
	4. $\forall y\forall z[(Aa \& (Pya \& Cza)) \rightarrow (Va \leftrightarrow \sim \exists u(Iu \& (Tyu \& \sim Tzu)))]$	UI 1
	5. $\forall z[(Aa \& (Pba \& Cza)) \rightarrow (Va \leftrightarrow \sim \exists u(Iu \& (Tbu \& \sim Tzu)))]$	UI 4
	6. $(Aa \& (Pba \& Cca)) \rightarrow (Va \leftrightarrow \sim \exists u(Iu \& (Tbu \& \sim Tcu)))$	UI 5
	7. Aa	CE 2
	8. $Aa \& (Pba \& Cca)$	CI 3,7
	9. $Va \leftrightarrow \sim \exists u(Iu \& (Tbu \& \sim Tcu))$	MP 6,8
	10. $[Va \rightarrow \sim \exists u(Iu \& (Tbu \& \sim Tcu))] \& [\sim \exists u(Iu \& (Tbu \& \sim Tcu)) \rightarrow Va]$	Equiv 9
	11. $Va \rightarrow \sim \exists u(Iu \& (Tbu \& \sim Tcu))$	CE 10

	12.	$\exists u[Iu \ \& \ (Tbu \ \& \ \sim Tcu)]$	CE 2
	13.	$\sim Va$	MT
11,12			
	14.	$(Pba \ \& \ Cca) \rightarrow \sim Va$	CP 3-13
(27)	1.	$\forall x(Lcx \rightarrow \sim Lxx)$	PR
	2.	$\forall x[Lxx \rightarrow (Tx \vee Vx)]$	PR
	3.	$\forall x(Vx \rightarrow \sim Lcx)$	PR
		$\therefore \forall x[(Vx \vee Lxx) \rightarrow (\sim Tx \rightarrow \sim Lcx)]$	
	4.	$\sim \forall x[(Vx \vee Lxx) \rightarrow (\sim Tx \rightarrow \sim Lcx)]$	PA-RAA
	5.	$\exists x \sim [(Vx \vee Lxx) \rightarrow (\sim Tx \rightarrow \sim Lcx)]$	Q 4
	6.	$\sim [(Va \vee Laa) \rightarrow (\sim Ta \rightarrow \sim Lca)]$	PA-EI 5
	7.	$\sim [\sim (Va \vee Laa) \vee (\sim Ta \rightarrow \sim Lca)]$	Impl 6
	8.	$(Va \vee Laa) \ \& \ \sim (\sim Ta \rightarrow \sim Lca)$	DeM
	9.	$Va \vee Laa$	CE 8
	10.	$\sim (\sim Ta \rightarrow \sim Lca)$	CE 8
	11.	$\sim (Ta \vee \sim Lca)$	Impl 10
	12.	$\sim Ta \ \& \ Lca$	DeM 11
	13.	Lca	CE 12
	14.	$Lca \rightarrow \sim Laa$	UI 1
	15.	$\sim Laa$	MP 13,14
	16.	$Va \rightarrow \sim Lca$	UI 3
	17.	$\sim Va$	MT 13,16
	18.	$\sim Va \rightarrow Laa$	Impl 9
	19.	Laa	MP 17,18
	20.	$P \ \& \ \sim P$	IR 15,19
	21.	$P \ \& \ \sim P$	EI 6-20
	22.	$\forall x[(Vx \vee Lxx) \rightarrow (\sim Tx \rightarrow \sim Lcx)]$	RAA 4-21