

EXERCISES FOR SECTION 3.8

PART I: TAUTOLOGIES

1. This statement is *not* a tautology.

$$\frac{A \quad \vee \quad B}{F \quad F \quad F}$$

2. This statement is *not* a tautology.

$$\frac{A \quad \vee \quad \sim B}{F \quad F \quad F \quad T}$$

3. This statement *is* a tautology.

$$\frac{\sim (A \ \& \ \sim A)}{F \quad T \quad T \quad F \quad T} \times$$

4. This statement is *not* a tautology.

$$\frac{\sim (A \ \vee \ \sim A)}{F \quad T \quad T \quad FT \\ F \quad F \quad T \quad TF}$$

6. This statement *is* a tautology.

$$\frac{\sim (\sim (H \ \& \ \sim G) \ \& \ G) \ \vee \ G}{T \quad F \quad T \quad T \quad TF \quad F \quad F \quad F \quad F \quad \times \\ T \quad T \quad F \quad F \quad TF \quad F \quad F \quad F \quad F \quad \times}$$

7. This statement *is* a tautology.

$$\frac{(P \ \& \ Q) \ \rightarrow \ Q}{T \quad F \quad F \quad F} \times$$

8. This statement *is* a tautology.

$$\frac{(X \ \rightarrow \ Y) \ \vee \ (Y \ \rightarrow \ X)}{F \quad F \quad T \quad F \quad T \quad F \quad F} \times$$

9. This statement *is* a tautology.

$$\frac{\sim (Q \ \rightarrow \ R) \ \rightarrow \ (\sim Q \ \rightarrow \ R)}{T \quad F \quad T \quad F \quad F \quad T \quad F \quad F \quad F} \times$$

10. This statement *is* a tautology.

$((A \ \& \ B) \ \vee \ ((\sim A \ \& \ B) \ \vee \ (A \ \& \ \sim B))) \ \vee \ (\sim A \ \& \ \sim B)$															
F	F	T	F	TF	F	T	F	F	F	FT	F	TF	F	FT	×
T	F	F	F	FT	F	F	T	T	T	TF	F	FT	F	TF	×
T	T	T	T	T	T	T	T	T	T	T	F	FT	F	FT	×

11. This statement *is* a tautology.

$(P \ \rightarrow \ Q) \ \vee \ (P \ \rightarrow \ \sim \ Q)$								
T	F	T	F	T	F	F	T	×

12. This statement *is not* a tautology.

$\sim \ (H \ \rightarrow \ J) \ \rightarrow \ \sim \ (J \ \rightarrow \ H)$								
T	T	F	F	F	F	F	T	T

14. This statement *is* a tautology.

$[(G \ \& \ \sim \ H) \ \& \ (H \ \& \ \sim \ G)] \ \vee \ (P \ \rightarrow \ P)$									
				F					×

15. This statement *is* a tautology.

$[\sim \ W \ \& \ (M \ \rightarrow \ W)] \ \rightarrow \ \sim \ M$									
T	F	T	T	F	F	F	F	T	×

16. This statement *is not* a tautology.

$\sim \ (Q \ \& \ E) \ \rightarrow \ \sim \ (Q \ \vee \ E)$								
T	F	F	T	F	F	F	T	T

17. This statement *is* a tautology.

$(H \ \rightarrow \ J) \ \rightarrow \ ((H \ \rightarrow \ \sim \ J) \ \rightarrow \ \sim \ H)$											
T	T	F	F	T	T	T	F	F	F	T	×

18. This statement *is* a tautology.

$[J \ \rightarrow \ (H \ \& \ K)] \ \rightarrow \ [(J \ \rightarrow \ H) \ \& \ (J \ \rightarrow \ K)]$												
T	T			F	F	T	T	F	T	F	F	×
T	T	F	F		F	T	F	F	F	T	T	×
F	T				F	F	T	T	F	T		×

19. This statement is *not* a tautology.

\sim	A	&	$(\sim$	B	&	\sim	C)	\vee	(A	&	B)
F	T	F	T	F	F	F	T	F	T	F	F

20. This statement *is* a tautology.

[(X	&	Y)	\rightarrow	Z]	\rightarrow	[(X	\rightarrow	Z)	\vee	(Y	\rightarrow	Z)]
T	T	T	T	F	F	T	F	F	F	T	F	F

×

PART II: VALIDITY

21. This argument is *valid*.

A	&	B	A
F	T		F

×

22. This argument is *invalid*.

A	\vee	B	\sim A	\vee	\sim B
T	T	T	F	T	F

24. This argument is *invalid*.

\sim	(P	&	Q)	\sim P	&	\sim Q
T	T	F	F	F	T	F
T	F	F	T	T	F	F

25. This argument is *valid*.

\sim	H	\sim	J	\sim	(H	&	J)
T	T	T	T	F	T	T	T

×

26. This argument is *valid*.

Z	\vee	W	W	\rightarrow	Z	Z
F	T	T	T	T	F	F

×

27. This argument is *invalid*.

C	\vee	\sim	D	D	\rightarrow	C
F	T	F	T	T	F	F

28. This argument is *invalid*.

$$\frac{E \vee F \quad \sim \sim E \quad F}{T \quad T \quad F \quad T \quad F \quad T \quad F}$$

29. This argument is *valid*.

$$\frac{\sim G \rightarrow H \quad \sim H \quad G}{T \quad F \quad T \quad F \quad T \quad F \quad F} \times$$

30. This argument is *valid*.

$$\frac{\sim (A \rightarrow B) \quad \sim B}{T \quad T \quad F \quad T \quad F \quad T \quad \times}$$

$$T \quad F \quad F \quad T \quad F \quad T \quad \times$$

32. This argument is *invalid*.

$$\frac{F \rightarrow G \quad \sim F \rightarrow \sim G \quad \sim (F \& G)}{T \quad T \quad T \quad F \quad T \quad F \quad T \quad F \quad T \quad T \quad T}$$

33. This argument is *valid*.

$$\frac{(J \& K) \quad \sim K \vee J \quad \sim K \vee J}{F \quad T \quad T \quad F \quad T \quad T \quad F \quad F \quad F \quad F} \times$$

34. This argument is *valid*.

$$\frac{H \rightarrow J \quad \sim J \rightarrow (H \rightarrow J)}{T \quad T \quad F \quad T \quad F \quad F \quad T \quad F \quad F} \times$$

35. This argument is *invalid*.

$$\frac{\sim (K \& \sim L) \quad \sim K \quad L}{T \quad F \quad F \quad T \quad F \quad T \quad F \quad F}$$

36. This argument is *valid*.

$$\frac{\sim (\sim \sim K \& \sim \sim L) \quad \sim \sim K \quad \sim L}{T \quad T \quad F \quad T \quad T \quad T \quad F \quad T \quad T \quad F \quad T \quad F \quad T} \times$$

37. This argument is *valid*.

$J \rightarrow K$	$\sim (K \vee L)$	$\sim J$	\times
T T T	T T T	F T	

38. This argument is *invalid*.

$M \rightarrow N$	$N \rightarrow O$	$\sim M$	$\sim O$
F T T	T T T	T F	F T
F T F	F T T	T F	F T

39. This argument is *invalid*.

$\sim P \vee Q$	$\sim Q \vee R$	$\sim R$	P
T F T F	T F T F	T F	F

40. This argument is *valid*.

$\sim A$	$(C \vee D) \rightarrow \sim A$	$A \rightarrow \sim D$	\times
T F	T	T F F T	

42. This argument is *invalid*.

$H \rightarrow J$	$J \rightarrow \sim (L \& M)$	$\sim M$	$H \& J$
F T T	T T T T F F	T F	F F T
F T T	T T T F F F	T F	F F T
F T F	F T T T F F	T F	F F F
F T F	F T T F F F	T F	F F F

43. This argument is *valid*.

$H \& (G \vee F)$	$\sim (G \rightarrow F)$	$\sim H \rightarrow G$	\times
F T F	T F	T F F F	

44. This argument is *valid*.

$\sim [K \vee (K \rightarrow M)]$	$P \& \sim P$	\times
T F F T F	T F F T	
T F F T F	F F T F	\times
	F F F T	\times

45. This argument is *invalid*.

$\sim (S \& \sim L)$	$(L \& S) \vee (\sim L \& \sim S)$
T F F F T	T F F F F T F T F

46. This argument is *valid*.

R	\vee	$(S$	\rightarrow	$T)$		\sim	T	$\&$	\sim	R		\sim	$(S$	$\&$	$U)$		\times
F	T	T	F	F		T	F	T	T	F		F	T	T	T		\times

47. This argument is *invalid*.

\sim	$[H$	\rightarrow	$(K$	\rightarrow	$M)]$		K	$\&$	\sim	M		H	\rightarrow	$(K$	\rightarrow	$M)$		\times
T	T	F	T	F	F		T	T	T	F		T	F	T	F	F		\times

48. This argument is *valid*.

\sim	$[M$	\vee	$(N$	\vee	$O)]$		$(\sim$	M	\rightarrow	$P)$		$\&$	$(\sim$	O	\rightarrow	$P)$		P	\rightarrow	\sim	N		\sim	N	\rightarrow	P		\times
T	F	F	F	F	F		T	F	F	F		T	T	F	F	F		F	T	T	F		T	F	F	F		\times

49. This argument is *valid*.

$(E$	\vee	$G)$	\rightarrow	F		$(G$	\vee	$H)$	\rightarrow	F		$(G$	$\&$	$E)$	\vee	$(G$	$\&$	$H)$		F	$\&$	G		\times
F	T	T	F	F		T	T	F	F	F		T	F	F	F	F	F	F		T	F	F		\times
T	T	F	T	T		T	F	T	F	F		T	T	F	F	F	F	F		F	F	T		\times
T	T	T	F	F		T	F	F	F	F		T	F	F	F	F	F	F		F	F	T		\times

50. This argument is *valid*.

$(A$	$\&$	$B)$	\rightarrow	$(C$	\rightarrow	$(D$	$\&$	$E))$		\sim	$(D$	\rightarrow	$E)$		$\&$	C		\sim	$(A$	$\&$	$B)$		\times
T	T	T	T	T	F	T	F	F		T	T	F	F		T	T		F	T	T	T		\times