

Solution for USAMTS Problem 4/1/16

Jason Ma (Username jjm20, ID 5702)

December 3, 2004

Problem: The interior angles of a convex polygon form an arithmetic progression with a common difference of 4° . Determine the number of sides of the polygon if its largest interior angle is 172° .

Solution:

Let s be the number of sides in the polygon. Thus, the sum of the degrees of the interior angles would be

$$172 + (172 - 4) + (172 - 8) + (172 - 12) + \dots + (172 - 4(s - 1))$$

That, in turn, simplifies to

$$172s - \frac{4s(s - 1)}{2} = 172s - 2s(s - 1) = 172s - 2s^2 + 2s = 174s - 2s^2$$

The sum of the interior angles of a polygon is defined as $180(s - 2)$. Thus,

$$180(s - 2) = 174s - 2s^2$$

Distributing and moving terms,

$$s^2 + 3s - 180 = 0$$

Factoring,

$$(s - 12)(s + 15) = 0$$

Thus, $s = 12$ or $s = -15$. $s = -15$ is an extraneous root; thus the answer is

$$s = 12$$