Binomial Distribution

The binomial distribution describes the probability distribution of a discrete variable, such as the number of times that a certain event A occurs in n independent trials.

Let π = P(A); then the probability that A will occur s times out of n is given by

$$P(s) = \frac{n!}{s!\,(n-s)!}\pi^{s}(1-\pi)^{n-s}$$

The first factor, the "binomial coefficient,"

$$\frac{n!}{s!\,(n-s)!} = \frac{n(n-1)(n-2)\cdots(n-s+1)}{s(s-1)\cdots 3\cdot 2\cdot 1}$$

is the number of combinations of elementary events (s out of n), and the second factor

$$\pi^{s}(1-\pi)^{n-s}$$

is the probability of each elementary event.

Two-child families ($n = 2$, $\pi = 0.488$)

s		$\frac{n!}{s!\,(n-s)!}$	$\pi^s(1-\pi)^{n-s}$	$P(s)$
0	BB	1	0.262	0.262
1	BG, GB	2	0.250	0.500
2	GG	1	0.238	0.238
				1.000

Three-child families ($n = 3$, $\pi = 0.488$)

s		$\frac{n!}{s!(n-s)!}$	$\pi^s(1-\pi)^{n-s}$	$P(s)$
0	BBB	1	0.134	0.134
1	BBG, BGB, GBB	3	0.128	0.384
2	GGB, GBG, BGG	3	0.122	0.366
3	GGG	1	0.116	0.116
				1.000

Four-child families ($n = 4$, $\pi = 0.488$)

s		$\frac{n!}{s!(n-s)!}$	$\pi^s(1-\pi)^{n-s}$	$P(s)$
0	BBBB	1	0.0687	0.069
1	BBBG, BBGB, BGBB, GBBB	4	0.0655	0.262
2	BBGG, BGBG, BGGB, GBGB, GBBG, GGBB	6	0.0624	0.375
3	GGGB, GGBG, GBGG, BGGG	4	0.0595	0.238
4	GGGG	1	0.0567	0.057
				1.000