# Hypothesis Testing

Daniel A. Menascé
Department of Computer Science
George Mason University

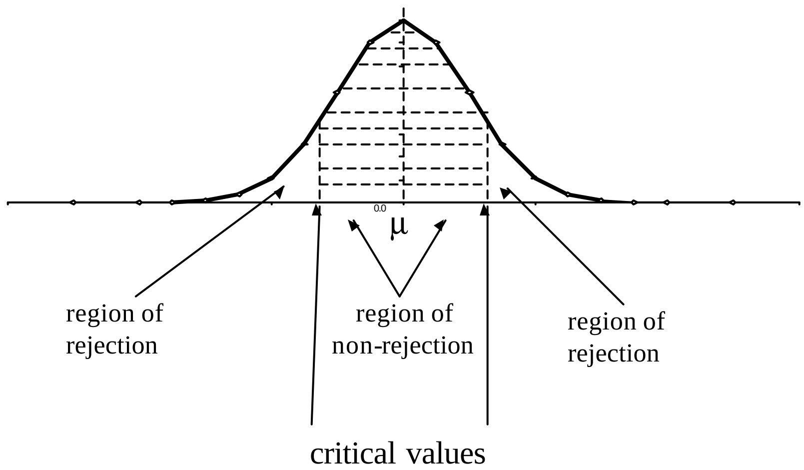
1



# Hypothesis Testing

- Purpose: make inferences about a population parameter by analyzing differences between observed sample statistics and the results one expects to obtain if some underlying assumption is true.
- Null hypothesis: $H_0 : m = x$
- Alternative hypothesis: $H_1 : m \neq x$
- If the null hypothesis is rejected then the alternative hypothesis is accepted.

2




Test statistic: $Z = \dfrac{\overline{X} - m}{\dfrac{s}{\sqrt{n}}}$

3



# Risks in Decision Making

- Type I Error occurs if $H_o$ is rejected when it is true.
  - Pr [$H_o$ is rejected | true] = $\alpha$
- Type II Error occurs if $H_o$ is not rejected when it is false.
  - Pr[$H_o$ is not rejected | false] = $\beta$
- Confidence coefficient:
  - Pr [$H_o$ not rejected | true]= 1- $\alpha$
- Power of the test:
  - Pr[$H_o$ is rejected |false]= 1-$\beta$

4

| | Actual Situation | |
|---|---|---|
| | $H_o$ true | $H_o$ false |
| Accept $H_o$ | Correct decision<br>Confidence=1-$\alpha$ | Type II Error:<br>Pr[Type II]=$\beta$ |
| Reject $H_o$ | Type I Error<br>P[Type I]=$\alpha$ | Correct Decision<br>Power=1-$\beta$ |





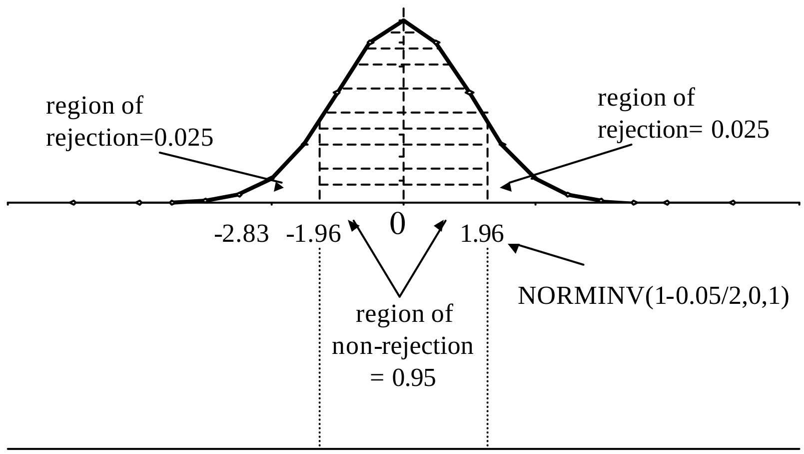
# Example of Hypothesis Testing

- A sample of 50 files from a file system is selected. The sample mean is 12.3Kbytes. The standard deviation is known to be 0.5 Kbytes.

  Ho: $\mu = 12.5$ Kbytes

  $H_1$: $\mu \neq 12.5$ Kbytes

  Confidence: 0.95




$$Z = \frac{12.3 - 12.5}{\frac{0.5}{\sqrt{50}}} = -2.83$$

Reject $H_o$

7



# Solving the Problem with PhStat

| Z Test of Hypothesis for the Mean | |
|---|---|
| | |
| **Null Hypothesis m=** | **12.5** |
| **Level of Significance** | **0.05** |
| **Population Standard Deviation** | **0.5** |
| **Sample Size** | **50** |
| **Sample Mean** | **12.3** |
| Standard Error of the Mean | 0.070710678 |
| Z Test Statistic | -2.828427125 |
| | |
| **Two-Tailed Test** | |
| **Lower Critical Value** | **-1.959961082** |
| **Upper Critical Value** | **1.959961082** |
| ***p*-Value** | **0.00467786** |
| **Reject the null hypothesis** | |

8

## Steps in Hypothesis Testing

1. State the null and alternative hypothesis.
2. Choose the level of significance $\alpha$.
3. Choose the sample size *n*. Larger samples reduce allow us to detect even small difference between sample statistics and true population parameters. For a given $\alpha$, increasing *n* decreases $\beta$.
4. Choose the appropriate statistical technique and test statistic to use (Z or t).





## Steps in Hypothesis Testing

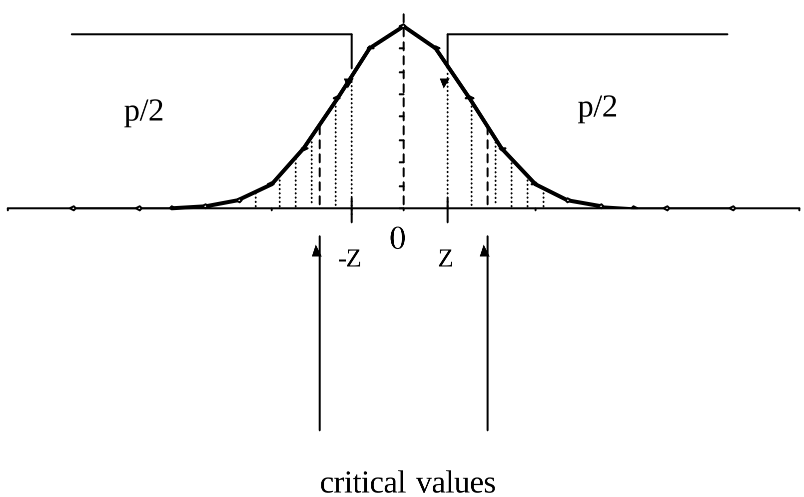
5. Determine the critical values that divide the regions of acceptance and non-acceptance.
6. Collect the data and compute the sample mean and the appropriate test statistic (e.g., Z).
7. If the test statistic falls in the non-reject region, $H_o$ cannot be rejected. Else $H_o$ is rejected.

# The p-value Approach

- p-value: observed level of significance. Defined as the probability that the test statistic is equal to or more extreme than the result obtained from the sample data, given that $H_o$ is true.

11






If $p \geq \alpha$ then do not reject $H_o$, else reject $H_o$.

12

# Computing p-values with PhStat

| Z Test of Hypothesis for the Mean | |
|---|---|
| | |
| **Null Hypothesis** m= | **12.5** |
| **Level of Significance** | **0.05** |
| **Population Standard Deviation** | **0.5** |
| **Sample Size** | **50** |
| **Sample Mean** | **12.3** |
| Standard Error of the Mean | 0.070710678 |
| Z Test Statistic | -2.828427125 |
| | |
| **Two-Tailed Test** | |
| **Lower Critical Value** | **-1.959961082** |
| **Upper Critical Value** | **1.959961082** |
| ***p*-Value** | **0.00467786** |
| **Reject the null hypothesis** | |

The null hypothesis is rejected because p (0.0047) is less than the level of significance (0.05).





# Steps in Determining the p-value.

1. State the null and alternative hypothesis.
2. Choose the level of significance $\alpha$.
3. Choose the sample size $n$. Larger samples reduce allow us to detect even small difference between sample statistics and true population parameters. For a given $\alpha$, increasing $n$ decreases $\beta$.
4. Choose the appropriate statistical technique and test statistic to use (Z or t).

## Steps in Determining the p-value.

5. Collect the data and compute the sample mean and the appropriate test statistic (e.g., Z).
6. Calculate the p-value based on the test statistic
7. Compare the p-value to $\alpha$.
8. If $p \geq \alpha$ then do not reject $H_o$, else reject $H_o$.



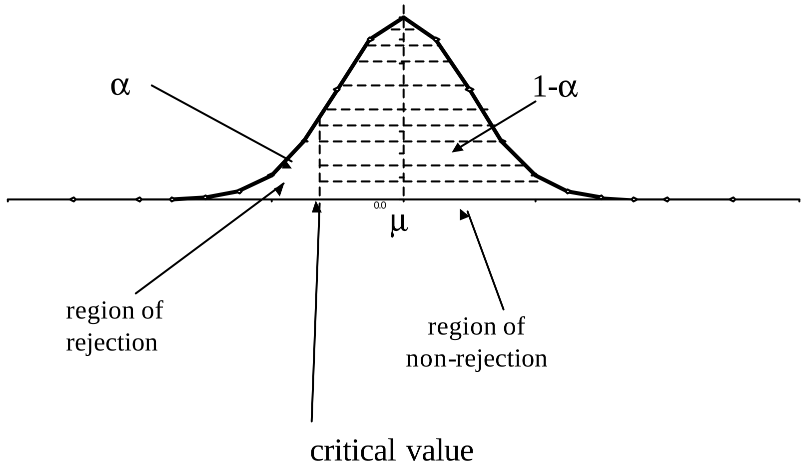


## One-tailed Tests

- Null hypothesis is an inequality.

  $H_o \geq 3.5$

  $H_1 < 3.5$




Test statistic: $Z = \dfrac{\overline{X} - m}{\dfrac{s}{\sqrt{n}}}$





# Example of One-Tailed Test

- A sample of 50 files from a file system is selected. The sample mean is 12.35Kbytes. The standard deviation is known to be 0.5 Kbytes.

  Ho: $\mu \geq 12.3$ Kbytes

  $H_1$: $\mu < 12.3$ Kbytes

  Confidence: 0.95

# Example of One-Tailed Test

$$Z = \frac{\overline{X} - m}{s / \sqrt{n}} = \frac{12.35 - 12.3}{0.5 / \sqrt{50}} = 0.707$$

Critical value =NORMINV(0.05,0,1)= -1.645.
Region of non-rejection: Z ≥ –1.645.
So, do not reject $H_o$.





# One-tailed Tests with PhStat

| Z Test of Hypothesis for the Mean | |
|---|---|
| | |
| **Null Hypothesis m=** | **12.3** |
| **Level of Significance** | **0.05** |
| **Population Standard Deviation** | **0.5** |
| **Sample Size** | **50** |
| **Sample Mean** | **12.35** |
| Standard Error of the Mean | 0.070710678 |
| Z Test Statistic | 0.707106781 |
| | |
| **Lower-Tail Test** | |
| **Lower Critical Value** | **-1.644853** |
| ***p*-Value** | **0.760250013** |
| **Do not reject the null hypothesis** | |

## Hypothesis Tests with Unknown $\sigma$

- If the population is assumed to be normally distributed the sampling distribution for the mean follows a t distribution with n-1 degrees of freedom.
- t statistic for unknown $\sigma$:

$$t = \frac{\overline{X} - m}{\frac{s}{\sqrt{n}}}$$

21



## Example of Hypothesis Testing

- A sample of 50 files from a file system is selected. The sample mean is 12.3Kbytes. The sample standard deviation is 0.5 Kbytes.

  Ho: $\mu = 12.35$ Kbytes

  $H_1$: $\mu \neq 12.35$ Kbytes

Confidence: 0.95

22

| t Test of Hypothesis for the Mean | |
|---|---|
| | |
| **Null Hypothesis** m= | **12.35** |
| **Level of Significance** | **0.05** |
| **Sample Size** | **50** |
| **Sample Mean** | **12.3** |
| **Sample Standard Deviation** | **0.5** |
| Standard Error of the Mean | 0.070710678 |
| Degrees of Freedom | 49 |
| *t* Test Statistic | -0.707106781 |
| | |
| **Two-Tailed Test** | |
| **Lower Critical Value** | **-2.009574018** |
| **Upper Critical Value** | **2.009574018** |
| ***p*-Value** | **0.482849571** |
| **Do not reject the null hypothesis** | |

The t test statistic is between the lower and critical values.
So, do not reject the null hypothesis.

23