CS 672
Basic Performance Modeling Concepts

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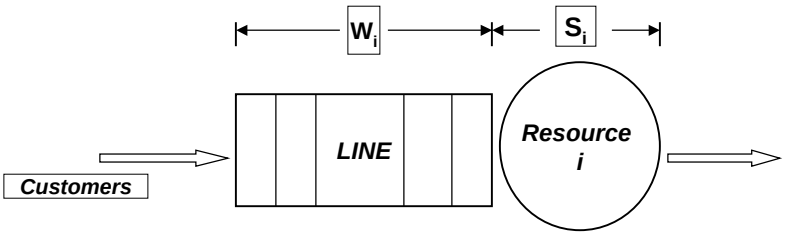
Outline

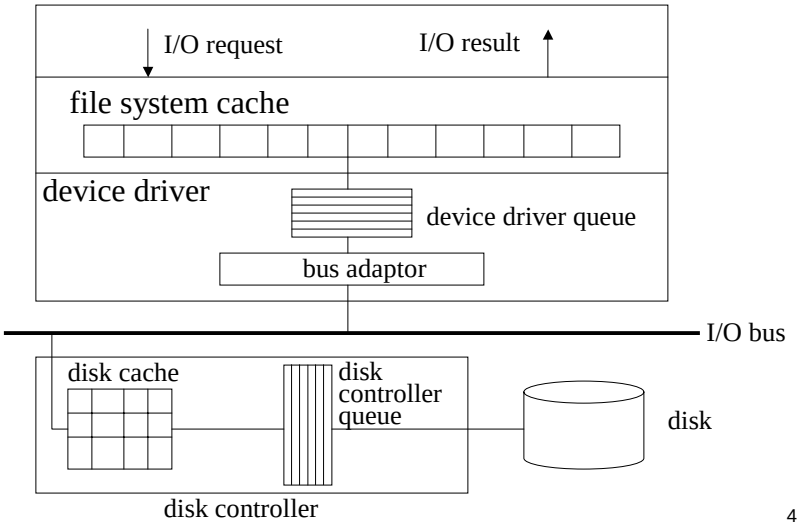
- Single Queue
- Computation of Service Times
- Service Demands
- Operational Laws

A Resource and its Queue



S_i : service time
W_i : waiting time
resource: CPU, disk, network, etc.

Computing Disk Service Times



Computing Disk Service Times

$$s_d = ContrTime + P_{miss}(Seek + Latency + TransferT)$$

$$TransferT = \frac{BlockSize}{TransferRate}$$

Computing Disk Service Times
Types of Workloads

Random Workload:
10, 201, 15, 1023, 45, 39, 782

Sequential Workload:
4, 102, 103, 104, 105, 106, 25, 88, 32, 33, 34, 35, 36, 37, 38, 29, 15

run length= 5 run length= 7

Computing Disk Service Times

Random Workload:

$$P_{miss} = 1$$
$$RunLength = 1$$
$$SeekTime = S_{rand}$$
$$Latency = 1/2 \times \text{Re}volutionTime$$

Computing Disk Service Times

Sequential Workload:

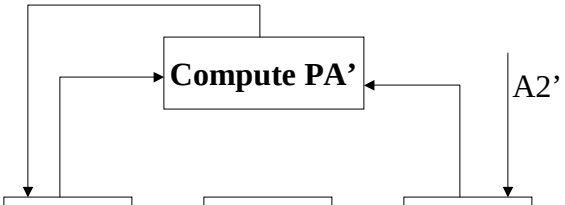
$$P_{miss} = 1 / RunLength$$
$$SeekTime = S_{rand} / RunLength$$
$$Latency = \frac{1/2 + (RunLength - 1)[(1 + U_d)/2]}{RunLength} \times \text{Re}volutionTime$$

$$U_d = \lambda_d \times S_D$$

Disk Arrays

PA	A1	A2	A3	A4
B1	PB	B2	B3	B4
C1	C2	PC	C3	C4
D1	D2	D3	PD	D4
E1	E2	E3	E4	PE

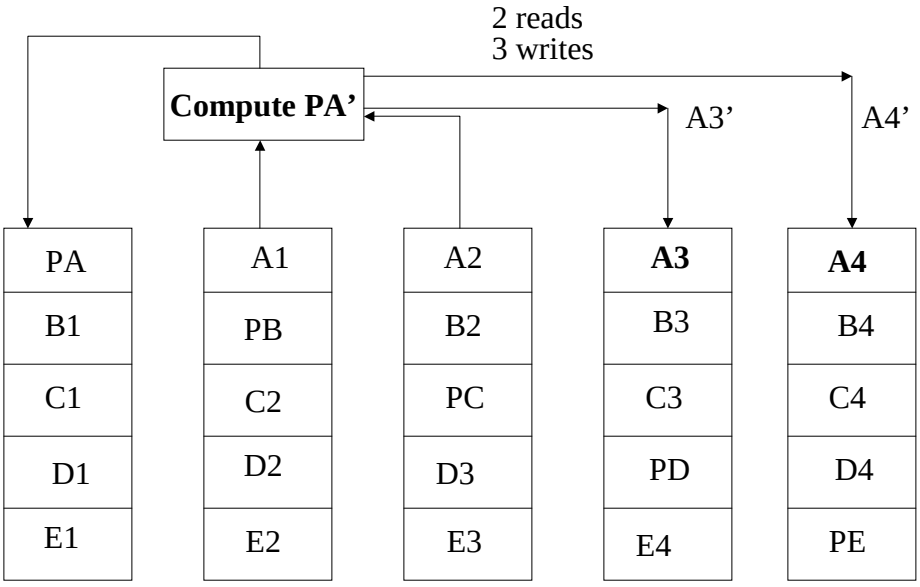
Disk Arrays - Write One Stripe Unit



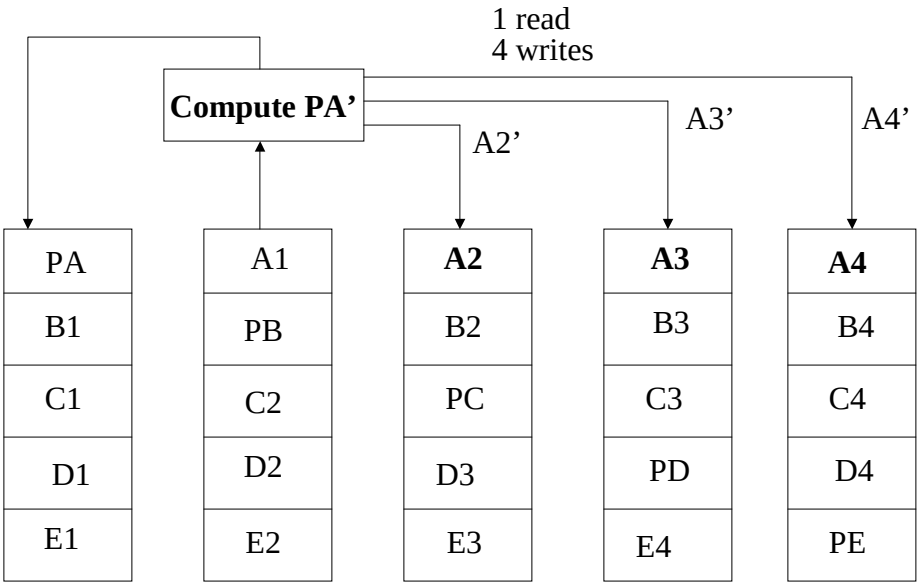
2 reads
2 writes

PA	A1	**A2**	A3	A4
B1	PB	B2	B3	B4
C1	C2	PC	C3	C4
D1	D2	D3	PD	D4
E1	E2	E3	E4	PE

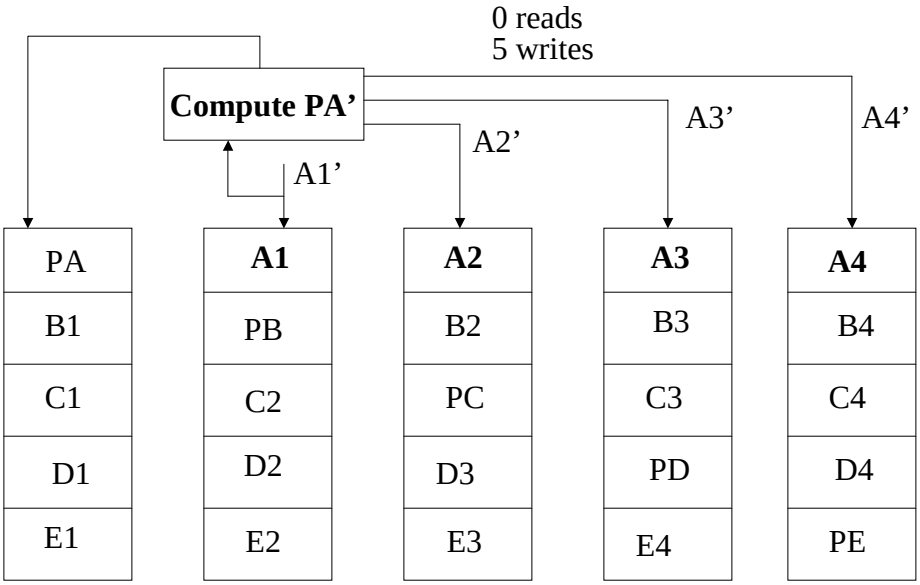
Disk Arrays - Write Two Stripe Units



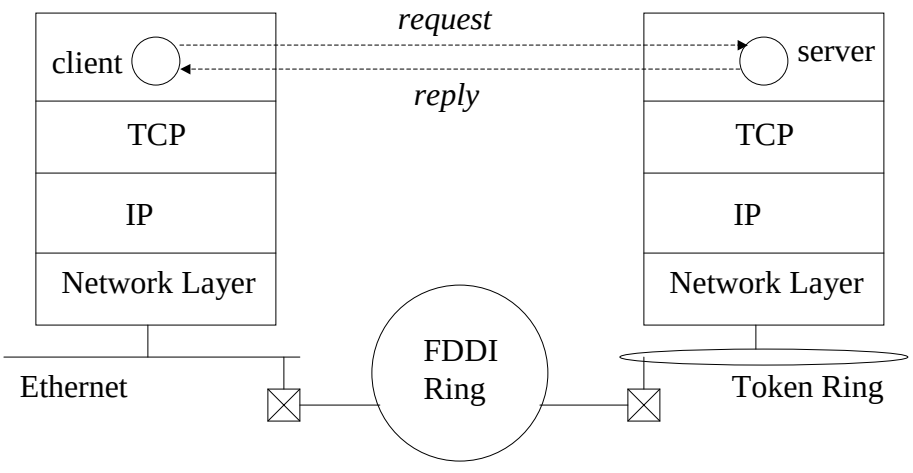
Disk Arrays - Write Three Stripe Units



Disk Arrays - Write Four Stripe Units



Network Service Times

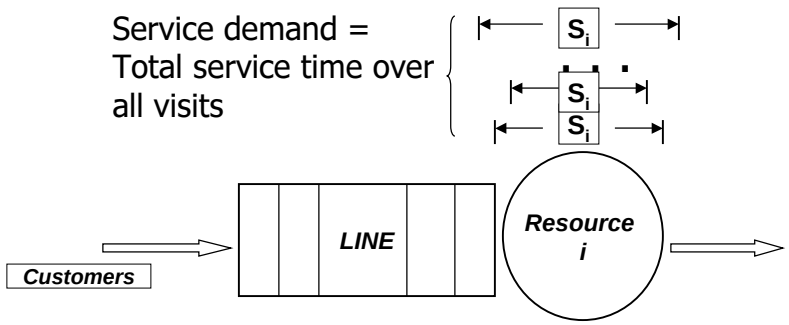


Network Service Times

18 B (with trailer)	20 B	20 B		
Frame Header	IP Header	TCP Header	Client Request	Frame Trailer

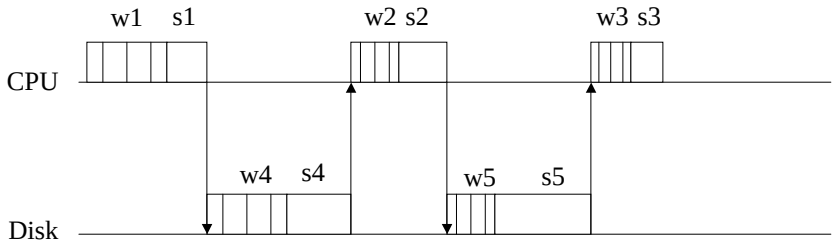
MTU=1500 bytes
Client Message Size = 2500 bytes
No Datagrams = $\lceil 2500 / (1500–20–20) \rceil = 2$
Total Overhead = 2 * (18+20+20)=116 bytes
Message Service Time = [2500+116]*8/10,000,000=0.02098 sec

Service Demand (D_i)



S_i : service time
D_i : service demand

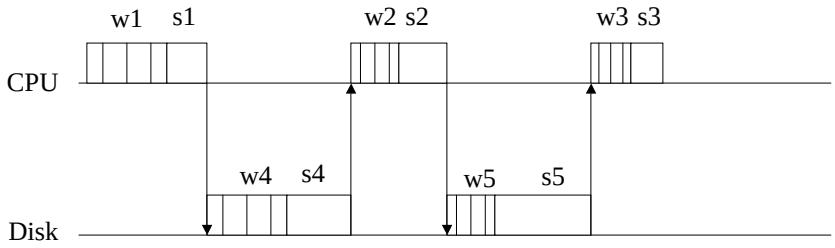
Service Demand



Service demand at the CPU = s1 + s2 + s3
Service demand at the disk = s4 + s5

Waiting time Service time

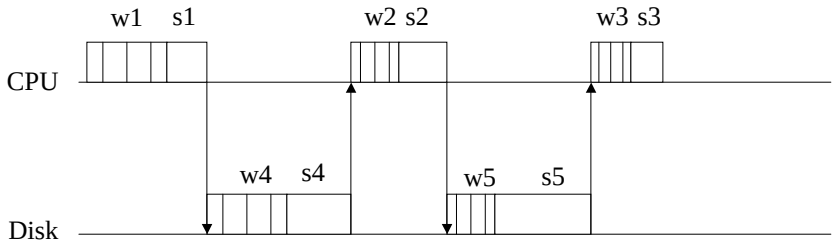
Queuing Time



Queuing time at the CPU = w1 + w2 + w3
Queuing time at the disk = w4 + w5

Waiting time Service time

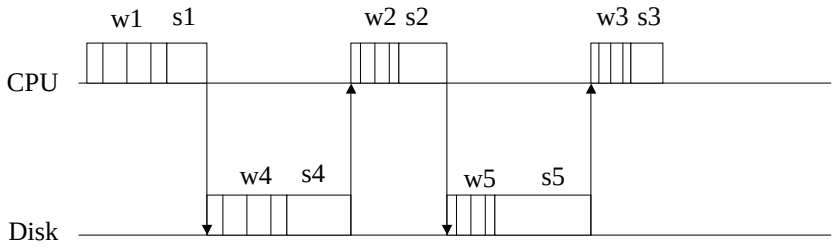
Residence Time



Residence time at the CPU = w1 + s1 + w2 + s2 + w3 + s3
Residence time at the disk = w4 + s4 + w5 + s5

Waiting time Service time

Response Time



Response time = Residence time at the CPU + Residence time at the disk

Waiting time Service time

Queuing Basic Concepts

- Total time spent by a request during the j^{th} visit to a resource *i*:
  - Service time (S_i^j): period of time a request is receiving service from resource *i*, such as CPU or disk.
  - Waiting time (W_i^j): the time spent by a request waiting access to resource *i*

Basic Queuing Concepts

- Service Demand (D_i) is the sum of all service times for a request at resource *i*

  $D_{scpu} = S^1_{scpu} + S^2_{scpu}$

- Queuing Time (Q_i) is the sum of all waiting times for a request at resource *i*

  $Q_{scpu} = W^1_{scpu} + W^2_{scpu}$

Basic Queuing Concepts

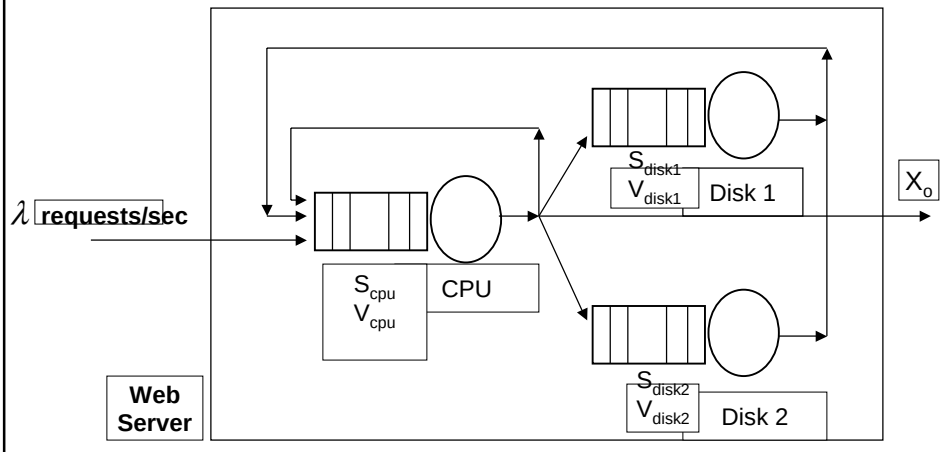
- Residence Time (R'_i) at resource *i* is the sum of service demand plus queuing time.

$$R'_i = Q_i + D_i$$

- Response time (R_r) of a request *r* is the sum of that request's residence time at all resources.

$$R_{server} = R'_{cpu} + R'_{disk}$$

A Web Server and its Queues



A Web Server and its Queues: parameters and notation (1)

- V_i: average number of visits to queue *i* by a request;
- $S_{i:}$ average service time of a request at queue *i* per visit to the resource;
- λ_i average arrival rate of requests to queue *i*
- D_i service demand of a request at queue *i*,
- $D_i = V_i \times S_i$

A Web Server and its Queues: parameters and notation (2)

- $N_{i:}$ average number of requests at queue *i*, waiting or receiving service from the resource
- $X_{i:}$ average throughput of queue *i*, i.e. average number of requests that complete from queue *i* per unit of time
- $X_{o:}$ average system throughput, defined as the number of requests that complete per unit of time.

Basic Performance Results

Utilization Law

- The utilization (U_i) of resource i is the fraction of time that the resource is busy.

$$U_i = X_i * S_i = \lambda_i * S_i$$

Utilization Law: example

- A network segment transmits 1,000 packets/sec. Each packet has an average transmission time equal to 0.15 msec.
- What is the utilization of the LAN segment?

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$$U_{LAN} = X_{LAN} * S_{LAN} = 1{,}000 * 0.00015 = 0.15 = 15\%$$

Basic Performance Results

Forced Flow Law

- By definition of the average number of visits V_i, each completing request has to pass V_i times, on the average, by queue *i*. So, if X_o requests complete per unit of time, $V_i * X_o$ requests will visit queue *i.*

$$X_i = V_i * X_o$$

Forced Flow Law: example

- **Database transactions perform an average of 4.5 I/O operations on the database server. During a one-hour monitoring period, 7,200 transactions were executed.**
- **What is the average throughput of the disk?**
- **If each I/O takes 20 msec on the average, what is the disk utilization?**

Forced Flow Law: example

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- **What is the average throughput of the disk?**
- **If each I/O takes 20 msec on the average, what is the disk utilization?**

$$X_{server} = 7{,}200 / 3{,}600 = 2 \text{ tps}$$
$$X_{disk} = V_{disk} * X_{server} = 4.5 * 2 = 9 \text{ tps}$$
$$U_{disk} = X_{disk} * S_{disk} = 9 * 0.02 = 0.18 = 18\%$$

Basic Performance Results

Service Demand Law

- The service demand D_i is related to the system throughput and utilization by the following:

$$D_i = V_i * S_i = (X_i/X_o)(U_i/X_i) = U_i / X_o$$

Service Demand Law: example

- A Web server running on top of a Unix system was monitored for 10 minutes. It was observed that the CPU was 90% busy during the monitoring period. The number of HTTP requests counted in the log was 30,000.
- What is the CPU service demand of an HTTP request?

Service Demand Law: example

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- What is the CPU service demand of an HTTP request?

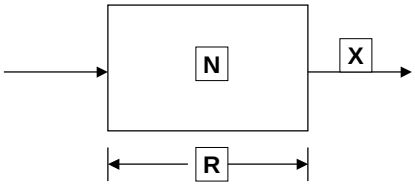
$$U_{cpu} = 90\%$$

$$X_{server} = 30{,}000 / (10*60) = 50 \text{ requests/sec}$$

$$D_{cpu} = V_{cpu} * S_{cpu} = U_{cpu} / X_{server} = 0.90 / 50 = 0.018 \text{ sec}$$

Basic Performance Results

Little's Law



- The average number of customers in a "black box" is equal to average time each customer spends in the "box" times the throughput of the "box".

$$N = R * X$$

Little's Law Example I

- An NFS server was monitored during 30 min and the number of I/O operations performed during this period was found to be 32,400. The average number of active requests (N_{req}) was 9.
- What was the average response time per NFS request at the server?

Little's Law Example I

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"black box" = NFS server

X_{server} = 32,400 / 1,800 = 18 requests/sec

$R_{req} = N_{req} / X_{server} = 9 / 18 = 0.5$ sec

Little's Law Example II

- The average delay experienced by a packet when traversing a network segment is 50 msec. The average number of packets that cross the network per second is 512 packets/sec (network throughput).
- What is the average number of packets in transit in the network?

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"black box" = network segment

$N_{packets} = R_{packet} * X_{network}$

$N_{packets} = 0.05 * 512 = 25.6$ packets

Little's Law Example III

- The disk of a Web server receives requests at a rate of 20 requests/sec. The average disk service time, considering both random and sequential requests, is 8.02 msec.
- What is the average disk utilization?

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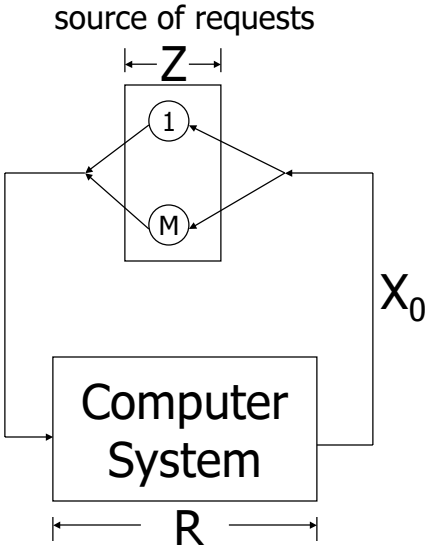
"black box" = disk

$\lambda_{disk} = X_{disk} = 20$ requests/sec

$S_{request} = 0.00802$ sec

$U_{disk} = S_{request} * X_{disk} = 0.00802 * 20 = 16.04\%$

Response Time Law



$$R = M/X_0 - Z$$

R: avg. response time
Z: avg. think time
X_0: avg. throughput
M: number of sources of requests.

Response Time Law Example

- A database server is capable of processing 20 requests/sec. The average think time is 15 sec. What is the maximum number of client machines that can be supported so that the average response time does not exceed 2 seconds?

Response Time Law Example

- A database server is capable of processing 20 requests/sec. The average think time is 15 sec. What is the maximum number of client machines that can be supported so that the average response time does not exceed 2 seconds?
- Z = 15 sec, X_0 = 20 req/sec. So,
- $M = (R + 15) * 20 \leq (2 + 15) * 20 = 340$

Summary of Basic Results

- ☐ Basic Concept of Queuing Theory and Operational Analysis
  - ☐ terminology and notation
  - ☐ service time and service demand
  - ☐ waiting time and queuing time
- ☐ Basic Performance Results and Examples
  - ☐ utilization law: $U_i = X_i * S_i$
  - ☐ forced flow law: $X_i = V_i * X_0$
  - ☐ service demand law: $D_i = V_i * S_i = U_i / X_0$
  - ☐ Little's Law: $N = R * X$
  - ☐ Response Time Law: $R = M/X_0 - Z$