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Dynamic stability of the viscoelastic rotating shaft subjected to random excitation

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Abstract

The dynamic stability problem of a viscoelastic Voigt–Kelvin rotating shaft subjected to action of axial forces at the ends is studied. The shaft is of circular cross-section, it rotates at a constant rate about its longitudinal axis of symmetry. The effect of rotatory inertia of the shaft cross-section is included in the present formulation. Each force consists of a constant part and a time-dependent stochastic function. Closed form analytical solutions are obtained for simply supported boundary conditions. By using the direct Liapunov method almost sure asymptotic stability conditions are obtained as the function of stochastic process variance, retardation time, angular velocity, and geometric and physical parameters of the shaft. Numerical calculations are performed for the Gaussian process with a zero mean and variance σ^2 as well as for harmonic process with amplitude H.

Keywords: Random loading; Liapunov functional; Almost sure stability; Rotatory inertia; Gaussian and harmonic process

1. Introduction

Rotating shafts, as elements of construction, often can take position to lose stability. The stability problem of rotating shafts arises when shafts are required to run smoothly at high speed. Destabilizing factors can be compressive force, the normal inertia force, as well as certain types of damping. So internal damping has this effect, while external damping generally has a stabilizing influence on the system.

The dynamic stability of rotating shafts, with omission of the compressive force, was first analyzed by Bishop [1] using a modal approach. The same problem using the direct Liapunov method was examined by Parks and Pritchard [2].

Shaw and Shaw [3] considered instabilities and bifurcations in non-linear rotating shaft made of viscoelastic Voigt–Kelvin material without compressive force.

Uniform stochastic stability of the rotating shafts, when the axial force is a wide-band Gaussian process with zero mean was studied by Tylikowski [4]. The rotating shaft subjected to axial forces with simultaneous internal damping (Voigt–Kelvin model) and external viscous damping was analyzed by the same author [5].

Tylikowski and Hetnarski [6] examined the influence of the activation through the change of the temperature on dynamic stability of the shape memory alloy hybrid rotating shaft.

Young and Gau [7,8] investigated dynamic stability of a pre twisted cantilever beam with constant and non-constant spin rates, subjected to axial random forces. By using stochastic averaging method, they determined mean-square stability condition in Ref. [7] and first and second moment stability conditions in Ref. [8].

In the present paper almost sure stability of the rotating viscoelastic Voigt–Kelvin shaft without accounting external damping is investigated. The axial force is stochastic process with known density function. Problem is solved by direct Liapunov method, and stability regions are given as function of geometric and physics parameters of the shaft.

2. Problem formulation

Let us consider a shaft rotating about its longitudinal axis with angular velocity $\bar{\Omega}$, shown in Fig. 1. In this Figure

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Notation

A	area of cross-section
I	axial moment of inertia
E	Young's modulus
f_{cr}	dimensionless Euler's critical force
f_o	dimensionless constant component of axial force
$f(t)$	dimensionless stochastic component of axial force
$\bar{F}$	axial force
l	length of the shaft
r	radius of gyration
p	probability density function
P	probability
t	dimensionless time
T	time
X, Y, Z	shaft coordinates
z	dimensionless axial shaft coordinate
u, v	flexural displacements in X and Y direction, respectively
V	Liapunov's functional
α_i	retardation time
ζ	dimensionless retardation time
$\bar{\Omega}$	angular velocity
Ω	dimensionless angular velocity
ρ	density
σ^2	variance of stochastic loading
$E\{\cdot\}$	mathematical expectation
$\|\|\cdot\|\|$	distance of solution from the trivial solution

(X, Y, Z) is rotating coordinate system where Z-axis coincides with longitudinal axis of the rotating shaft.

According to Young and Gau [7], governing differential equations can be written in the form

$$\rho A\left(\frac{\partial^2 u}{\partial T^2} - 2\bar{\Omega}\frac{\partial v}{\partial T} - \bar{\Omega}^2 u\right) - \rho I\frac{\partial^4 u}{\partial T^2 \partial Z^2} + EI\alpha_i\frac{\partial^5 u}{\partial T \partial Z^4} + EI\frac{\partial^4 u}{\partial Z^4} + \bar{F}(T)\frac{\partial^2 u}{\partial Z^2} = 0, \quad (1)$$

$$\rho A\left(\frac{\partial^2 v}{\partial T^2} + 2\bar{\Omega}\frac{\partial u}{\partial T} - \bar{\Omega}^2 v\right) - \rho I\frac{\partial^4 v}{\partial T^2 \partial Z^2} + EI\alpha_i\frac{\partial^5 v}{\partial T \partial Z^4} + EI\frac{\partial^4 v}{\partial Z^4} + \bar{F}(T)\frac{\partial^2 v}{\partial Z^2} = 0, \quad (2)$$

where u, v are flexural displacements in the X and Y direction, ρ is mass density, A is area of the cross-section of shaft, I is axial moment of inertia, E is Young modulus of elasticity, α_i is retardation time, T is time and Z is the axial coordinate.

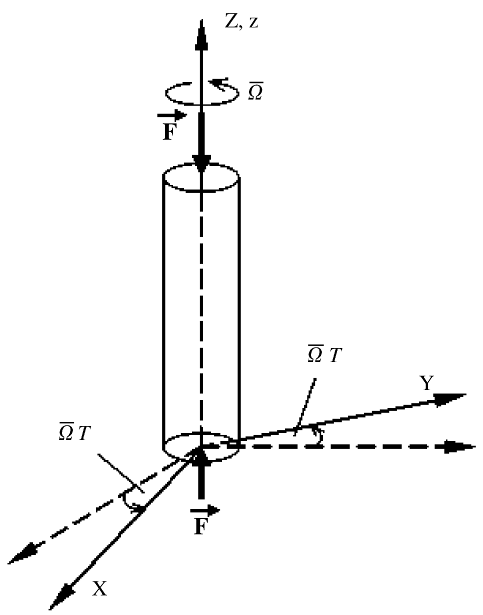

Fig. 1. The rotating shaft and co-ordinate systems.

Using the following transformations:

$$Z = z\ell, \quad e^2 = \frac{I}{A\ell^2}, \quad k_t = \sqrt{\frac{\rho A \ell^4}{EI}}\, T = k_t t,$$
$$f_0 + f(t) = \frac{\bar{F}(t)\ell^2}{EI}, \quad 2\varsigma = \frac{\alpha_i}{k_t}, \quad \Omega = \bar{\Omega} k_t, \quad (3)$$

where ℓ is the length of the shaft and ζ is reduced retardation time, we get governing equations as

$$\frac{\partial^2 u}{\partial t^2} - 2\Omega\frac{\partial v}{\partial t} - \Omega^2 u - e^2\frac{\partial^4 u}{\partial t^2 \partial z^2} + 2\varsigma\frac{\partial^5 u}{\partial t\, \partial z^4} + \frac{\partial^4 u}{\partial z^4} + (f_0 + f(t))\frac{\partial^2 u}{\partial z^2} = 0, \quad (4)$$

$$\frac{\partial^2 v}{\partial t^2} + 2\Omega\frac{\partial u}{\partial t} - \Omega^2 v - e^2\frac{\partial^4 v}{\partial t^2 \partial z^2} + 2\varsigma\frac{\partial^5 v}{\partial t \partial z^4} + \frac{\partial^4 v}{\partial z^4} + (f_0 + f(t))\frac{\partial^2 v}{\partial z^2} = 0, \quad (5)$$

$z \in (0, \ 1)$.

Boundary conditions for the simply supported shaft are

$$u(t,0) = u(t,1) = \frac{\partial^2 u}{\partial z^2}(t,0) = \frac{\partial^2 u}{\partial z^2}(t,1) = 0,$$
$$v(t,0) = v(t,1) = \frac{\partial^2 v}{\partial z^2}(t,0) = \frac{\partial^2 v}{\partial z^2}(t,1) = 0. \quad (6)$$

The purpose of the present paper is the investigation of almost sure asymptotic stability of the rotating shaft subjected to stochastic time-dependent axial loads. To estimate perturbated solutions it is necessary to introduce a measure of distance $\|\cdot\|$ of solutions of Eqs. (4) and (5) with nontrivial initial conditions and the trivial one. Following Kozin [9], the equilibrium state of Eqs. (4) and

(5) is said to be almost sure stochastically stable, if:

$$P\{\lim_{t\to\infty} ||\mathbf{w}(.,t)|| = 0\} = 1, \tag{7}$$

where $\mathbf{w} = \mathrm{col}(u, v)$ matrix column.

3. Stability analyses

With the purpose of applying the Liapunov method, we can construct the functional by means of the Parks–Pritchard's method [2]. Thus, let us write Eqs. (4) and (5) in the formal form $\mathfrak{L}\mathbf{w} = 0$, where $\mathfrak{L}$ is the matrix

$$\mathfrak{L} = \begin{bmatrix} \ell_{11} & \ell_{12} \\ \ell_{21} & \ell_{22} \end{bmatrix}, \tag{8}$$

with elements

$$\ell_{11} = \ell_{22} = \frac{\partial^2}{\partial t^2} - \Omega^2 - e^2\frac{\partial^4}{\partial t^2\partial z^2} + 2\varsigma\frac{\partial^5}{\partial t\,\partial z^4} + \frac{\partial^4}{\partial z^4} + (f_0 + f(t))\frac{\partial^2}{\partial z^2}, \tag{9}$$

$$\ell_{12} = -\ell_{21} = -2\Omega\frac{\partial}{\partial t},$$

and introduce the linear operator:

$$\Re = \begin{bmatrix} 2\left(\frac{\partial}{\partial t} + 2\varsigma\frac{\partial^4}{\partial z^4} - e^2\frac{\partial^3}{\partial t\partial z^2}\right) & -2\Omega \\ 2\Omega & 2\left(\frac{\partial}{\partial t} + 2\varsigma\frac{\partial^4}{\partial z^4} - e^2\frac{\partial^3}{\partial t\partial z^2}\right) \end{bmatrix}, \tag{10}$$

which is a formal derivative of the operator $\mathfrak{L}$ with respect to $\partial/\partial t$.

Integrating the scalar product of the vectors $\mathfrak{L}\mathbf{w}\,\Re\mathbf{w}$ on rectangular $C = [z\colon 0\leqslant z\leqslant 1]\times[\tau\colon 0\leqslant\tau\leqslant t]$ with respect to Eqs. (4) and (5), it is clear

$$\int_0^1\int_0^\tau \mathfrak{L}\boldsymbol{w}\Re\boldsymbol{w}\,\mathrm{d}z\,\mathrm{d}\tau = 0. \tag{11}$$

After applying partial integration to Eq. (11), the sum of two integrals may be obtained. In the first, integration is only on the spatial domain, and it is chosen to be the Liapunov functional:

$$\begin{aligned} \boldsymbol{V} = \int_0^1 \Bigg\{ &\left(\frac{\partial u}{\partial t} - \Omega v + \varsigma\frac{\partial^4 u}{\partial z^4} - e^2\frac{\partial^3 u}{\partial t\,\partial z^2}\right)^2 \\ &+ \left(\frac{\partial v}{\partial t} - \Omega u + \varsigma\frac{\partial^4 v}{\partial z^4} - e^2\frac{\partial^3 v}{\partial t\,\partial z^2}\right)^2 \\ &- (f_0 + \Omega^2 e^2)\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] \\ &+ (1 - f_o e^2)\left[\left(\frac{\partial^2 u}{\partial z^2}\right)^2 + \left(\frac{\partial^2 v}{\partial z^2}\right)^2\right] \\ &+ e^2\left[\left(\frac{\partial^3 u}{\partial z^3}\right)^2 + \left(\frac{\partial^3 v}{\partial z^3}\right)^2\right] \\ &+ \varsigma^2\left[\left(\frac{\partial^4 u}{\partial z^4}\right)^2 + \left(\frac{\partial^4 v}{\partial z^4}\right)^2\right]\Bigg\}\,\mathrm{d}z. \end{aligned} \tag{12}$$

Since it is evident

$$\boldsymbol{V}|_0^t - \int_0^t \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = 0, \tag{13}$$

then the second integral in Eq. (11) is a time derivative of the functional Eq. (12) along Eqs. (4) and (5):

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = -2\int_0^1 \Bigg\{ &\varsigma\left[\left(\frac{\partial^3 u}{\partial t\,\partial z^2}\right)^2 + \left(\frac{\partial^3 v}{\partial t\,\partial z^2}\right)^2 + e^2\left(\frac{\partial^4 u}{\partial t\,\partial z^3}\right)^2 + e^2\left(\frac{\partial^4 v}{\partial t\,\partial z^3}\right)^2\right] \\ &+ \varsigma\left[\left(\frac{\partial^4 u}{\partial z^4}\right)^2 + \left(\frac{\partial^4 v}{\partial z^4}\right)^2\right] - \Omega^2\varsigma\left[\left(\frac{\partial^2 u}{\partial z^2}\right)^2 + \left(\frac{\partial^2 v}{\partial z^2}\right)^2\right] \\ &- f_0\varsigma\left[\left(\frac{\partial^3 u}{\partial z^3}\right)^2 + \left(\frac{\partial^3 v}{\partial z^3}\right)^2\right] \\ &+ f(t)\frac{\partial^2 u}{\partial z^2}\left(\frac{\partial u}{\partial t} - \Omega v + \varsigma\frac{\partial^4 u}{\partial z^4} - e^2\frac{\partial^3 u}{\partial t\,\partial z^2}\right) \\ &+ f(t)\frac{\partial^2 v}{\partial z^2}\left(\frac{\partial v}{\partial t} + \Omega u + \varsigma\frac{\partial^4 v}{\partial z^4} - e^2\frac{\partial^3 v}{\partial t\,\partial z^2}\right)\Bigg\}\,\mathrm{d}z. \end{aligned} \tag{14}$$

Functional $\boldsymbol{V}$ will be a Liapunov functional if it is a positive definite. By using well known Steklov's inequalities:

$$\int_0^1\left[\left(\frac{\partial^{n+1}u}{\partial z^{n+1}}\right)^2 + \left(\frac{\partial^{n+1}v}{\partial z^{n+1}}\right)^2\right]\mathrm{d}z \geqslant \pi^2\int_0^1\left[\left(\frac{\partial^n u}{\partial z^n}\right)^2 + \left(\frac{\partial^n v}{\partial z^n}\right)^2\right]\mathrm{d}z, \quad n = 1,2,3,\ldots, \tag{15}$$

and omitting dynamical terms, we can write:

$$V \geqslant [\pi^2(1 - f_0 e^2) + \pi^4 e^2 - (f_0 + \Omega^2 e^2)]\int_0^1\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]\mathrm{d}z, \tag{16}$$

so, the positive definite condition reduces to relation:

$$f_0 \leqslant \pi^2 - \frac{\Omega^2 e^2}{1 + \pi^2 e^2}. \tag{17}$$

4. Stability under constant axial force

If $f(t) = 0$, and using relations (15), the first derivative of Liapunov functional (14) will be negative definite when

$$f_0 \leqslant \pi^2 - \frac{\Omega^2}{\pi^2}. \tag{18}$$

The relation (18) is stronger than (17) and represents the dynamic stability condition of the rotating shaft under constant force. We may observe that if $f_o = f_{cr} = \pi^2$, where f_{cr} is Euler's critical force, then $\Omega = 0$.

In the absence of axial force ($f_o = 0$), we find

$$\Omega \leqslant \pi^2 \Rightarrow \bar{\Omega} \leqslant \frac{\pi^2}{\ell^2}\sqrt{\frac{EI}{\rho A}} = \omega_1, \quad (19)$$

where ω_1 denotes the first natural frequency of the shaft at rest. The angular velocity Ω may be larger if $f_o < 0$ (i.e., f_o is tensile force).

5. Almost-sure stability

Let a scalar function $\lambda(t)$ be defined as

$$\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}t} \leqslant \lambda(t). \quad (20)$$

As a maximum point is a particular case of the stationary point, we may write:

$$\delta(\dot{V} - \lambda V) = 0. \quad (21)$$

By using the associated Euler's equations we obtain:

$$\left(\lambda\ell_e^{(2)} + 2\varsigma\frac{\partial^4 \ell_e}{\partial z^4}\right)\frac{\partial u}{\partial t} + \left[\lambda\varsigma\frac{\partial^4\ell_e}{\partial z^4} + f(t)\frac{\partial^2\ell_e}{\partial z^2}\right] \times u - \Omega\lambda\ell_e v = 0,$$
$$\left(\lambda\ell_e^{(2)} + 2\varsigma\frac{\partial^4 \ell_e}{\partial z^4}\right)\frac{\partial v}{\partial t} + \Omega\lambda\ell_e u + \left[\lambda\varsigma\frac{\partial^4\ell_e}{\partial z^4} + f(t)\frac{\partial^2\ell_e}{\partial z^2}\right] v = 0,$$
$$\left[\lambda\varsigma\frac{\partial^4\ell_e}{\partial z^4} + f(t)\frac{\partial^2\ell_e}{\partial z^2}\right]\frac{\partial u}{\partial t} + \Omega\lambda\ell_e\frac{\partial v}{\partial t} + \ell_{uv}u = 0,$$
$$-\Omega\lambda\ell_e\frac{\partial u}{\partial t} + \left[\lambda\varsigma\frac{\partial^4\ell_e}{\partial z^4} + f(t)\frac{\partial^2\ell_e}{\partial z^2}\right]\frac{\partial v}{\partial t} + \ell_{uv}v = 0, \quad (22)$$

where

$$\ell_e = 1 - e^2\frac{\partial^2}{\partial z^2},\ \ell_e^{(2)} = \left(1 - e^2\frac{\partial^2}{\partial z^2}\right)\left(1 - e^2\frac{\partial^2}{\partial z^2}\right) = 1 - 2e^2\frac{\partial^2}{\partial z^2} + e^4\frac{\partial^4}{\partial z^4},$$
$$\ell_{uv} = \lambda\left(\Omega^2 + 2\varsigma^2\frac{\partial^8}{\partial z^8} + \frac{\partial^4\ell_e}{\partial z^4} + f_o\frac{\partial^2\ell_e}{\partial z^2} + \Omega^2 e^2\frac{\partial^2}{\partial z^2}\right) - 2\Omega^2\varsigma\frac{\partial^4}{\partial z^4} + 2\varsigma\frac{\partial^8}{\partial z^8} + 2\varsigma f_o\frac{\partial^6}{\partial z^6} + 2\varsigma f(t)\frac{\partial^6}{\partial z^6}. \quad (23)$$

After simplifying, we get two equations:

$$\left[\left(\lambda\varsigma\frac{\partial^4\ell_e}{\partial z^4} + f(t)\frac{\partial^2\ell_e}{\partial z^2}\right)^{(2)} + \lambda^2\Omega^2\ell_e^{(2)} \left(\lambda\ell_e^{(2)} + 2\varsigma\frac{\partial^4\ell_e}{\partial z^4}\right) - \ell_{uv}\right]\begin{Bmatrix} u \\ v \end{Bmatrix} = 0. \quad (24)$$

According to the boundary condition (10), we may write the solution in the form

$$u(z,t) = \sum_{m=1}^{\infty} U_m T_m(t)\sin\alpha_m z,$$
$$v(z,t) = \sum_{m=1}^{\infty} V_m T_m(t)\sin\alpha_m z, \quad (25)$$

where $\alpha_m = m\pi$ and from Eq. (24) we obtain algebraic equation:

$$A_m\lambda_m^2 + 2B_m\lambda_m + C_m = 0, \quad (26)$$

where

$$A_m = (1 + e^2\alpha_m^2)\alpha_m^2[\varsigma^2\alpha_m^6 + (1 + e^2\alpha_m^2)(\alpha_m^2 - f_0) - e^2\Omega^2],$$
$$B_m = 2\varsigma^2\alpha_m^6[\varsigma^2\alpha_m^6 + (1 + e^2\alpha_m^2)(\alpha_m^2 - f_o) - e^2\Omega^2],$$
$$C_m = -\alpha_m^4(1 + e^2\alpha_m^2)f^2(t) + 4\varsigma^2\alpha_m^8[\alpha_m^2(\alpha_m^2 - f_0) - \alpha_m^2 f(t) - \Omega^2]. \quad (27)$$

Hence, from Eq. (26)

$$\lambda_m = \frac{\alpha_m}{1 + e^2\alpha_m^2} \times \left\{\sqrt{\frac{[2\varsigma^2\alpha_m^6 + (1 + e^2\alpha_m^2)f(t)]^2 + 4\varsigma^2\Omega^2\alpha_m^4}{\varsigma^2\alpha_m^6 + (1 + e^2\alpha_m^2)(\alpha_m^2 - f_o) - e^2\Omega^2}} - 2\varsigma\alpha_m^3\right\}. \quad (28)$$

By solving the differential inequality (17), we obtain the following estimation of the functional:

$$V(t) \leqslant V(0) \quad \exp\left[-t\left(\frac{1}{t}\int_0^t \lambda(\tau)\mathrm{d}\tau\right)\right]. \quad (29)$$

Therefore, it can be stated that the trivial solution of Eq. (9) is almost sure asymptotically stable if

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t \lambda(\tau)\,\mathrm{d}\tau \leqslant 0, \quad (30)$$

or, when the process $f(t)$ is ergodic and stationary:

$$E\{\lambda(t)\} \leqslant 0, \quad (31)$$

where E denotes the operator of the mathematical expectation, and

$$\lambda(t) = \max_m \lambda_m(t). \quad (32)$$

6. Numerical results and discussion

The relations (30), (31) and (32) give us possibility to obtain minimal retardation time ζ guaranteeing the asymptotic and almost sure asymptotic stability called critical retardation time. The domain where the retardation times are greater than the critical retardation time is called the stability region or almost sure stability region. The stability regions are given as functions of loading variance, retardation time, angular velocity, dimensionless parameter $e = r/\ell$ where $r = \sqrt{I/A}$ is the radius of gyration and constant component of the axial loading for Gaussian and harmonic process.

Knowledge of the probability density function *p*(*f*) for the process *f*(*t*) gives us possibility to obtain more precise results, (see Kozin, [9]). The boundaries of the almost sure stability are calculated by using the corresponding Gauss–Cristofel quadratures, and presented with a full line for Gaussian process, and a dashed line for the harmonic one. For Gaussian process we take the parameters of Gauss–Hermite quadrature, and for harmonic process we set $f(t) = H\cos(\omega t + \theta)$, where H, ω are fixed amplitude and frequency, and θ is a uniformly distributed random phase on the interval $[0,2\pi)$. In order to compare both processes the variance of harmonic process $\sigma^2 = H^2/2$, is used, and we take the Gauss–Chebyshev quadrature, (see Pavlović et al. [10]). Calculations were performed for the first mode, ($m = 1$).

In Fig. 2 stability regions are plotted as a function of the angular velocity when the influence of rotatory inertia is neglected ($e = 0$) and axial loading is absent $f_o = 0$. As expected, the increase of the angular velocity leads to stability regions decreasing.

Fig. 3 illustrates effect of cross-section rotatory inertia on almost sure stability when angular velocity is fixed ($\Omega = 4$). Even in case of very short shafts ($e = 0.1$) rotatory inertia neglecting ($e = 0$) causes error less than 5%, while for e = 0.05 error is less than 1.2%.

The relations (30), (31) and (32) also give us possibility to obtain critical angular velocity guaranteeing the almost sure stability of the shaft.

In Figs. 4–6 stability regions are given as functions of retardation time, constant component of axial force and parameter *e*. Stability regions are larger when retardation time increases and constant component of axial loading changes from pressure ($f_o = 4$) to tension ($f_o = -4$). Generally speaking, in technical problems influence of rotatory inertia on dynamic stability of rotating shafts can be neglected, except in case of very short shafts. In that

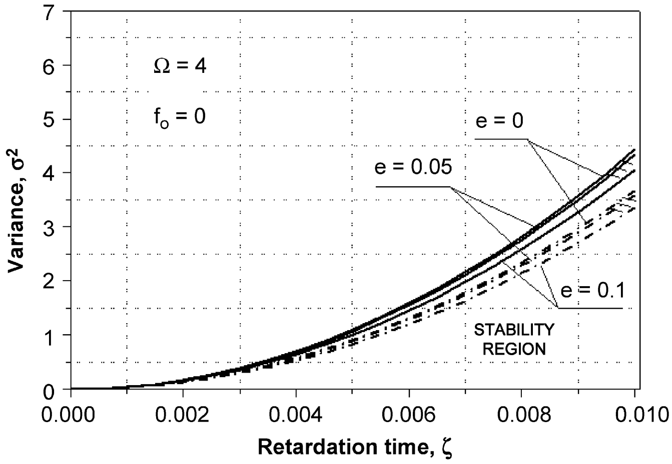

Fig. 3. Influence rotatory inertia on stability regions.

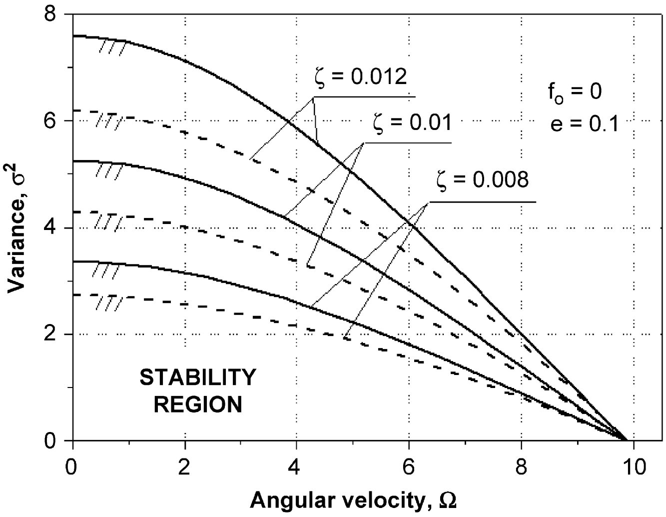

Fig. 4. Influence of retardation time on stability regions.

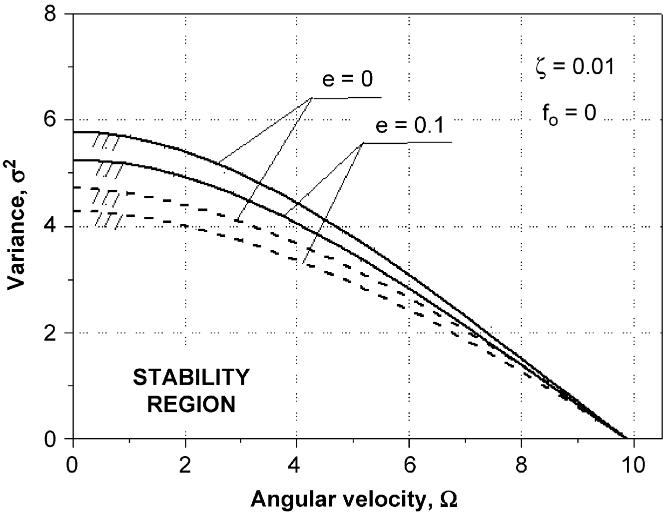

Fig. 5. The variance of stochastic process as a function of angular velocity and rotatory inertia.

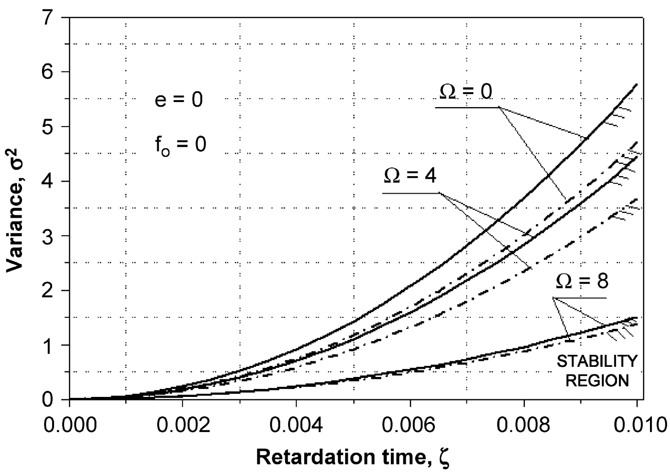

Fig. 2. Influence of angular velocity on stability regions.

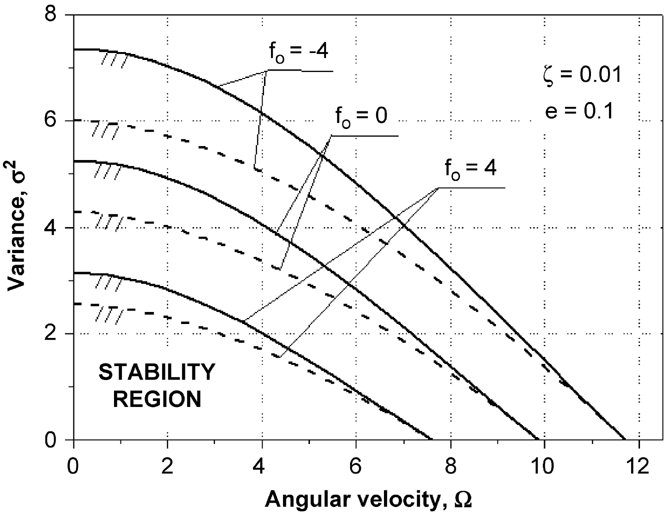

Fig. 6. Influence of the statical force on stability regions.

case influence of transverse shear should be also taken into account.

7. Conclusions

The dynamic stability of a viscoelastic rotating shaft subjected to the axial random forces at the ends is analyzed. By taking into account rotatory inertia of the shaft cross-section, almost sure asymptotic stability conditions are obtained. Stability regions are calculated for Gaussian and harmonic processes, and shown in variance–retardation time and variance–angular velocity coordinate systems.

According to previous, we can emphasize the following conclusions:

1. In the classical case, when the rotatory inertia of cross-section is neglected, increasing of the angular velocity causes rapidly decreasing of the stability regions.
2. The rotatory inertia neglecting of the viscoelastic Voigt–Kelvin rotating shaft causes error less than 5%.
3. Stability regions are noticeable larger when retardation time increases.
4. Critical angular velocity and stability regions can be enlarged by applying tension axial loading at the ends of the rotating shaft.

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