

# DETERMINATION OF TIME CONSTANT OF A THERMOMETER

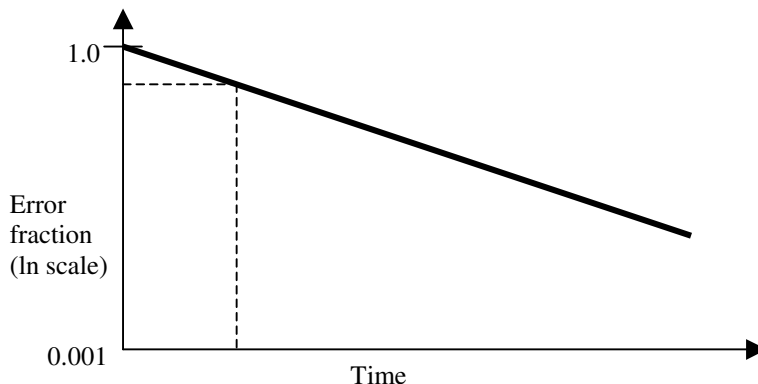
(Corrected 22 Feb 2006)

## Principle:

The behavior of a thermometer can be modeled as a first-order differential equation  $\tau \dot{y} + y = KF(t)$ , where  $\tau$  is called the time constant of the system and provides a measure of the system response. For a step input, the solution can be expressed as  $\Gamma(t) = \frac{y(t) - y_{\infty}}{y_0 - y_{\infty}} = e^{-t/\tau}$ , where  $\Gamma$  is the error fraction of the output signal. The time constant can be determined by recording the system's response from  $t=0$  until a steady response is achieved.

## Procedure:

1. Maintain the thermosistor bath at the required constant temperature  $y_{\infty}$ .
2. Note the initial temperature of the thermometer  $y_0$ .
3. Dip the thermometer into the bath and simultaneously start a stopwatch.
4. Read out the temperature at as short intervals as possible and note the corresponding times in the stopwatch.
5. Continue until the temperature reads the steady state value.
6. Plot the graph of  $\ln(\text{error fraction})$  vs time
7. Fit a straight line through the points using the method of least squares and determine the slope  $m$ .
8. The time constant is  $\tau = -1/m$ .



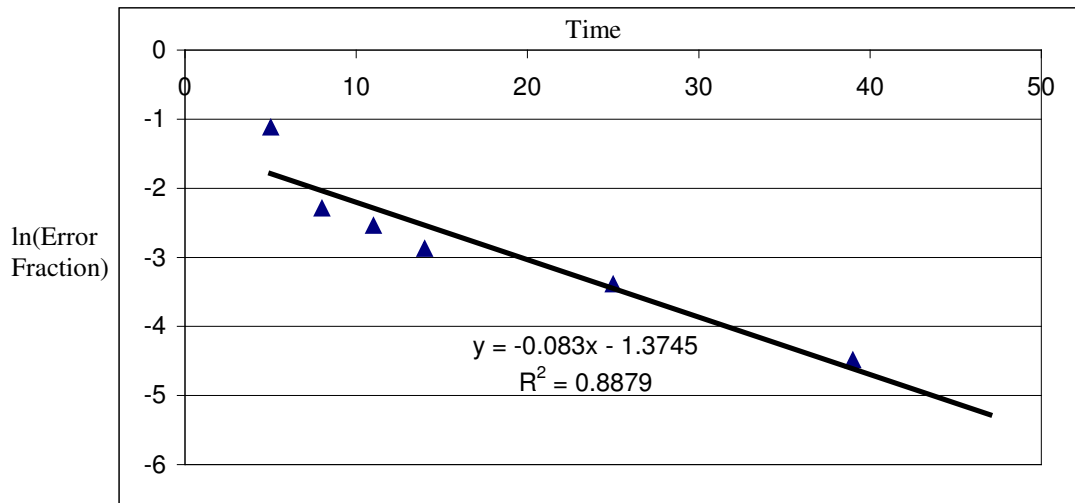
## Discussion:

1. What is the benefit of taking logarithms?
2. Is the plot linear? If not, what does it mean?

## Reference:

1. Figliola, Richard S & Beasley, Donald E, "Theory and Design for Mechanical Measurements" Third Edition, John Wiley & Sons New York, 2000.

Temp	Time	Error fraction	Ln(Error fraction)
60	5	0.325843	-1.12134
70	8	0.101124	-2.29141
71	11	0.078652	-2.54273
72	14	0.05618	-2.8792
73	25	0.033708	-3.39002
74	39	0.011236	-4.48864
74.5	47	0	



Time constant =  $-1 / -0.083 = 12$  seconds