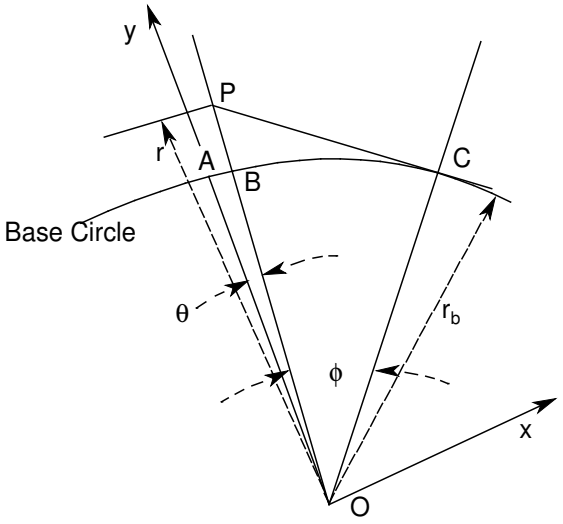
PROCEDURE TO DETERMINE THE THEORETICAL PROFILE OF AN INVOLUTE GEAR

(Revised 22 Feb 2006)

1. Draw the Base Circle with centre O and radius r_b. Let A be the point from which the gear tooth starts.
2. Imagine a string wound around the Base Circle with its free end at A. If the string is unwound keeping it taut, the end A will trace an involute profile.
3. Let P be any point on the involute profile. Then OP makes an angle θ with respect to OA. CP is tangential to the base circle and angle COP= ϕ



4. Then the radius r= OP = $r_b \sec\phi$ or $\phi = \cos^{-1}\left(\frac{r_b}{r}\right)$.
5. From triangle OPC, OC is perpendicular to CP. Then $\tan\phi = \frac{CP}{OC}$, or $CP = r_b \tan\phi$.
6. CP is also the length of the thread unwound, $CP = r_b(\theta + \phi)$.
7. Equating the equations in steps 5 and 6, $\theta = \tan\phi - \phi$.
8. For a given value of r ($>r_b$), we can calculate ϕ from 4 and θ from 7.
9. We can determine the x and y coordinates of the profile as $x = r * \sin\theta$ and $y = r * \cos\theta$
10. The involute function for various angles and the profile for a base circle radius of 19.5 is shown in the next page.
11. Plot the points choosing same scale (say, 20:1) for x and y axis. Join points with a smooth line.
12. To determine the profile error of a given gear, trace the actual profile of a tooth in the same scale (say, 20:1) and compare with the theoretical profile obtained.

r	phi	theta	x	y
19.50	0.0000	0.0000	0.0000	19.5000
19.70	0.1426	0.0010	0.0192	19.7000
19.90	0.2008	0.0027	0.0546	19.8999
20.10	0.2450	0.0050	0.1009	20.0997
20.30	0.2817	0.0077	0.1562	20.2994
20.50	0.3136	0.0107	0.2194	20.4988
20.70	0.3422	0.0140	0.2900	20.6980
20.90	0.3681	0.0176	0.3674	20.8968
21.10	0.3919	0.0214	0.4512	21.0952
21.30	0.4141	0.0254	0.5411	21.2931
21.50	0.4347	0.0296	0.6370	21.4906
21.70	0.4542	0.0340	0.7386	21.6874
21.90	0.4725	0.0386	0.8457	21.8837
22.10	0.4900	0.0434	0.9583	22.0792
22.30	0.5065	0.0483	1.0762	22.2740
22.50	0.5223	0.0533	1.1992	22.4680
22.70	0.5374	0.0585	1.3274	22.6612
22.90	0.5519	0.0638	1.4606	22.8534
23.10	0.5658	0.0693	1.5987	23.0446
23.30	0.5792	0.0748	1.7416	23.2348
23.50	0.5921	0.0805	1.8894	23.4239
23.70	0.6045	0.0863	2.0419	23.6119
23.90	0.6165	0.0921	2.1991	23.7986
24.10	0.6281	0.0981	2.3610	23.9841