Reliability-based Performance of RC Buildings Subjected to Earthquakes

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Abstract

Reliability is proposed as a quantitative measure of seismic performance of reinforced concrete buildings subjected to earthquakes. The procedures in the proposed methodology comprise of modeling of reinforced concrete buildings, modeling of earthquake, reliability analysis, and computational procedure. The reliability or probability of failure is attributed by the uncertainty in earthquake mechanism, characteristics of soil media, characteristics of earthquake phenomena. Damage of structures is used in the formulation of Limit State Function. A practical numerical example is given to demonstrate the methodology.

1. Introduction

Earthquake is a detrimental natural phenomenon and usually leads to damage of structures. Under an earthquake, the damage of building structures can vary from moderate degree, i.e. repairable, to disastrous degree, i.e. total collapse, which may result in considerable loss of lives. Therefore, the performance of buildings when subjected to earthquakes needs be thoroughly evaluated. Among construction materials, reinforced concrete is in popular use. The reinforced concrete structures are, however, vulnerable to the reversal of combined stresses with high magnitudes, such as those induced by earthquakes. The seismic performance of reinforced concrete buildings is thus of paramount importance from the viewpoint of engineering, economy, and societal impact.

In reality an earthquake cannot be predicted deterministically. Correspondingly, a quantitative measure of seismic performance may be defined in terms of the failure probability of a building for a prescribed critical damage level. The probability of failure is obtained from the so-called reliability analysis. In this paper a methodology for evaluation of seismic performance, utilizing reliability measure, will be presented. A numerical example is accompanied herein in order to demonstrate the procedure. Finally, some concluding remarks relevant to the presented methodology will be given.

2. Methodology

2.1 Modeling of Reinforced Concrete Buildings under Earthquake Excitation

In order to evaluate the seismic performance of a reinforced concrete building, the responses of the structure relevant to the performance index are required first. Consequently, the building structure subjected to earthquake is modeled by a dynamical system

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{F}_r\left(\mathbf{X}, \dot{\mathbf{X}}\right) = -\mathbf{M}\mathbf{I}\ddot{\mathbf{X}}_g \qquad (1)$$

in which $\mathbf{M}$ is a mass matrix, $\mathbf{C}$ is a damping matrix, $\mathbf{F}_r\left(\mathbf{X}, \dot{\mathbf{X}}\right)$ is a non-linear restoring-force matrix, $\mathbf{I}$ is the identity matrix, and $\ddot{\mathbf{X}}_g$ is a ground acceleration vector. The non-linear restoring-force matrix is formed by the non-linear restoring-forces in the structural elements building up the structural model. Under strong earthquake excitation, the load-deformation of structural members may show hysteretic behavior. A hysteretic model will therefore be used in the modeling of the non-linear restoring-force of structural members. Among considerable numbers of hysteretic model (see e.g. [2], [3], [5], [16], and [23]), the Three-Parameter model is suitable for its application to reinforced concrete members. This is because the model can characterize the stiffness degradation, the strength deterioration, and the pinching behavior, which happen to the reinforced concrete members under cyclic loading with high magnitudes [16]. The parameters, which correspond to the stiffness degradation, the strength deterioration, and the pinching behavior, are denoted respectively by α, β, and γ. The complete description of the Three-Parameter hysteretic model and the influence of each control parameter on the shape of hysteresis loop can be found in [16]. The schematic diagram of the force-displacement relation for the Three-Parameter model is shown in Fig. 1.

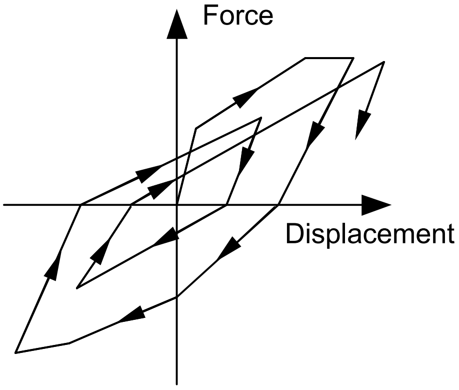

Fig. 1 Force-displacement relation following the Three-Parameter model.

2.2 Modeling of Earthquake

The records of ground motion at site are in principle needed in the reliability analysis. However, there is always the inadequacy in the number of the recorded ground motion. This leads to the notion of using synthetic earthquake. One of the well-recognized earthquake models is the Kanai-Tajimi (KT) model ([9] and [22]). The Kanai-Tajimi model yields seismic waves that are generated at a firm base and transmitted through a soil layer to the free ground surface. Correspondingly, the model comprises of a mass, supported by a linear spring and a dashpot in parallel (Fig. 2).

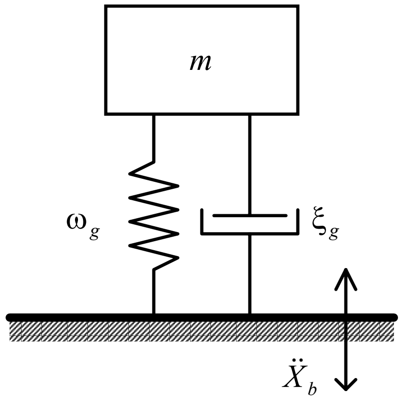

Fig. 2 Kanai-Tajimi model.

The equation of motion for the Kanai-Tajimi model is given by

$$\ddot{X}_m + 2\zeta_g\omega_g\dot{X}_m + \omega_g^2 X_m = 2\zeta_g\omega_g\dot{X}_b + \omega_g^2 X_b \quad (2)$$

where subscripts *m* and *b* stand for mass and base, respectively. ω_g and ζ_g denote the natural frequency and damping ratio of the soil layer. To account for the uncertainty in the generating mechanism of seismic waves at the firm base, the base motion is modeled by a Gaussian white noise process. As a result, the one-sided power spectral density function of the ground motion $G(\omega)$ is obtained as

$$G(\omega) = \frac{G_o\left(1 + 4\zeta_g^2(\omega/\omega_g)^2\right)}{\left(1-(\omega/\omega_g)^2\right)^2 + 4\zeta_g^2(\omega/\omega_g)^2} \quad (3)$$

in which G_o is the one-sided power spectral density function of the white noise process. Apart from the generating mechanism, the uncertainty exists inherently in the natural frequency ω_g and the damping ratio ζ_g. Therefore, ω_g and ζ_g are also random variables.

The ground motion from the KT model is of stationary stochastic process. However, the real ground motion is observed to be non-stationary stochastic processes. To include the non-stationary characteristic, a modulation function is applied to the stationary process obtained from the KT model. Amin and Ang [1] proposed a modulation function of the form

$$I(t)=\begin{cases} 0 & t\le 0 \\ (t/t_1)^2 & 0<t\le t_1 \\ 1 & t_1<t\le t_2 \\ \exp(-c_\varphi(t-t_2)) & t>t_2 \end{cases} \quad (4)$$

in which t_1, t_2, c_φ are the model parameters. The value $T_s = t_2 - t_1$ is the strong motion duration. An example of the modulation function is shown in Fig. 3. Using the proposed modulation function, the ground acceleration $\ddot{X}_g$ is obtained from

$$\ddot{X}_g(t) = I(t)\ddot{X}_m(t) \quad (5)$$

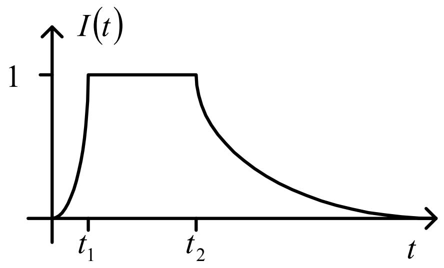

Fig. 3 Modulation function [1].

2.3 Damage Index

The seismic performance of a building structure may be reflected by its structural damage. Accordingly, a damage index is established as a measure of damage level. There are many proposed definitions of damage index [19]. For the purpose of analysis demonstration, the well-known Park-Ang damage index will be adopted here as the indicator of damage level. For a spring-mass system under a repeated cyclic loading, the damage in the spring is due not only to the effect from the loading magnitude, but also to that from the frequency content. Correspondingly, Park and Ang [15] developed a damage index, defined as

$$D = \frac{\delta_M}{\delta_u} + \frac{\beta}{Q_y \delta_u}\int dE \quad (6)$$

in which D is the Park-Ang damage index of a spring, δ_M is the maximum deformation under earthquake, δ_u is the ultimate deformation under monotonic loading, Q_y is the calculated yield strength, and dE is the incremental absorbed hysteretic energy. It should be noted that the results obtained by Banon and Veneziano [26] do not support the assumptions used in the formulation of the Park-Ang damage index.

When a structure is modeled as a discrete spring-mass system, the overall structural damage D_T may be defined as

$$D_T = \sum \lambda_i D_i \; ; \; \lambda_i = E_i \Big/ \sum E_i \quad (7b)$$

in which D_i is the damage index of the ith spring, computed from eq.(6). E_i is the total energy (including the potential energy) absorbed by the respective spring.

2.4 Reliability Analysis of Dynamical System

The reliability of a system is defined as the probability that the system performs its assigned function over a specified period of time and under specified operating conditions. This probability is called more specifically as the probability of success or safe. Equivalently, the probability of failure can be used to reflect the reliability of a system, which is usually done the civil engineering. Thus, the probability of failure will be adopted here as a quantitative measure of seismic performance. In order to identify the state of a system, a boundary separating the safe and failure domains has to be set up first. Such a boundary - Limit State Surface - is explicitly defined from the so-called Limit State Function. For the evaluation of the seismic performance of a building structure, the Limit State Function may be defined as

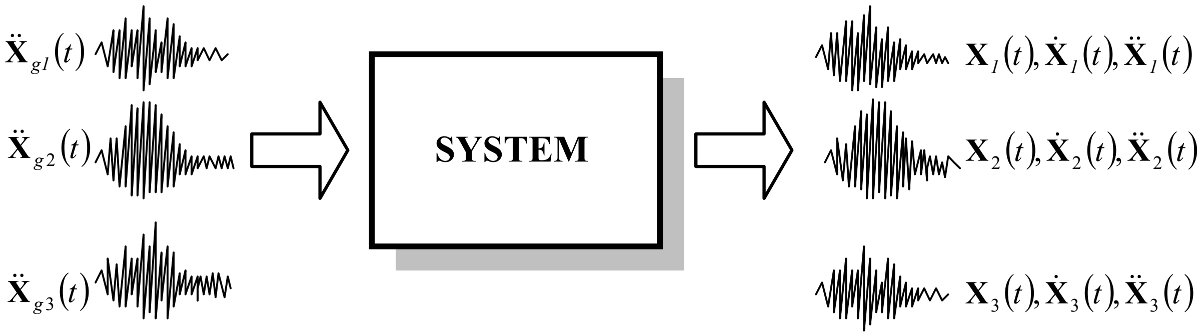

Fig. 4 Monte Carlo simulation method for analysis of stochastic dynamical system.

$$g(D_T, d_{T,c}) = d_{T,c} - D_T \qquad (8)$$

in which $d_{T,c}$ is the critical damage index. From eq.(8), $g(D_T, d_{T,c}) < 0$ represents the failure domain while $g(D_T, d_{T,c}) \geq 0$ means the safe domain. The Limit State Surface is readily obtained as $g(D_T, d_{T,c}) = 0$. For a building structure subjected to a random earthquake, as represented by eqs.(1) and (2), its dynamical responses are random. Consequently, the overall damage index D_T is also a random variable. The probability of failure P_f is then

$$P_f = P[g(D_T, d_{T,c}) < 0] \qquad (9)$$

$$P_f = \int_{D_f} f_{D_T}(d_T)\,dd_T \qquad (10)$$

where D_f is the failure domain, and $f_{D_T}(d_T)$ is the probability density function of the random variable D_T.

3. Computational Procedure

3.1 Solution Concept

To compute P_f, $f_{D_T}(d_T)$ has to be known first. Although a specific distribution type of $f_{D_T}(d_T)$ can be readily employed, its distribution parameters, e.g. mean and standard deviation, are still needed for the unique definition of $f_{D_T}(d_T)$. However, it is not possible to obtain accurately the distribution parameters of D_T by the analytical methods due to the high non-linearity in the considered dynamical system. Numerical methods of solution are thus unavoidable. For such purpose, the Monte Carlo simulation method (MCS) is introduced. The concept of MCS is illustrated in Fig. 4. According to Fig. 4, an ensemble of ground-acceleration histories is first simulated. The numerical solutions of the dynamic responses and of the damage index are then computed respectively for each synthetic history. This yields a sample of D_T. Finally, the distribution parameters are derived from the sample. Based on the obtained distribution parameters, $f_{D_T}(d_T)$ is determined and P_f is computed.

According to the MCS method, the synthetic ground acceleration is the indispensable input. Therefore, the simulation of ground acceleration will be described in Subsection 3.2.

3.2 Simulation of Ground Motion

Based on the Kanai-Tajimi model and the Amin-Ang modulation function, the simulation of ground motion proceeds as follows:

1. Specify the natural frequency ω_g and the damping ratio ζ_g of the soil layer at site.
2. Specify the desired Peak Ground Acceleration (PGA) $a_{\max}$.
3. Specify the strong motion duration T_s.
4. Compute the root-mean-square ground acceleration a_{RMS}, using the relation [24]

$$Z_P = \frac{a_{\max}}{a_{RMS}} \quad (11)$$

in which Z_P is a specified peak factor.

5. Create $G(\omega)$ from [24]

$$a_{RMS}^2 = \int_0^\infty G(\omega)d\omega \quad (12)$$

6. Simulate the ground acceleration $\ddot{X}_g(t)$ using the expression [20]

$$\ddot{X}_g(t) = I(t)\sum_{i=1}^{n} A_i \sin(\omega_i t + \phi_i) \quad (13)$$

in which $A_i^2 = 2G(\omega_i)\Delta\omega_i$ and ϕ_i is a random variable with uniform distribution between 0 and 2π. The geometrical interpretation of A_i^2 is illustrated in Fig. 5.

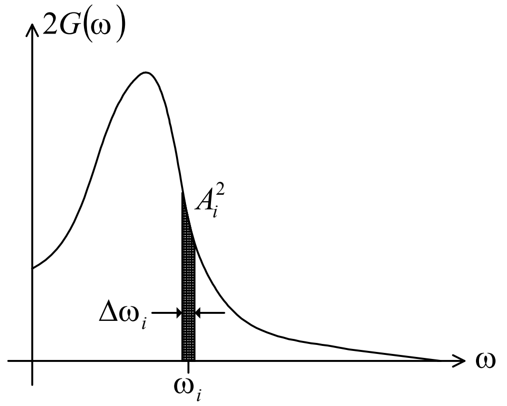

Fig. 5 Geometrical interpretation of A_i^2.

4. Numerical Example

A 5-storey reinforced concrete (RC) building is constructed on the stiff soil. The building consists of parallel rigid frames with the separating distance of 4.0 m. A typical plane frame is considered (Fig. 6). The uniformly distributed load in the first, second, third, fourth, and fifth floor is 38.4, 52.3, 52.3, 52.3, 44.9 T/m, respectively. The damping ratio is equal to 0.05 for RC buildings [17]. The dimension of structural members is shown in Fig. 8. The compressive strength of concrete is 240 ksc. The yield strengths of the round bar and of the deformed bar are 2,400 and 3,000 ksc, respectively.

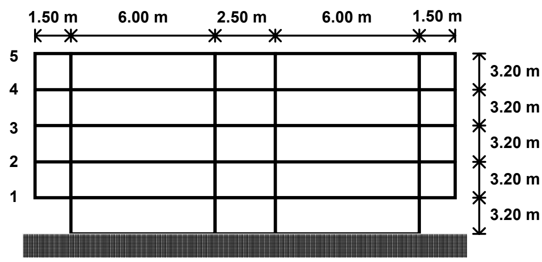

Fig. 6 A typical plane frame of the 5-storey RC building.

The building structure is modeled as a system of macro beam and column elements, each of which has two nodes at both ends (Fig. 7). Flexure and shear components are considered. Axial component is included exclusively in the column element.

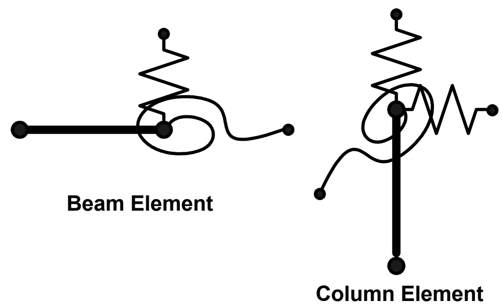

Fig. 7 Beam and column elements.

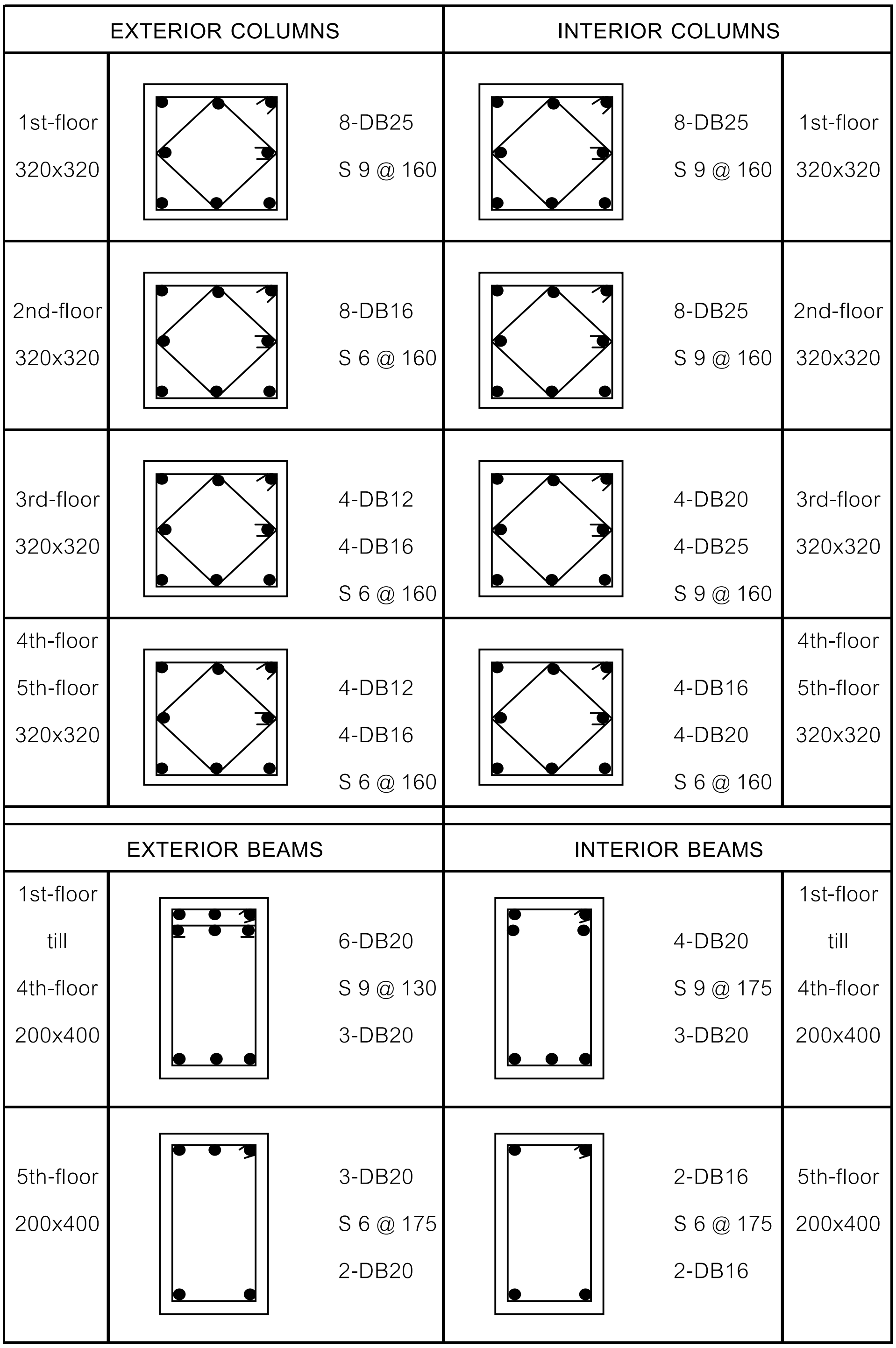

Fig. 8 Cross-sections of structural members (*mm*).

To realistically account for the uncertainty in the characteristics of earthquake, their relevant parameters are treated as random variables. The natural frequency of the stiff soil was found a gamma distribution function [11] with $\mu_{\Omega g} = 3.03$ Hz and $COV_{\Omega g} = 0.43$ [18], where $\mu_{\Omega g}$ and $COV_{\Omega g}$ are its mean and coefficient of variation, respectively. The damping ratio is log-normally distributed [11] with $\mu_{\Xi g} = 0.313$ and $COV_{\Xi g} = 0.44$ [18]. According to [12], Moayyad and Mohraz performed a statistical analysis on the data of the strong motion duration and obtained $\mu_{Ts} = 6.9$ s and $COV_{Ts} = 0.42$. In addition, Rahman and Grigoriu [19] took a log-normal distribution function for the strong motion duration. Vanmarcke and Lai [24] had studied the peak factor and showed that it had a log-normal distribution function with $\mu_{Zp} = 2.67$ and $COV_{Zp} = 0.14$. Following the study of Rahman and Grigoriu [19], the parameters in the Amin-Ang modulation function are chosen as $t_1 = 0.15T_s$, $t_2 = 1.15T_s$, and $C_\varphi = 2.0/T_s$.

For the hysteretic model of the reinforced concrete members, the stiffness degradation is selected equal to 2.0 according to the suggestion of Kunnath et al. [10]. The strength deterioration is set as 0.15, following the conclusion of Cosenza et al. [4]. This value was used also by Singhal and Kiremidijian [21], and Gilmore [6]. Based on the assumption of good reinforcement detailing, the parameter corresponding to the pinching behavior is fixed at 1.0 (see e.g. [10] and [19]).

Following the Monte Carlo simulation method, the sample points in the sample space of all input random variables are required. Correspondingly, the input random variables are generated according to their respective distributions. A sample point is obtained by assembling sampled values from each random variable and forming a sample vector (point). The process of creating all such sample points is carried out by the so-called Latin hypercube sampling technique (LHS) ([7] and [8]). Each sample point is then used in generating ensemble of ground-motion histories. All simulated ensembles are conditional on a specified PGA because $G(\omega)$ is a function of PGA. Consequently, the results from MCS are conditional on the specified PGA. In this example, the sample size of 20 is employed for the random variables of earthquake parameters. The corresponding sampled values are shown in Table 1.

Each sample history of ground acceleration due to a specific sample point of the earthquake random variables is set as an input for eq.(1). The dynamic responses resulting from such an input ground acceleration are calculated using the Newmark method [13] with the integration time step of 0.005 s. The computation of response history is doned by IDARC2D [25]. Based on the response ensemble, a sample of overall damage indices D_T, as computed from eq.(7), its statistical mean, and standard deviation (see Table 2) are obtained. The probability density function of the overall damage index was found to be a log-normal distribution [14].

To compute the probability of failure P_f, the limit state function needs to be established. According to the calibrated damage index with the observed strurctural damage in nine RC buildings subjected to earthquake [15], the damage from seismic ground motion is unreparable when the corresponding overall damage index is greater than 0.4. This value of damage index will be adopted here as the critical damage index. Thus the limit state function becomes

$$g(D_T) = 0.4 - D_T \tag{14}$$

The conditional probabilities of failure for respective PGA's are shown in Fig. 9. Specifying a level of acceptable risk, the seismic performance of the analyzed building can be evaluated. For example, if the level of acceptable risk is set at $P_f = 1.0 \times 10^{-6}$, this exemplified 5-story RC building will show satisfactory performance under the earthquakes with PGA less than $0.15g$ only.

Table 1 Sampled values of basic random variables

No.	T_s (s)	ω_g (Hz)	ζ_g	Peak Factor
1	10.2	2.90	0.25	2.82
2	11.4	2.00	0.31	2.67
3	6.80	1.35	0.24	2.77
4	5.60	2.47	0.27	3.02
5	9.30	1.62	0.13	3.11
6	4.70	4.17	0.39	2.32
7	5.80	3.10	0.35	2.85
8	6.50	2.60	0.21	2.15
9	4.40	3.90	0.42	2.62
10	3.00	4.55	0.16	2.73
11	8.00	5.05	0.29	2.43
12	5.00	1.81	0.63	3.24
13	7.60	1.05	0.19	2.25
14	14.0	2.75	0.22	2.37
15	6.30	5.90	0.18	2.48
16	5.30	3.45	0.46	3.45
17	8.60	2.30	0.33	2.58
18	7.30	2.15	0.37	2.53
19	3.50	3.25	0.28	2.00
20	4.00	3.65	0.52	2.94

Table 2 The conditional mean μ_{D_T} and standard deviation σ_{D_T} of D_T

PGA	μ_{D_T}	σ_{D_T}
0.050*g*	0.004	0.0021
0.075*g*	0.013	0.0070
0.100*g*	0.022	0.0119
0.150*g*	0.040	0.0214
0.200*g*	0.057	0.0311
0.250*g*	0.075	0.0408
0.300*g*	0.093	0.0504
0.350*g*	0.111	0.0601
0.400*g*	0.129	0.0698

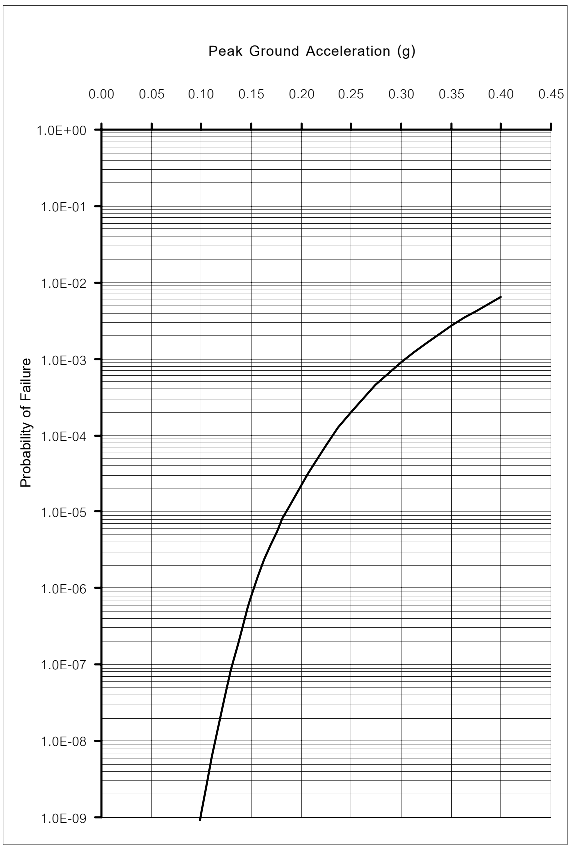

Fig. 9 Probabilities of failure for $d_{T,c} = 0.4$

5. Concluding Remarks

A methodology for evaluation of seismic performance is presented here. The failure probability of an RC building subjected to an earthquake is employed as a quantitative measure of the performance. With this quantitative measure, the seismic performance of structures can be evaluated and compared. Setting the failure probability as a design constraint, the concept can be integrated to the optimal design of an RC building. Finally, it should be noted that the considered uncertainty in this paper is limited to the earthquake only. The uncertainty in the system properties has not been taken into account yet but may be included in the analysis for its completeness.

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