# Reliability analysis of structures subjected to earthquake

# with dynamic soil-pile-structure interaction

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ABSTRACT: The reliability analysis of structures subjected to earthquake, including dynamic soil-structure interaction, is described. The substructure of interest belongs to the class of pile foundation. The pile-to-pile interaction is considered in the analysis through the use of dynamic interaction factor. It has been found that the pile-to-pile interaction can be neglected in determining the safety level of structures with typical configurations of foundations. As a result, the analysis procedure becomes more efficient.

## 1 INTRODUCTION

The analysis of civil engineering structures is normally performed without considering the mechanical interaction between superstructures and their substructures or foundations. The base of a superstructure is assumed completely fixed or hinged. Thus, the mechanical effects due to the existence of foundation on the responses of structure are ignored. For the purpose of realistic design, the structure-foundation interaction should not be neglected in the analysis. With respect to this awareness, efforts have been put on the problems of soil-structure interaction. However, most of these efforts belong either to static cases or to shallow foundations. On the other hand, the reliability analysis of structure including soil-structure interaction is rarely addressed although a number of publications are available on the stochastic analysis of soil-structure interaction, e.g. (Jin et al. 2000) or (Lutes et al. 2000). Consequently, the purpose of this paper is to describe a procedure for reliability analysis of structure including dynamic soil-structure interaction of soil-pile-structure type. In addition, the effect of pile-to-pile interaction on the safety level of structure has also been addressed.

## 2 PILE-SOIL-PILE SYSTEM

### 2.1 *Dynamic interaction factor*

The response of a pile in a pile group subjected to dynamic loading is naturally affected by the surrounding piles. The deformation of the surrounding piles creates waves that propagate and hit on the target single pile. Thus, there exists an interaction among piles forming a pile group. The dynamic response of a pile cap consisting of several piles can be more realistically improved when the pile-to-pile interaction is taken into account. A number of models have been developed to compute the dynamic of pile groups accounting for pile-to-pile interaction, see e.g. (Novak 1991). Among available models of pile group behavior, the model proposed by Makris and Gazetas (Makris & Gazetas 1992) can describe the complicated behavior of pile-to-pile interaction through simple analytical formulas. The model is derived from a dynamic beam on Winkler foundation in combination with wave propagation theory. The pile-to-pile interaction is expressed through the dynamic interaction factor.

According to the Makris-Gazetas model, the dynamic pile-to-pile interaction factors are derived for two types of loading; namely head-type loading and seismic-type excitation. In case of seismic-type excitation, the displacement "$x_{j/i}$" at the head of pile "$j$" due to the displacement "$x_i$" at the head of pile "$i$" is obtained from

$$x_{j/i} = \alpha_{ji} x_i \qquad \text{(no summation on } i\text{)} \tag{1}$$

where $\alpha_{ji}$ is the dynamic interaction factor, which is given as follows:

$$\alpha_{ji} = \psi(r,\theta)\left(\frac{k_x + i\omega c_x}{E_\mathrm{p} I_\mathrm{p} \delta^4 + k_x + i\omega c_x - m\omega^2} - 1\right) \tag{2}$$

where $m$ is the mass per unit length of pile, $c_x$ is the Winkler-dashpot damping coefficient, $k_x$ is the Winkler-spring stiffness, $E_\mathrm{P}$ is Young modulus of the pile material, $I_\mathrm{p}$ is the moment of inertia of the pile, $\delta = \omega/V_\mathrm{s}$, $\omega$ is the excitation frequency, and $V_\mathrm{s}$ is the shear wave velocity. $\psi(r,\theta)$ is the attenuation function that can be approximated by (Makris & Gazetas 1992)

$$\psi(r,\theta) \approx \psi(r,0)\cos^2\theta + \psi\left(r,\frac{\pi}{2}\right)\sin^2\theta \tag{3}$$

$$\psi(r,0) = \left(\frac{r_0}{r}\right)^{1/2} \exp\left(\frac{-\beta_\mathrm{s}\omega(r-r_0)}{V_\mathrm{La}}\right)\exp\left(\frac{-\mathrm{i}\omega(r-r_0)}{V_\mathrm{La}}\right) \tag{4.1}$$

$$\psi\left(r,\frac{\pi}{2}\right) = \left(\frac{r_0}{r}\right)^{1/2} \exp\left(\frac{-\beta_\mathrm{s}\omega(r-r_0)}{V_\mathrm{s}}\right)\exp\left(\frac{-\mathrm{i}\omega(r-r_0)}{V_\mathrm{s}}\right) \tag{4.2}$$

where $r$ is the center-to-center distance between pile "$i$" and pile "$j$", $\theta$ is the angle between the direction of loading and the line connecting two pile-centers, $r_0$ is the radius of pile "$i$", $\beta$ is the damping ratio of the soil, and $V_\mathrm{La}$ is the Lysmer's analogue wave velocity. From eq. (2) to eq. (4), one can see that the dynamic interaction factor is frequency-dependent. This frequency-dependent characteristic is attributed to the interaction between the wave propagating from other neighboring piles and the pile hit by the wave.

### 2.2 *Dynamic pile-group stiffness*

The dynamic stiffness of a pile group can be derived from the total effect of single pile forming the pile group. The derivation presented here follows the mechanical model and is based on the assumptions made by (Makris et al. 1994). Let $x_j$ be the lateral displacement of pile "$j$" in a group of $N$ piles. Taking into account the pile-to-pile interaction, the resulting lateral displacement "$x_{i,\mathrm{Tp}}$" of pile "$i$" is obtained as

$$x_{i,\mathrm{Tp}} = \sum_{j=1}^{N} \alpha_{ij} x_j \,; \qquad i = 1,\ldots,N \tag{5}$$

Note that $\alpha_{ii} = 1$. If $R_j$ is the force at the head of pile "$j$" and $k_\mathrm{p}$ is the lateral dynamic stiffness of a single pile, then eq. (5) becomes

$$x_{i,\mathrm{Tp}} = \sum_{j=1}^{N} \alpha_{ij} \frac{R_j}{k_\mathrm{P}}; \qquad i = 1,\ldots,N \tag{6}$$

When the pile cap is rigid, the lateral displacement of the pile cap "$x_\mathrm{C}$" is equal to that of each pile; i.e. $x_\mathrm{C} = x_{i,\mathrm{Tp}}$ ($i$= 1,…,$N$), one obtains

$$r_i = x_\mathrm{C} k_\mathrm{P} \sum_{j=1}^{N} \varepsilon_{ij} \qquad ; i = 1,\ldots,N \tag{7}$$

where $\varepsilon_{ij}$ is an element ($i,j$) in the inverse of matrix containing $\alpha_{ij}$. If $F_\mathrm{C}$ be the external force acting on the pile cap, the equilibrium condition yields

$$F_\mathrm{C} = \sum_{i=1}^{N} r_i \tag{8}$$

If the total reaction force from all piles is written as $x_\mathrm{C}k_\mathrm{g}$, where $k_\mathrm{g}$ denotes the dynamic stiffness of pile group, it can be shown that

$$k_\mathrm{g} = k_\mathrm{p} \sum_{i=1}^{N} \sum_{j=1}^{N} \varepsilon_{ij} \tag{9}$$

The dynamic stiffness of pile group is also frequency-dependent because it is the function of dynamic interaction factor.

## 3 SOIL-PILE-STRUCTURE SYSTEM UNDER EARTHQUAKE EXCITATION

A mechanical model of a soil-pile-structure system subjected to base excitation is shown in Figure 1. The structure is assumed linear. The equation of motion is, thus,

$$\boldsymbol{m}\ddot{\boldsymbol{X}} + \boldsymbol{c}\dot{\boldsymbol{X}} + \boldsymbol{k}(\omega)\boldsymbol{X} = -\ddot{X}_e \boldsymbol{m}\boldsymbol{\Lambda} \tag{10}$$

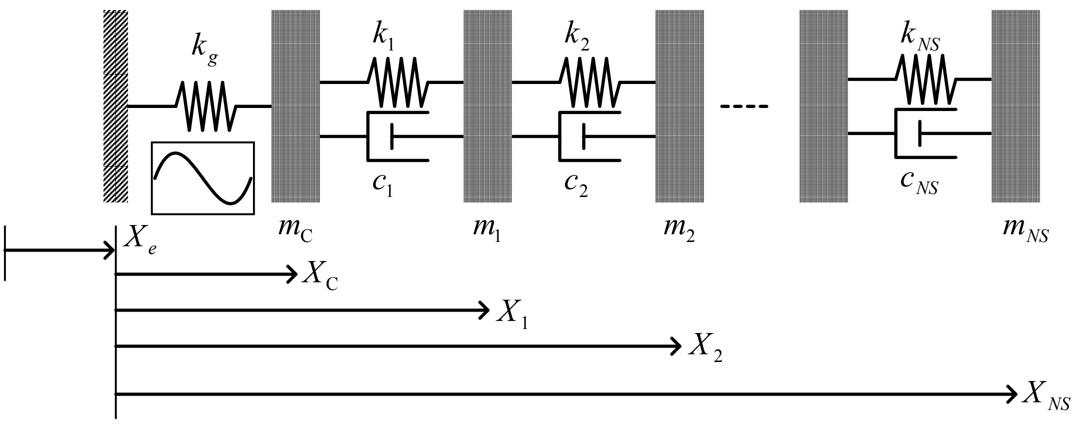


Figure 1. Soil-pile-structure system subjected to base excitation.

in which

$$\boldsymbol{m} = \text{diag}\begin{bmatrix} m_1 & m_2 & \cdots & m_{\text{NS}} & m_{\text{C}} \end{bmatrix} \quad \text{: mass matrix} \tag{11.1}$$

$$\boldsymbol{c} = \begin{bmatrix} c_1 & -c_1 & 0 & 0 & 0 \\ -c_1 & c_1 + c_2 & -c_2 & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & -c_{\text{NS}-1} & c_{\text{NS}-1} + c_{\text{NS}} & -c_{\text{NS}} \\ 0 & 0 & 0 & -c_{\text{NS}} & c_{\text{NS}} \end{bmatrix} \quad \text{: damping matrix} \tag{11.2}$$

$$\boldsymbol{k}(\omega) = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & -k_{\text{NS}-1} & k_{\text{NS}-1} + k_{\text{NS}} & -k_{\text{NS}} \\ 0 & 0 & 0 & -k_{\text{NS}} & k_{\text{NS}} + k_{\text{g}}(\omega) \end{bmatrix} \quad \text{: stiffness matrix} \tag{11.3}$$

$$\boldsymbol{X} = \begin{bmatrix} X_1 & X_2 & \ldots & X_{\text{NS}} & X_{\text{C}} \end{bmatrix}^T \tag{11.4}$$

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 1 & \ldots & 1 & 1 \end{bmatrix}^T \tag{11.5}$$

In eq. (10), $\ddot{X}_e$ is the ground acceleration at the end of the bearing piles and *NS* is the total number of degree-of-freedom of structure. Subscript C denotes the pile cap. $\boldsymbol{X}$ is the displacement vector of each structure mass and pile cap, measured relative to the base displacement.

The solution procedures for equations with frequency-dependent parameters in general cases combine the approaches used in time domain and frequency domain together. Among such procedures, the so-called hybrid frequency-time domain (HFTD) procedure has been successfully applied to various types of problem, e.g. (Darbre & Wolf 1988) and (Chávez & Fenves 1995). The HFTD procedure comprises of the following steps:

1. The pseudo-forces at the *i*th iteration are computed. The pseudo-force is defined as the difference between the non-linear and linear internal forces using the responses at the (*i-1*)th iteration.
2. The *i*th-iteration pseudo-loads resulting from the pseudo-forces are calculated and then transformed to frequency domain.
3. The system of equations is solved in the frequency domain.
4. The displacements in frequency domain are transformed to the time domain.
5. The convergence criteria are checked. If the convergence is not achieved, return to step 1. The new responses are used in the calculation of the pseudo-forces.

It should be noted that the total duration of response is divided into segments of time and the solution is obtained for a segment at a time. After the convergence has been achieved, the solution proceeds to the next segment. The updating of the computed responses is done only in the present segment, not in the previous segments where convergence is reached already.

In the present study, the computation of pseudo-force is not necessary because the internal-force terms are in a linear form. To solve eq. (10) in the frequency domain, Fourier transform is applied to eq. (10), which yields

$$\left[\left(\boldsymbol{k} - \omega^2 \boldsymbol{m}\right) + i\omega \boldsymbol{c}\right]\hat{\boldsymbol{X}}(\omega) = -\ddot{X}_e(\omega)\boldsymbol{m}\boldsymbol{\Lambda}\,;\; i = \sqrt{-1} \tag{12}$$

The solution in the frequency domain is obtained as

$$\hat{\boldsymbol{X}}(\omega) = -\hat{\ddot{X}}_e(\omega)\boldsymbol{H}(\omega)\boldsymbol{m\Lambda} \tag{13}$$

Applying the inverse Fourier transform to eq. (13), the solution in the time domain is written as

$$\boldsymbol{X}(t) = IFT\left(-\hat{\ddot{X}}_e(\omega)\boldsymbol{H}(\omega)\boldsymbol{m\Lambda}\right) \tag{14}$$

where $IFT(\cdot)$ is the inverse Fourier transform operator. Since there is no computation of pseudo-force, therefore there is no trial-error on pseudo-force or response. Consequently, the computed responses as shown in eq. (14) are the final solution.

## 4 RELIABILITY ANALYSIS OF SOIL-PILE-STRUCTURE SYSTEM

### 4.1 *Stochastic earthquake excitation*

In reliability analysis, the description of uncertainty has to be first identified. The uncertainty can be attributed to the variability nature in the values of system parameters or earthquake excitation. To account for the seismic risk from the variability in earthquake excitation, only the ground acceleration at the end of bearing piles is thus considered as a stochastic quantity in this study. The ground acceleration is modeled as a non-stationary white noise process, obtained from a filtered white noise process with a modulation function. The filter employs the modified Tanai-Kajimi model; (Kanai 1957) and (Tajimi 1960), which has the filter equation as (Schuëller et al. 1998)

$$\begin{bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \\ \dot{Y}_3 \\ \dot{Y}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\Omega_{1g}^2 & -2\zeta_{1g}\Omega_{1g} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \Omega_{1g}^2 & 2\zeta_{1g}\Omega_{1g} & -\Omega_{2g}^2 & -2\zeta_{2g}\Omega_{2g} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} + \begin{bmatrix} 0 \\ W(t) \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

where $W(t)$ is a stationary Gaussian white noise process with 2-side power spectral $G_0$. The ground acceleration $\ddot{X}_e$ is defined as

$$\ddot{X}_e(t) = g(t)[\Omega_{1g}^2 Y_1(t) + 2\zeta_{1g}\Omega_{1g}Y_2(t) - \Omega_{2g}^2 Y_3(t) - 2\zeta_{2g}\Omega_{2g}Y_4(t)] \tag{16}$$

in which $g(t)$ is a temporal modulation function.

### 4.2 *Limit State Function*

A limit state function in terms of displacement response is established as

$$g(X_{\max}, x_{\text{cri}}) = x_{\text{cri}} - X_{\max} \tag{17}$$

in which $x_{\text{cri}}$ is the critical displacement and $X_{\max}$ is defined as

$$X_{\max} = \max_{t\in[0,T]} |X(t)| \tag{18}$$

where $X(t)$ is the displacement at time $t$ and $T$ is the total time duration of the response history.

## 4.3 *Computation of Failure Probability*

The probability density function of $X_{max}$ cannot be determined from analytical procedures. As pointed out by (Schuëller et al. 1998), Mote Carlo simulation is the most generally applicable tool for problems in stochastic dynamics. Therefore, Monte Carlo simulation will be utilized here.

A specific number of ground acceleration histories is realized and input to eq. (10). The response history $X(t)_j$ for the $j$th realization of ground acceleration history $\ddot{X}_e(t)_j$ is computed according to the procedures in section 3 above. The probability density function of $X_{max}$ is then estimated from the ensemble of $X_{max,\ j}$. Based on the obtained probability density function, the probability of failure is computed.

# 5 NUMERICAL EXAMPLE

## 5.1 *Example Description*

An 8-story and 3-bay building subjected to earthquake is considered. The values of all parameters in building and foundation model are given in Table 1 and Table 2, respectively. The pile foundation used for the building is illustrated in Figure 2. The exemplified structure and foundation is of typical configurations, which can be found in general residential buildings.

Table 1. Parameters in building model.

| i | $m_i$ (kg) | $c_i$ (Ns/m) | $k_i$ (N/m) |
|---|---|---|---|
| 1 | $48.6 \times 10^3$ | $66.1 \times 10^5$ | $916 \times 10^5$ |
| 2,…, 8 | $58.7 \times 10^3$ | $72.7 \times 10^5$ | $916 \times 10^5$ |

Table 2. Parameters in foundation model.

| Parameter | (unit) | Value |
|---|---|---|
| $\rho_s$ | (kg/m$^3$) | 1600 |
| $V_s$ | (m/s) | 100 |
| $V_{La}$ | (m/s) | 217 |
| $\nu_s$ | | 0.500 |
| $\beta_s$ | | 0.600 |
| $E_s$ | (N/m$^2$) | $648 \times 10^6$ |
| $G_s$ | (N/m$^2$) | $216 \times 10^6$ |
| $k_x(\omega)$ | (N/m) | $778 \times 10^3$ |
| $m_x$ | (kg/m) | 116 |
| $E_p$ | (N/m$^2$) | $2.57 \times 10^9$ |
| $I_p$ | (m$^4$) | $1.86 \times 10^{-4}$ |
| $k_x^{[1]}$ | (N/m) | $229 \times 10^3$ |

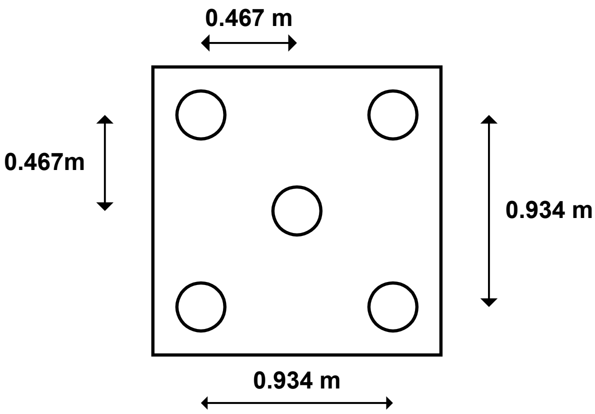


Figure 2. Plan of pile foundation.

To account for the non-stationary characteristics of earthquake excitation, the temporal modulation $g(t)$ as proposed by (Amin & Ang 1968) is applied

$$g(t) = \begin{cases} 0 & t \le 0 \\ (t/t_1)^2 & 0 < t \le t_1 \\ 1 & t_1 < t \le t_2 \\ exp(-c_\varphi(t - t_2)) & t > t_2 \end{cases} \tag{19}$$

in which $t_1$, $t_2$, and $c_\varphi$ are the model parameters. All parameters of earthquake model are shown in Table 3.

Table 3. Parameters in earthquake model.

| Parameter (unit) | Value |
|---|---|
| $G_o$ | 1.00 x $10^{-2}$ |
| $\zeta_{1g}$ | 0.600 |
| $\Omega_{1g}$ (rad/s) | 9.26 |
| $\zeta_{2g}$ | 0.313 |
| $\Omega_{2g}$ (rad/s) | 19.0 |
| $t_1$ (s) | 5.00 |
| $t_2$ (s) | 12.0 |
| $c_\varphi$ | 0.286 |

The limit state function in this example considers the excessive response of the first floor relative to the foundation. Consequently, the limit state function reads

$$g(X_{R1,max}, x_{R1,cri}) = x_{R1,cri} - X_{R1,max} \tag{19}$$

where $x_{R1,cri}$ and $X_{R1,max}$ are the critical and maximum relative displacements of the first floor to the foundation, respectively.

There have been numerous studies on the analysis of soil-pile-interaction, e.g. see (Yang 1982), but these studies have not considered the pile-to-pile interaction. It is, therefore, instructive to learn the effects of pile-to-pile interaction on the safety level of structures. For this purpose, two cases will be considered, i.e. cases with and without pile-to-pile interaction.

5.2 *Numerical Results and Discussion*

The numerical results are obtained from using Monte Carlo simulation with a sample size of 3000 in both cases. The ensemble of $X_{max}$ is obtained from the ensemble of $X(t)$. The failure probabilities $P_f$'s for different values of $x_{R1,cri}$'s in both cases are plotted and compared in Figure 3.

It can be seen from Figure 3 that there is no observable difference in the failure probabilities between both cases. Thus, for the foundations with similar configurations to the study case, the inclusion of soil-pile interaction in the analysis may be unnecessary.

## 6 CONCLUDING REMARKS

Reliability analysis of structure subjected to earthquake, including dynamic soil-pile-structure interaction, is described. The effect of pile-to-pile interaction on the response and probability of failure is accounted. It has been shown through a case study that such an interaction can be neglected in the analysis of building structures with typical foundations. Consequently, the reliability analysis of structures considering dynamic soil-pile-structure interaction is expected to be more efficient.

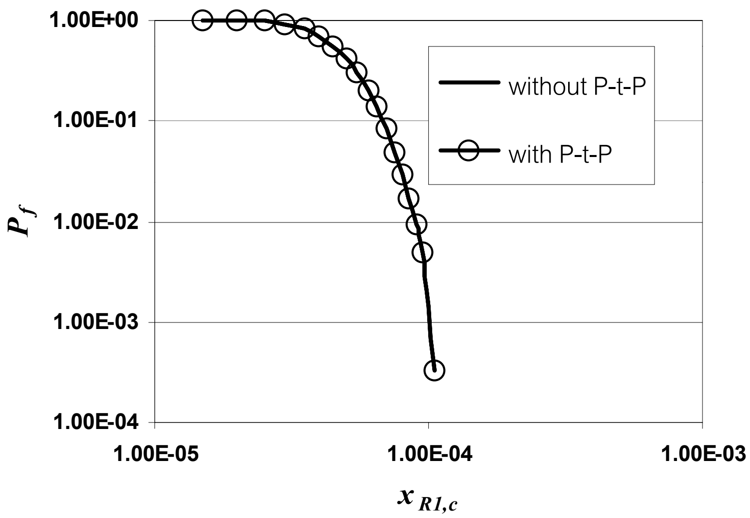


Figure 3. Probability of failure.

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