How Far to Where?

Activity Involving an Application of the Ambiguous Case of the Law of Sines

Interface 2008
Tan-Tar-A Resort
Osage Beach, Missouri

February 26, 2008

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Ambiguous Map Lab

(Law of Sines)

- On the map you are given, find two cities with identical names (cities 1 and 2).

- Use a ruler to draw a line through the two cities. Locate another city on the same line (city A). If possible, locate city A not between the two cities; the problem will still work, though, as long as A is not at the midpoint between 1 and 2.

- Construct the perpendicular bisector of the segment between the two identically named cities. Locate a city on the perpendicular bisector (city B).

- Use a ruler to draw a line connecting cities A and B. Find the distance between those cities by measuring and using the map's scale.

- Verify by measuring that cities 1 and 2 are approximately the same distance from city B. (*Remember a theorem from geometry which says that any point on the perpendicular bisector of a segment is equidistant from its endpoints.)* Use the map's scale to compute that distance.

- On a piece of patty paper draw perpendicular axes to represent north-south-east-west lines. Place the origin of the axes on city A and find angle bearings to cities B and to either city 1 or city 2.

- Using the measurements you have calculated, create a problem asking for the distance from city A to city 1 (or 2). This is an ambiguous question with two unique solutions because two cities have that same name.

- Now solve your own problem – giving both solutions. First use the Law of Sines. Then verify your results by using ruler measurements and the map's scale.

- Turn in one page with only the statement of your problem. Also turn in your solution and all work you've done in this activity. You will exchange your problem with other students and check each other's work.

In order to do this project, you need a map which has two cities with a common name, either in the same state or across state lines. The maps used in this presentation are portions of an Arkansas highway map.

The original idea for the Ambiguous Map Lab came from:

Jay Yohe and Beth Kluz
Susquehanna Township High School
Harrisburg, Pennsylvania

http://www.susq-town.org/yohe/Ambiguous/index.htm#Objectives

Below is a grading rubric which I copied from the website. You could adapt the rubric to your particular problem. For example, I have my students do the measurements in miles rather than in kilometers.

Points Issued	Criteria
20 points	All steps are shown on a map for the creation/construction of the problem. The problem is stated, the solution is provided and the answer is accurate within 5 kilometers.
15 points	All steps are shown on a map for the creation/construction of the problem. The problem is stated, the solution is provided but the answer has an error of greater than 5 kilometers but less than 10 kilometers.
10 points	Most steps are shown on a map for the creation/construction of the problem and an attempt at stating and solving the problem were made, but there were a few errors that led to an error in calculations.
5 points	The process for creating the problem is incomplete, and the statement of the problem along with the solution is missing details. There are major errors in the calculations.
0 points	Virtually no effort was placed on solving the Ambiguous Map problem.

The demonstration applet for construction of a perpendicular bisector came from Math Open Reference -- http://www.mathopenref.com/index.html.
This site is a great online resource for demonstrations of geometry concepts, including several other constructions.

How Far to Where?

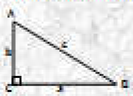
Ambiguous Map Problem

(application of the Ambiguous Case of the Law of Sines)

Students should have done some work with the Ambiguous Case of the Law of Sines before doing this project.

Review of Ambiguous Case of the Law of Sines

To solve a right triangle, you can use the Pythagorean Theorem and/or basic trig ratios.



$a^2 + b^2 = c^2$

$\sin A = \frac{a}{c}$ $\csc A = \frac{c}{a}$

$\cos A = \frac{b}{c}$ $\sec A = \frac{c}{b}$

$\tan A = \frac{a}{b}$ $\cot A = \frac{b}{a}$

In an oblique triangle (one in which there are no right angles) neither of those methods will work.

Solving an oblique triangle requires using either the Law of Sines or the Law of Cosines, depending on which information you are given.

Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

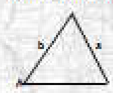
The Law of Sines can be used when you are given two angles and one side of an oblique triangle (ASA or AAS).

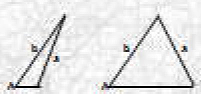
The Law of Sines can also be used to solve an oblique triangle when you are given two sides and the angle opposite one of the sides (SSA).

Suppose you are given the measures shown below for two sides and an angle of an oblique triangle.

The pieces could be put together to form this triangle.

But they could also be put together to form this triangle.



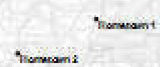


Since SSA does not always define a unique triangle, this is called the **Ambiguous Case** of the Law of Sines.

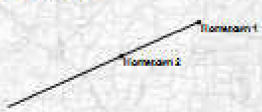
We are going to create a map problem involving the ambiguous case where two unique triangles are possible with the same given measures.

Students could work on this project individually or in pairs.

On the map you are given, find two cities with identical names (1 and 2).

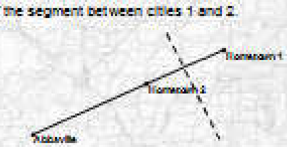


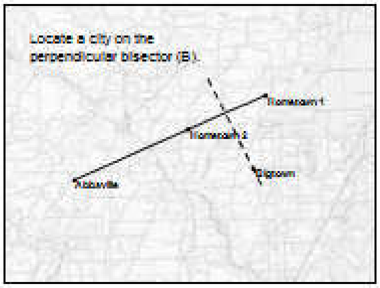
Use a ruler to draw a line through the two cities.

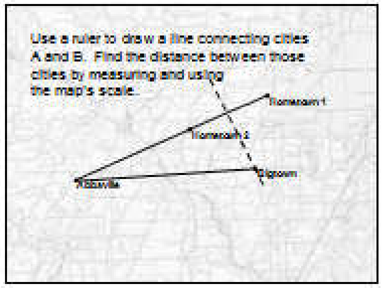


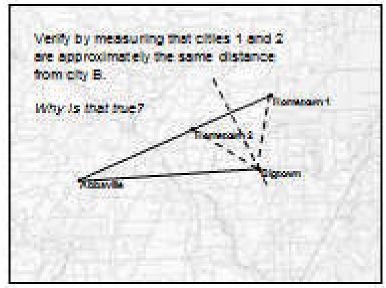
Locate another city on the same line – preferably not between the two cities (A).

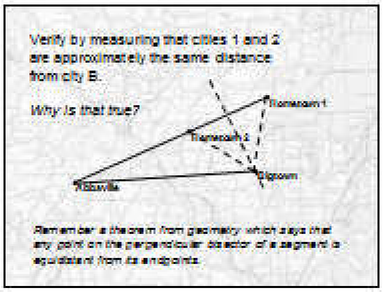
Construct the perpendicular bisector of the segment between cities 1 and 2.

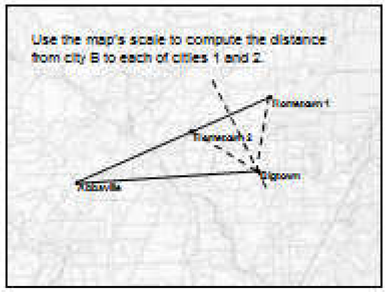


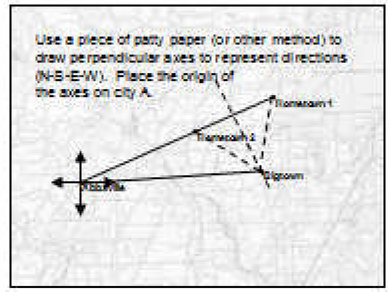


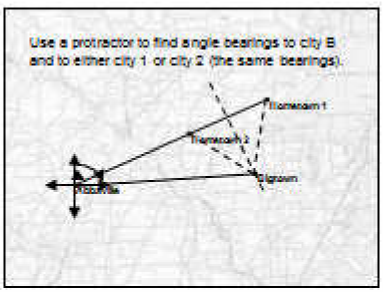


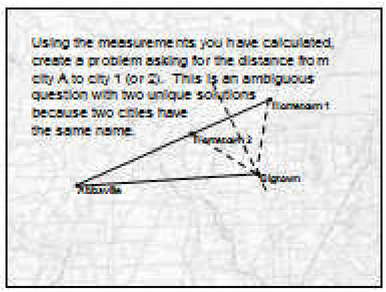












State your problem – in a creative way if you can. Be sure you include the basic facts, something like the following.

Hometown is 50 degrees east of north of Abbaville.
Bigtown is 86 degrees east of north of Abbaville.
Bigtown is 48 miles from Abbaville.
Hometown is 20 miles from Bigtown.

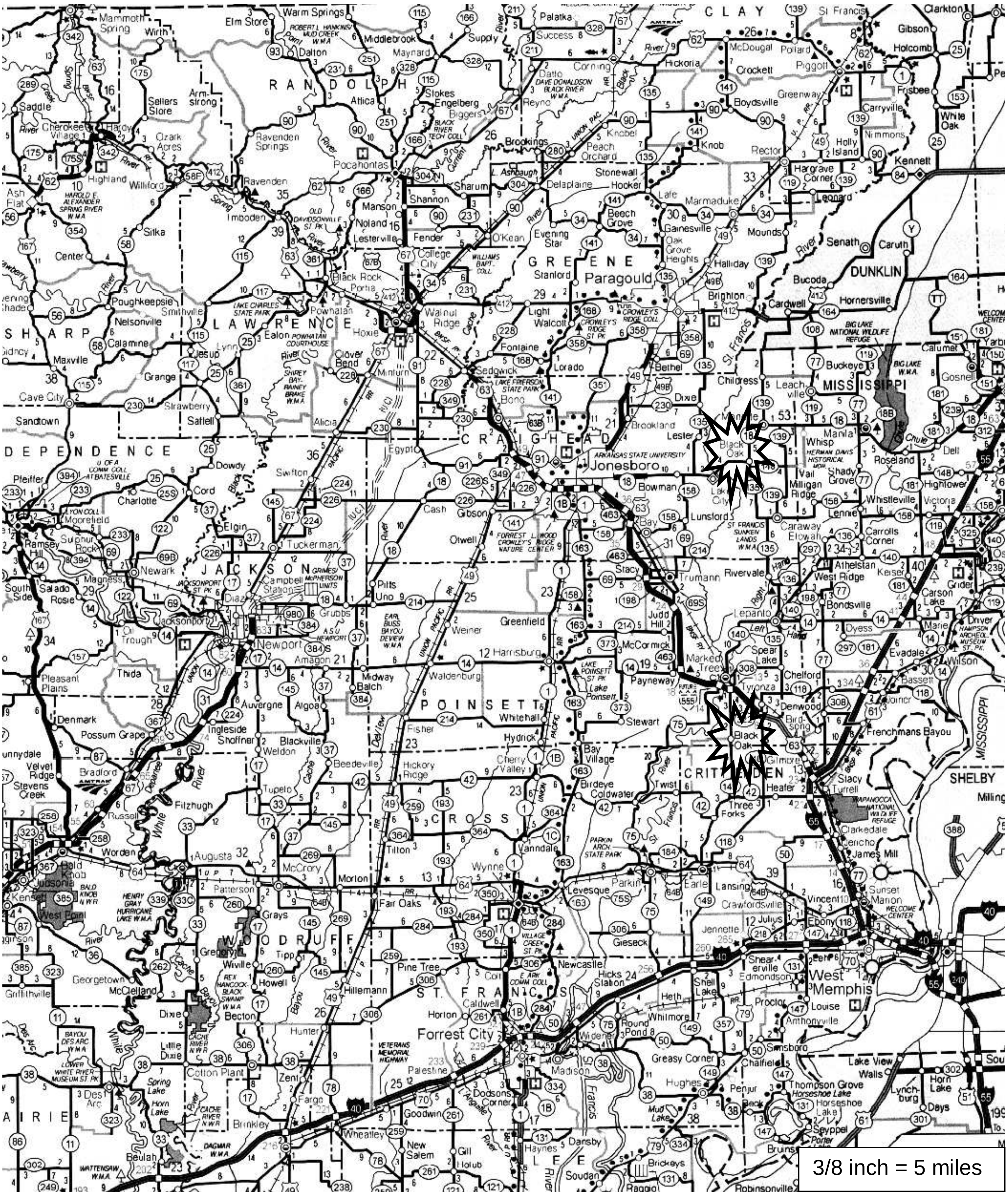
How far is Abbaville from Hometown?

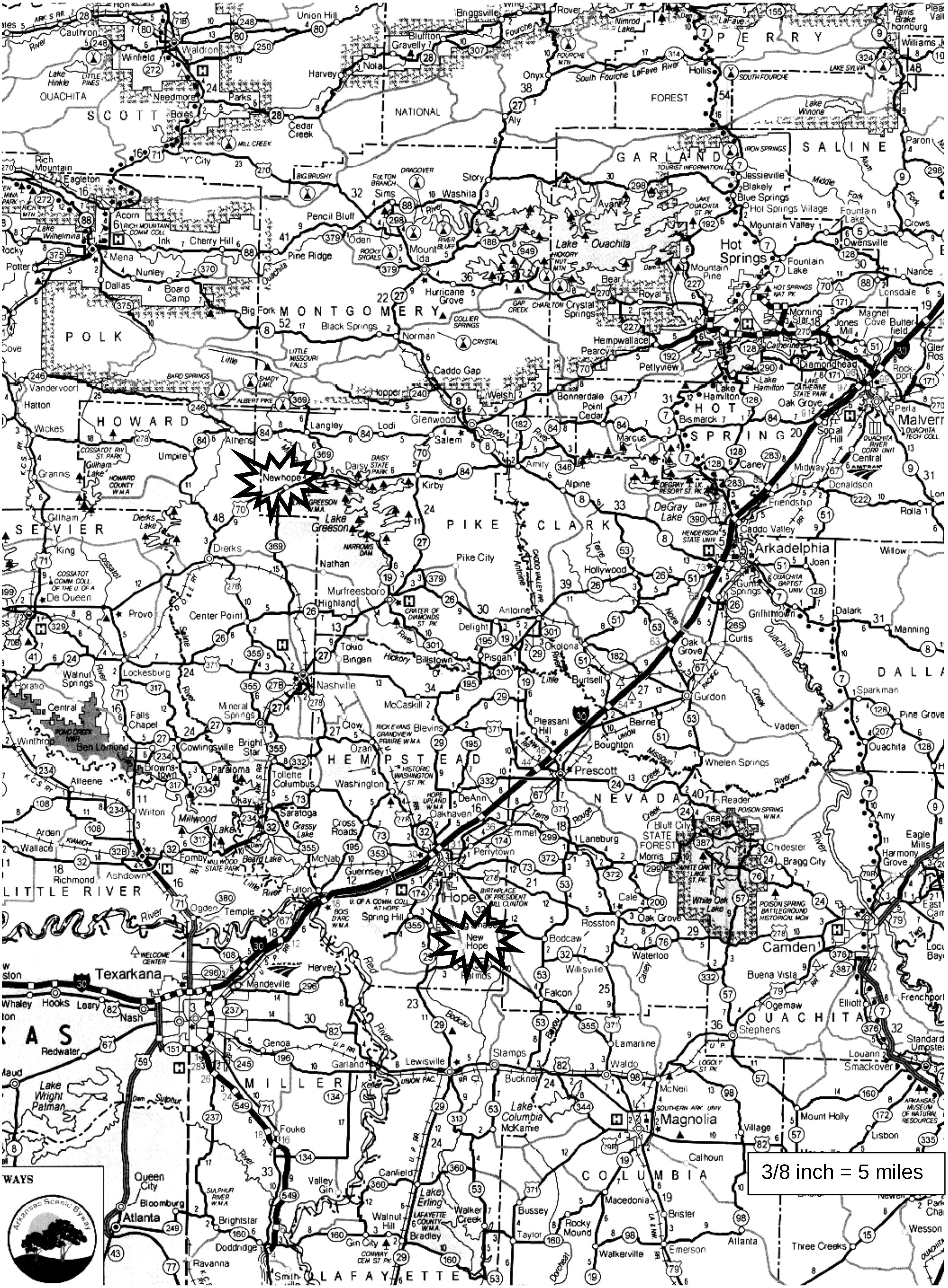
Remember that there are two solutions to your question because there are two cities with the same name (Hometown in my example).

Work out both solutions, using the Law of Sines. Then check your solutions with accurate ruler measures and your map's scale.

I would have each student (or pair of students) turn in all work they had done on the activity.

I would also have them turn in one sheet of paper that contains only the statement of their problem. This would be used to exchange problems with others in the class, giving students more practice in solving problems with the Law of Sines and to check accuracy and clear statement of their problems.


CLAY
RANDOLPH
LAWRENCE
GREENE
CRAIGHEAD
JACKSON
POINSETT
MISSISSIPPI
CRITTENDEN
CROSS
WOODRUFF
ST. FRANCIS
LEE
DUNKLIN
SHELBY
Black Oak
Black Oak
3/8 inch = 5 miles


3/8 inch = 5 miles
Newhope
New Hope
SCOTT
POLK
MONTGOMERY
GARLAND
SALINE
PERRY
HOWARD
PIKE
CLARK
HOT SPRING
SEVIER
HEMPSTEAD
NEVADA
LITTLE RIVER
MILLER
COLUMBIA
LAFAYETTE
OUACHITA
DALLAS
NATIONAL
FOREST
Texarkana
Hope
Arkadelphia
Hot Springs
Magnolia
Camden
Prescott
Nashville
Murfreesboro
Glenwood
Mena
Waldron
De Queen
Malvern
Lake Ouachita
Lake Greeson
DeGray Lake
Millwood Lake
Arkansas Scenic Byway