BEARING CAPACITY – SHALLOW AND DEEP FOUNDATIONS

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1.0 INTRODUCTION

A foundation is defined as the supporting base of a structure which forms the interface across which the loads are transmitted to the underlying soil or rock. In most cases foundations in civil engineering are constructed of plain or reinforced concrete, notable exceptions being roads, embankments and dams.

Foundations are classified according to the depth of founding, D (depth of base of foundation below ground level) compared to the width of the foundation, B.

Shallow foundations: are placed at shallow depths i.e. D< B or where D is less than about 3m (i.e. within reach of normal excavation plant).

Deep foundations: are placed at greater depths i.e. D>3m or D>B.

Pile foundations: transmit the loads to greater depths through steel or reinforced concrete columns (see separate note set)

2.0 SHALLOW FOUNDATIONS (D<B or D<3m) NOTE: only a ‘general guide’

2.1 Design criteria

i) Adequate depth
The depth of founding, D, must be sufficient to prevent conditions on the ground surface from affecting the foundation e.g. climatic changes – rainfall, temperature, freezing/ thawing.

Also the depth must be adequate to resist:
- Changes resulting in ground water level movement and
- The effects of horizontal loads and overturning moments.

ii) Stability of the supported structure
Failure of a foundation could occur due to inadequate bearing capacity of the soil beneath the foundation (leading to shear failure), overturning or sliding of the foundation.

iii) Limiting settlement
The acceptable settlement of a structure must not be exceeded by the settlement of its foundation. Types of settlement:

- Immediate (undrained settlement)
  Occurs during construction as dead and structural loading is imposed.

- Consolidation (time dependent)
  Settlement (+ differential settlement) develops during the life of a structure.

2.2 Spreading load

Shallow foundations convert a localised load (e.g. a point load transmitted by a column or a line load, such as cavity wall brick construction) into a pressure by spreading the load over the area of a foundation.
Thus for a column (point) load, P kN acting on a rectangular pad foundation, length, L by width, B the bearing pressure, q is given by;

\[ q = \frac{P}{L \times B} \text{ kN/m}^2 \]

For a line load of S kN/lin m, acting on a strip foundation width, B the applied pressure, q is given by;

\[ q = \frac{S}{B \times 1\text{m}} \text{ kN/m}^2 \]

2.3 Types of foundations

The shape and arrangement of foundations are determined by the type and location of loads from a structure, e.g. strip foundations are placed beneath a cavity wall. Reinforced concrete is the usual construction material.

Strip Foundations

*Plain concrete* is normally adequate for lightly load cavity wall used in housing. Note: To avoid tensile forces in the concrete, thickness \( T = \text{projection}\ P \).

*Reinforced concrete* enables wide foundations (which distribute high loads) to be placed at shallow depth. Bending stresses are resisted by transverse reinforcement.

*Deep strip* foundations using plain concrete take wall loads through a weak surface layer to stronger soil at depth. However, these foundations require brickwork to be constructed in a confined (narrow) space – which is expensive to complete.
Trench fill is an alternative to deep strip foundations. Concrete is placed in the trench when a suitable bearing stratum is reached. Although brickwork commences close to ground level, there is a considerable cost in concrete.

Pad Foundations

Plain concrete is only an economic option where the loading is relatively light as T must equal P otherwise excessively thick pads are needed which is not economic.

Reinforced concrete enables relatively wide but shallow foundations, often designed to be square plan area to make the reinforcing cage easier to construct and place. Rectangular pads are used for eccentric/inclined loading (longer dimension parallel to direction of inclination/eccentricity)

Combined pad foundations are adopted close to a site boundary to enable the balancing effect of an internal column to be incorporated.

Continuous pad exists when pads and the columns they support are fairly closely spaced. Extending the reinforcing between pads ensures longitudinal stiffness (resists differential settlement)
Pad and ground beam: here smaller isolated pads are connected by ground beams to provide structural rigidity.

2.4 Ground failure modes
Shear failure is defined as when the soil divides into separate blocks or zones, which move along slip surfaces. Three principal modes of shear failure may be defined:

a) General shear failure
A continuous slip surface occurs up to ground level. Soil above failure surface in state of plastic equilibrium, with heaving on either side. Failure is sudden and catastrophic and accompanied by tilting of the footing, see curve ‘a’ below.

Associated with low compressibility soils e.g. dense sands or stiff over-consolidated clays.

b) Local shear failure
Significant compression under footing causes only a partial development of plastic equilibrium. Failure surface is not continuous. Some minor heaving at ground level but no catastrophic failure, see curve ‘b’ below.

Occurs in moderately compressible soils – medium dense/compact sands.

c) Punching shear failure
Slip surfaces almost vertical, large vertical displacements. No heaving, tilting or catastrophic failure. Compression increases the density of the soil, see curve ‘c’.

Weak, highly compressible soils – loose sands, partially saturated clays, peats.
2.5 Definitions of bearing capacity

**Concepts of bearing capacity**

**Ultimate Bearing Capacity**, $q_{ult}$ (alternatively $q_f$, for failure)

Is the intensity of bearing pressure at which the supporting ground is expected to fail in shear, i.e. a building will collapse.

[ In Eurocode 7 the equivalent value is defined as the *ultimate limit state design vertical capacitance*, $Q_d$ and is expressed as a load (force) and not as a pressure or stress].

**Safe bearing capacity**, $q_s$

$$q_s = \frac{q_{ult}}{F}$$

Where $F =$ factor of safety (normally 3.0) [prior to EC7]

Vesic, 1975, suggested minimum factors of safety for shallow foundations which take into account the extent of the site investigation, likelihood of maximum design load and the consequences of failure, as shown below;
### Table: Characteristics and Extent of Site Investigation

<table>
<thead>
<tr>
<th>Category</th>
<th>Characteristics of category</th>
<th>Extent of site investigation</th>
<th>Typical structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Maximum design load: <em>likely to occur often.</em> Consequences of failure: <em>disastrous.</em></td>
<td>Thorough: 3.0, Limited: 4.0</td>
<td>Railway bridges, Warehouses, Blast furnaces, Reservoir embankments, Retaining walls / silos</td>
</tr>
<tr>
<td>B</td>
<td>Maximum design load: <em>may occur occasionally.</em> Consequences of failure: <em>serious.</em></td>
<td>Thorough: 2.5, Limited: 3.5</td>
<td>Highway bridges, Light industrial, Public buildings</td>
</tr>
<tr>
<td>C</td>
<td>Maximum design load: <em>unlikely to occur.</em></td>
<td>Thorough: 2.0, Limited: 3.0</td>
<td>Apartments, Office buildings</td>
</tr>
</tbody>
</table>

**NOTE:** Under EC7 it is very unlikely to simply adopt an 'engulfing' value for F

---

**Allowable bearing capacity, \( q_a \)**

Is the bearing pressure that will cause acceptable settlement of the structure, i.e. if settlement is excessive the safe bearing capacity value will need to be reduced (by increasing F until settlement is acceptable). Settlement may be either long term consolidation (clays) or immediate (sands and gravels) Thus any foundation design must include:

1. Bearing capacity analysis
2. Settlement analysis – varies according to type of structure / nature of soils

---

### 2.6 Ultimate Bearing Capacity

The development of an expression to determine the ultimate bearing capacity concerns an infinitely long strip foundation, placed on the surface of a thick layer of soil, see diagram below. The overburden pressure due to an actual founding depth \( D_r \) is considered as a surcharge \( \sigma_0 = \gamma D \) and any soil strength above this level is not taken into account.

As the ultimate bearing capacity is reached, a wedge of soil AFB is displaced downwards and the adjacent sectors (AFE and BGF) are forced sideways and zones CEA and BGD are displaced upwards.
Terzaghi (1943) derived the following equation for an infinitely long strip foundation of width \(B\):

\[
q_{ult} = cN_c + \sigma_0 N_q + 0.5B\gamma N_\gamma
\]

or

\[
q_{ult} = cN_c + \gamma D N_q + 0.5B\gamma N_\gamma
\]

where:
- \(c\) = cohesive strength of soil
- \(\sigma_0\) = overburden pressure = \(\gamma D\)
- \(B\) = width of foundation
- \(\gamma\) = unit weight of soil
- \(N_c\) = bearing capacity factor (cohesion)
- \(N_q\) = bearing capacity factor (surcharge and friction)
- \(N_\gamma\) = bearing capacity factor (self weight and friction),
  (bearing capacity factors are related to \(\phi\) value only)

Values of the bearing capacity factors have been obtained by several authors (e.g. Prandtl, Reissner and Brinch Hansen) by adopting different rupture figures and are usually presented in tabular form (see next page).

**NOTE:** For saturated clay soils the initial loading presents undrained conditions in which \(\phi = 0^\circ\) and from the table \(N_c = 5.14, N_\gamma = 0\) and \(N_q = 1\), so the above equation simplifies;

\[
q_{ult} = c_u 5.14 + \gamma D
\]
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<th>$N_c$</th>
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<td>266.9</td>
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</tr>
</tbody>
</table>

**Bearing Capacity Factors**

(Prandtl, Reissner, Brinch Hansen)
2.6.1 Net Bearing Capacity, $q_n$ or $q_{net}$

Soil excavated to depth, $D$ in order to construct a foundation, causes a relief in vertical stress of $\gamma D$. If the excavation is subsequently backfilled the overburden pressure, $\sigma_o$, is restored and net bearing capacity applies.

For example, net ultimate bearing capacity, $q_{n,ult}$, is the net change in total stress experienced by the soil at the base of the foundation i.e.;

$$q_{n,ult} = (\text{total applied stress}) - (\text{stress removed due to the excavation})$$

$$q_{ult} = q_{ult} - \sigma_o$$

$$q_{ult} = q_{ult} - \gamma D$$

The factor of safety, $F$ must be applied to the net and not the gross ultimate bearing capacity;

$$q_s = \frac{q_{ult} - \gamma D}{F} + \gamma D$$

In terms of effective stress (eg. For a foundation placed in free draining sand when the W.T. is above the foundation base):

$$q_{n}' = q_f - \sigma_o'$$

where:

$$\sigma_o' = \gamma D - \gamma_w h_w$$

$$h_w = \text{height of water table above the foundation base}$$

Class example 1

A strip foundation is required to support a load of 600kN/m run at a depth of 1.2m in a soil with the following properties;

$$\gamma = 18.0 \text{ kN/m}^3$$

$$c = 15.0 \text{ kN/m}^2$$

$$\phi = 32^\circ$$

Determine the required width of the foundation using a factor of safety of 3 against bearing capacity failure (net safe bearing capacity).

$$[1.36m]$$
In the above analysis, the foundation was assumed to be infinitely long. In order to use the expression for practical design purposes, three shape categories are defined:

- strip
- rectangular
- circular \( \equiv \) square

Consequently the Terzaghi equation changes to:

\[
q_{\text{ult}} = cNc_sc + \gamma DN_q s_q + 0.5\gamma BN_s s_{\gamma}
\]

And in terms of net ultimate bearing capacity, \( q_{\text{n ult}} \):

\[
q_{\text{n ult}} = cNc_sc + \gamma DN_q s_q + 0.5\gamma BN_s s_{\gamma} - D
\]

where \( s_c \), \( s_q \) and \( s_{\gamma} \) are shape factors (Vesic, 1975):

<table>
<thead>
<tr>
<th>Shape of footing</th>
<th>( s_c )</th>
<th>( s_q )</th>
<th>( s_{\gamma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Rectangle</td>
<td>1.0 + (B/L)(N_q/N_c)</td>
<td>1.0 + (B/L)tan( \phi' )</td>
<td>1.0 - (B/L)0.4</td>
</tr>
<tr>
<td>Circle or square</td>
<td>1.0 + (N_q/N_c)</td>
<td>1.0 + tan( \phi' )</td>
<td>0.6</td>
</tr>
</tbody>
</table>

### 2.7 Effect of water table level

**Water table at depth >= \( B + D \)**

\[
q_{\text{ult}} = cNc_sc + \gamma DN_q s_q + 0.5\gamma BN_s s_{\gamma}
\]

**Water table at depth \( D \) to \( (D + B) \)**

\[
q_{\text{ult}} = cNc_sc + \gamma DN_q s_q + 0.5\gamma_{\text{sub}} BN_s s_{\gamma}
\]
Water table at depth < D

\[ q_{ult} = cN_cS_c + \gamma_{sub}D\gamma_{s}\gamma + 0.5\gamma_{sub}B\gamma_f + \gamma_wh_w \]

NOTE: Above the water table soil is partially saturated, so \( \gamma \) applies. Below the water table soil is fully saturated, so \( \gamma_{sat} \) applies.

Also submerged density, \( \gamma_{sub} = \gamma_{sat} - \gamma_w \)

In terms of effective stress, e.g. for a foundation placed in free draining sand when the W.T. is above the foundation base:

\[ q_{net\ ult} = q_{ult} - \sigma_0' = q_{ult} - (\gamma D - \gamma_w h_w) \]

where:

\[ h_w = \text{height of water table above the foundation base} \]

Note the factor of safety, \( F \), remains the same for the case of net bearing pressures;

\[ F = \frac{q_{net\ ult}}{q_{ns\ safe}} = \frac{(q_{ult} - \sigma_0')}{(\gamma_{sat} - \sigma_0')} \]

Thus the net safe bearing capacity, \( q_{ns} \):

\[ q_{net\ safe} = \frac{q_{ult} - \sigma_0'}{F} + \sigma_0' \]

**Class example 2**

A square foundation (4.5m x 4.5m) is founded at 2.4m in a soil with the following properties:

\[ \gamma = 17.6 \text{ kN/m}^3 \]
\[ \gamma_{sat} = 20.4 \text{ kN/m}^3 \]
\[ c = 32.0 \text{ kN/m}^2 \]
\[ \phi = 28^\circ \]

Using a factor of safety of 3 against bearing capacity failure, determine the net safe bearing capacity in kN/m² when:

a) The water table is level with the foundation base

b) Percentage reduction in bearing capacity when the water table rises to 0.5m below G.L.

[829.3 kN/m²]
2.8 Bearing capacity of cohesive soils

The ultimate bearing capacity of saturated cohesive soils (clay and silt) with low permeability is most critical immediately after construction, before the excess porewater pressure has had time to dissipate i.e. undrained conditions. As time proceeds, consolidation occurs, the soil becomes stiffer and has more strength. Therefore design of foundations on fine grained soils should be in terms of undrained or total stress.

Skempton (1951) suggested for an undrained saturated clay ($\phi_u = 0^\circ$), the basic Terzaghi equation should be used, but with values of $N_c$ related to the shape and depth of the foundation;

$$q_{ult} = cN_c + \gamma D N_q + 0.5\gamma B N_{\gamma}$$

Since when $\phi = 0^\circ$; $N_q = 1$ and $N_{\gamma} = 0$

Terzaghi becomes;

$$q_{ult} = c_u N_c + \gamma D$$

And in net terms;

$$q_{ult} = c_u N_c + \gamma D - \gamma D = c_u N_c$$

Values of $N_c$ can be found from the chart (Skempton, 1951) below:
Class example 3

A rectangular foundation, 2.5m wide and 3.5m long, is placed at 1.7m below G.L. in a thick deposit of firm saturated clay. The water table is steady at a depth of 2.5m. Determine the ultimate bearing capacity and the net safe bearing capacity for;

a) Immediately after construction  
\[ [491.6 \text{ kN/m}^2; 188.2 \text{ kN/m}^2] \]

b) 40 years after construction.  
\[ [853.1 \text{ kN/m}^2; 308.7 \text{ kN/m}^2] \]

Soil parameters:  
\[ c_u = 65.0 \text{ kN/m}^2; \]
\[ \phi_u = 0^\circ \]
\[ c' = 3.0 \text{ kN/m}^2; \]
\[ \phi' = 27^\circ \]
\[ \gamma = 21.5 \text{ kN/m}^3 \]

HINT: Use Skempton’s design chart for part (a) and Terzaghi for (b)

2.9 Bearing capacity of granular soil

In the case of sands, the settlement is almost immediate and an allowable or permissible settlement of 25mm is usually applied.
Foundation design uses the *allowable bearing capacity*, \( q_a \), which satisfies the settlement condition and provides values of the Factor of Safety greater than the normal 3.0 – 4.0.

Because of the difficulty and expense of undisturbed sampling and lack of uniformity of sand/gravel deposits, in-situ test results are used to determine the allowable bearing capacity and to make settlement predictions.

In-situ (on-site) tests:
1. Plate bearing test
2. Standard penetration test (SPT) – sand/gravel beds
3. Dutch cone test – not used much in UK due to stony nature of ground

The standard penetration test results, N values, are corrected to allow for;

- pore water pressure and
- overburden pressure

and are then used to find the allowable bearing capacity, \( q_a \), from a chart.

Correction factor, \( CN \), for overburden pressure – this accounts for the confining pressure at the depth at which the N value has been taken and is read off a graph (Peck, Hanson & Thornburn, 1974):
Now a revised value for the number of blows, \( N_{\text{rev}} = CN \times N \)

The effects of pore water pressure at the location of the test are considered by further correcting the \( N_{\text{rev}} \) value:

\[
N_{\text{corr}} = 15 + 0.5(N_{\text{rev}} - 15)
\]

This corrected, \( N_{\text{corr}} \), value is then used to find the allowable bearing pressure (=capacity) \( q_a \) from the chart below (Terzaghi, K & Peck, R B 1967, p.491):
The effect of the water table may be taken into account by applying the following correction:

\[ C_w = 0.5 \left\{ 1 + \frac{D_w}{(D + B)} \right\} \]

where

- \( D_w \) = depth of water table below ground level
- \( D \) = depth of foundation below ground level
- \( B \) = width of foundation

Thus, allowable bearing pressure, \( q_a \) derived from the previous graph becomes a corrected allowable bearing pressure, \( q_{a \, \text{corr}} \):

\[ q_{a \, \text{corr}} = C_w \, q_a \]

(from graph)
Class example 4
A 3.0m square foundation is located at a depth of 1.8m in uniform sand, ($\gamma = 18.6$ kN/m$^3$; $\gamma_{sat} = 20.5$ kN/m$^3$). A site investigation revealed SPT values between 1.8m and 4.8m below ground level as shown below. Initially the water table level is steady at 3.5 m below ground level.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>10</td>
</tr>
<tr>
<td>2.8</td>
<td>13</td>
</tr>
<tr>
<td>3.8</td>
<td>17</td>
</tr>
<tr>
<td>4.8</td>
<td>20</td>
</tr>
</tbody>
</table>

If the water table then rises to 0.9m below ground level, determine the percentage reduction in the allowable bearing capacity for a maximum settlement of 25mm.

[37%]

3.0 DEEP FOUNDATIONS (D>B or D>3m)

3.1 Design criteria

If the ultimate bearing capacity of a deep foundation is estimated using Terzaghi’s equation for shallow foundations, the ultimate bearing capacity will be grossly under-estimated.

Meyerhof produced equations for the ultimate bearing capacity for deep foundations:

**Strip Foundation**

$$q_{ult} = c N_c + P_o N_q + 0.5 \gamma B N_y$$

**Rectangular Foundation**

$$q_{ult} = c N_c(1 + 0.2 \frac{B}{L}) + P_o N_q + 0.5 \gamma B N_y(1 - 0.2 \frac{B}{L})$$

Where;

- $N_c$, $N_q$ and $N_y$ = bearing capacity factors (symbols same as for shallow foundations)
- $P_o$ = average lateral pressure acting against the vertical face of the foundation over the height “$h$” above the base of the plastic failure zone (see below).
Typical Failure Zones for a Deep Foundation

3.2 Bearing capacity of granular soils (c = 0)

Large settlements generally occur in cohesionless soils before the ultimate bearing capacity is reached.

This presents a significant limitation to the allowable bearing capacity of a deep foundation in granular soil.

3.3 Bearing capacity of cohesive soils (c and c,ϕ)

For a deep strip foundation in a cohesive soil,

\[ q_{ult} = cN_c + P_o N_q + 0.5γBN_f \]

For a given ϕ value, the bearing capacity factors \( N_c \), \( N_q \), and \( N_f \) can be obtained from the chart (Meyerhof deep strip) below;
The lateral earth pressure coefficient $K_s$ which acts in the plastic failure zone is approximated to $K_a$, the active earth pressure coefficient. Hence if $D$ is the depth from the ground surface to the foundation base, then the average lateral pressure, $P_o$ is:

$$P_o = K_a \gamma \left[ D - \frac{h}{2} \right]$$

where;

$$K_a = \frac{(1 - \sin \phi)}{(1 + \sin \phi)}$$

Values of $h$ for corresponding values of $\phi$ are tabulated below. Hence knowing $h$, $P_o$ can be found and therefore $q_{ult}$ can be evaluated.

and
\[
h = \frac{e^{\theta \tan \phi} B}{2 \sin \left\{ \left( \frac{\pi}{4} \right) - \left( \frac{\phi}{2} \right) \right\}}
\]

where \( \theta = \frac{5\pi}{4} - \frac{\phi}{2} \)

Values of \( h \) for corresponding values of \( \phi \) are tabulated below. Hence knowing \( h \), \( P_s \) can be found and therefore \( q_f \) can be evaluated.

<table>
<thead>
<tr>
<th>( \phi^\circ )</th>
<th>( h )</th>
<th>( \phi^\circ )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.764B</td>
<td>21</td>
<td>3.715B</td>
</tr>
<tr>
<td>2</td>
<td>0.825B</td>
<td>22</td>
<td>4.044B</td>
</tr>
<tr>
<td>3</td>
<td>0.891B</td>
<td>23</td>
<td>4.406B</td>
</tr>
<tr>
<td>4</td>
<td>0.962B</td>
<td>24</td>
<td>4.805B</td>
</tr>
<tr>
<td>5</td>
<td>1.040B</td>
<td>25</td>
<td>5.246B</td>
</tr>
<tr>
<td>6</td>
<td>1.123B</td>
<td>26</td>
<td>5.735B</td>
</tr>
<tr>
<td>7</td>
<td>1.213B</td>
<td>27</td>
<td>6.277B</td>
</tr>
<tr>
<td>8</td>
<td>1.311B</td>
<td>28</td>
<td>6.897B</td>
</tr>
<tr>
<td>9</td>
<td>1.416B</td>
<td>29</td>
<td>7.550B</td>
</tr>
<tr>
<td>10</td>
<td>1.531B</td>
<td>30</td>
<td>8.299B</td>
</tr>
<tr>
<td>11</td>
<td>1.655B</td>
<td>31</td>
<td>9.317B</td>
</tr>
<tr>
<td>12</td>
<td>1.790B</td>
<td>32</td>
<td>10.077B</td>
</tr>
<tr>
<td>13</td>
<td>1.937B</td>
<td>33</td>
<td>11.134B</td>
</tr>
<tr>
<td>14</td>
<td>2.097B</td>
<td>34</td>
<td>12.325B</td>
</tr>
<tr>
<td>15</td>
<td>2.271B</td>
<td>35</td>
<td>13.673B</td>
</tr>
<tr>
<td>16</td>
<td>2.461B</td>
<td>36</td>
<td>15.202B</td>
</tr>
<tr>
<td>17</td>
<td>2.669B</td>
<td>37</td>
<td>16.941B</td>
</tr>
<tr>
<td>18</td>
<td>2.896B</td>
<td>38</td>
<td>18.926B</td>
</tr>
<tr>
<td>19</td>
<td>3.144B</td>
<td>39</td>
<td>21.201B</td>
</tr>
<tr>
<td>20</td>
<td>3.416B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Height, \( h \), above foundation base level, affected by plastic failure for deep strip footings in cohesive (c, \( \phi \)) soils

Meyerhof’s expression for the ultimate bearing capacity of a deep strip foundation in cohesive (c, \( \phi \)) soil agrees well with the observed behaviour of full scale foundations. It takes into account:

1. Depth \( D \) and breadth \( B \) of the foundation
2. The base of the foundation is assumed to be rough
3. Rough vertical side
4. Angle of shearing resistance, \( \phi \)
5. A lateral earth pressure coefficient \( K_s \) approximated to \( K_a \)

For deeper foundations (\( D/B >> 3 \)) in cohesive soil (\( \phi = 0^\circ \)) Meyerhof’s equation becomes:

\[
q_{ult} = c N_{cq} + \gamma D
\]

and

\[
q_s = \frac{c N_{cq}}{F} + \gamma D
\]
where \( N_{cq} \) is a bearing capacity factor based on foundation shape and roughness;

\[
\begin{align*}
N_{cq} &= 10.18 \quad \text{circular foundation, (rough)} \\
N_{cq} &= 9.52 \quad \text{circular foundation, (smooth)} \\
N_{cq} &= 8.85 \quad \text{strip foundation, (rough)} \\
N_{cq} &= 8.28 \quad \text{strip foundation, (smooth)}
\end{align*}
\]

Class example 5
A rectangular foundation (2.0m x 6.0m) is placed at 15m below ground level in a sandy clay soil (\( \gamma_{sat} = 21.3 \) kN/m\(^3\); \( c = 35.0 \) kN/m\(^2\); \( \phi = 25^\circ \)). The water table is steady at ground level.

If the shaft and base of the foundation are smooth, determine the net safe bearing capacity of the foundation.

Adopt a factor of safety of 3 against bearing capacity failure. \([2345.8 \text{ kN/m}^2]\)

4.0 OVERTURNING

So far only vertical concentric loading of the foundation has been considered. Many foundations have to be designed to cater for the effects of horizontal loading and moment loading as a result of eccentric or inclined loads.

The analysis involves resolving the loading into a single vertical load acting at the base of the foundation at some eccentricity, \( e \), from the centroid of the foundation. If a horizontal load exists, it is considered to act at the foundation base level, where it no longer contributes to the overturning moment but provides a resultant inclined loading.

Applying inclined and eccentric/concentric loads to a foundation with a horizontal base, leads to a reduction in the ultimate bearing capacity, \( q_{ult} \), compared with the value obtained for vertical loading.

Brinch Hansen (1970) developed a solution for eccentric and inclined loading for shallow and deep strip footings for all \( \phi \) conditions.

Meyerhof (1953) developed a solution (based on strip footing theory) for concentric inclined loading of shallow, circular, square and rectangular foundations for \( \phi = 0 \) conditions.

4.1 Eccentric loading of a strip foundation (Brinch Hansen, 1970)

This method uses a reduced foundation width, \( B' \) and an equivalent vertical pressure, \( P_v \).

If “\( e \)” is the eccentricity of the resultant load measured from the axis of symmetry of the foundation, then an effective foundation width \( B' \) for bearing capacity considerations is given by:

\[
B' = B - 2e \quad \text{where } B \text{ is the actual width.}
\]
The equivalent vertical pressure, \( q_e \), is given by:

\[
q_e = \frac{P_v + \lambda P_h}{B'}
\]

where:
- \( P_v \) = vertical load component/metre run
- \( P_h \) = horizontal load component/metre run
- \( B' \) = effective width of base/metre run
- \( \lambda \) = dimensionless constant related to \( \phi' \)

\[
\begin{array}{cccccc}
\tan \phi' / F & 0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\
\lambda & 1.4 & 1.8 & 2.3 & 2.8 & 3.3 & 3.9 \\
\end{array}
\]

where \( F \) is a factor of safety against bearing capacity failure.

Now,

\[
q_e \leq q_s
\]

For cohesive soil (\( \phi = 0^\circ \)) use;

\[
q_s = \frac{c_u N_c}{F} + \gamma D
\]

where \( N_c \) is related to \( D/B' \) and is found from Skempton’s graph for \( \phi = 0^\circ \)

For \( c, \phi \) soil use;

Use Terzaghi (\( q_{ult} = cN_c + \gamma D N_q + 0.5\gamma B'N_r \))

Class example 6

Check the bearing capacity of the strip foundation shown using a factor of safety of 3.0 against bearing capacity.

The soil parameters are:
- \( c = 28.0 \text{ kN/m}^2 \);
- \( \phi = 31^\circ \);
- \( \gamma = 16.9 \text{ kN/m}^3 \)
4.2 Concentric inclined loading of shallow foundations
\((\phi = 0^\circ \text{ conditions, Meyerhof, 1953})\)

Meyerhof has shown that for saturated undrained clays that:

\[
q_{\text{ult}} = c N_{cq} + \gamma D
\]

\[
q_{\text{net safe}} = \frac{c N_{cq}}{F} + \gamma D
\]

Where \(N_{cq}\) is a bearing capacity factor found from the chart below and is a function of:

1. Angle of inclination of the applied load to the vertical
2. Ratio \(D/B\)
3. Degree of base and side adhesion on the foundation
   (this is normally assumed to be zero)
Now having found the value of $q_a$, the foundation size ($L \times B$) can be calculated from:

$$\text{Total inclined applied load} = \text{area of foundation (}L \times B\text{)} \times q_a$$

### 4.3 Resistance to sliding along the base

The passive resistance of the soil is neglected since this may be affected by softening, weathering etc. The adhesion between the underside of the base and the clay is usually taken as $0.75 c_u$.

To prevent horizontal sliding,

$$B \cdot 0.75 c_u \geq F_{\text{sliding}} \cdot P_h \text{ / metre length}$$

($F_{\text{sliding}} = 1.5$ usually)
4.4 Design of heel rib

If for economic reasons, a wide base is not selected to counteract sliding, a heel rib may be included in the design.

1. Depth \( d = \text{balance of resistance to sliding} \)
   \[ \frac{2}{3} q_a \]
   \[ = \{ F.P_h - 0.75 c_u B \} \]

2. B.M. at junction of rib with heel = \( \{ F.P_h - 0.75 c_u B \} \frac{d}{2} \rightarrow A_{ul} \)

3. Check shear and bond.

Class example 7

Check the bearing capacity of the rectangular foundation shown (L = 6.0m) using a factor of safety of 3.0 against bearing capacity.

The soil parameters are;
\[ c_u = 138.0 \text{ kN/m}^2; \]
\[ \phi_u = 0^0 \]
\[ \gamma = 18.6 \text{ kN/m}^3 \]
5.0 SUMMARY

\[ q_{\text{ult}} = \text{ultimate bearing capacity} \]

\[ q_s = \text{safe bearing capacity} \]

\[ q_s = \frac{q_{\text{ult}}}{F} \]

where \( F \) = factor of safety (normally 3.0)

\[ q_n = \text{net bearing capacity} \]

\[ q_n = q_{\text{ult}} - \sigma_0 = q_{\text{ult}} - \gamma D \] (total stress)

\[ q_n = q_{\text{ult}} - \sigma_0' = q_{\text{ult}} - (\gamma D - \gamma_w h_w) \] (effective stress)

\[ q_{n\text{a}} = \text{net allowable bearing capacity} \]

\[ q_{n\text{a}} = \frac{q_{\text{ult}} - \gamma D}{F} + \gamma D \]

**Shallow Foundations**  \( D< B \); \( D< 3m \)

\( c, \phi \) soil

Terzaghi: \( q_{\text{ult}} = cN_c s_c + \gamma DN_q s_q + 0.5B\gamma N_s s_r \]

\[ [N_c ; N_q ; N'_\gamma ; s_c ; s_q ; s'_\gamma \]

are from tables\]

- modified when W.T. is present using \( \gamma_{\text{sub}} = \gamma_{\text{sat}} - \gamma_w \)

\( c \) soil (\( \phi = 0^\circ \))

Skempton: \( q_f = cN_c + \gamma D \] [\( N_c \) from graph]

\( \phi \) soil (\( c = 0 \))

\( q_a \) from graph using corrected S.P.T. (N) values

**Deep foundations**  \( D>B \); \( D>3m \)

\( c, \phi \) soil

Meyerhof:

Strip;

\[ q_{\text{ult}} = cN_c + p_o N_q + 0.5\gamma B N'_\gamma \]

Rectangular;

\[ q_{\text{ult}} = c (1 + 0.2 \frac{B}{L}) N_c + p_o N_q + 0.5\gamma B (1 - 0.2 \frac{B}{L}) N'_\gamma \]

\[ [N_c ; N_q ; N'_\gamma \]

from graph \( P_o = K_a\gamma [D - (h/2)] \)

\( h \) from table \( K_a = (1-\sin\phi)/(1+\sin\phi) \)

\( c \) soil (\( \phi = 0^\circ \))

Meyerhof: \( q_{\text{ult}} = c u N_c q + \gamma D \] [\( N_c q \) values given]

\( \phi \) soil (\( c = 0^\circ \))

Meyerhof: \( q_{\text{ult}} = 0.5\gamma B N_q \] [\( N_q \) from graph]

**Eccentric Loading** - strip foundation
Brinch Hansen: \( q_e \leq q_a \);

\[
q_e = \frac{P_v + \lambda P_h}{B'} \tag{\text{[} \lambda \text{ from table]} }
\]

\[
q_s = \frac{c_u N_c}{F} + \gamma D
\]

- for c soil: \( N_c \) from Skempton
- for \( c, \phi \) soil: \( N_c N_q N_f \) from table

**Inclined Loading**

Meyerhof: \( q_{ult} = cN_{cq} + \gamma D \) \( [N_{cq} \text{ from graph}] \)

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**REFERENCES**

Brinch Hansen, J (1970) *A revised and extended formula for bearing capacity* Bulletin No.28, Danish Geotechnical Institute, Copenhagen, pp5-11

Meyerhof, G G (1951) *The ultimate bearing capacity of foundations* Geotechnique v2 no.4


