THE IS-LM MODEL

TO GET AGGREGATE DEMAND:

$y = c + i + g$ (1)

This is an identity, so that in itself it doesn't help us understand the behaviour of the economy.

To understand the behaviour of *y*, GDP, we need to provide a theory that explains the behaviour of each of its components: consumption, investment and government spending.

Consumption.

Assume:

$c = C(y-t)$ $0<C'<1$ where C' = marginal propensity to consume. (2)

C (.) is a function which we won't (yet) specify.

That is, consumption depends on current disposable income in some way. As it stands we don't know exactly the shape of the consumption function,. However, the assumption that its derivative is between zero and one means that if disposable income were to rise by x%, consumption would rise, but by less than x%.

Note what this implies for savings:

$s = y - c - t = S(y-t)$

As before, savings depend on disposable income. Because when income rises consumption does not rise by the same magnitude, then $0<S'(y-t)<1$, i.e. if income rises, so do savings.

However, one would expect savings to be affected by interest rates. If so, then (indirectly) consumption will also be affected by interest rates (remember that keeping income fixed, if the amount of savings varies then so does consumption). Nevertheless, allowing consumption to depend on interest rates won't change the main conclusions of the IS-LM mode; it just complicates the maths. We will ignore this element for now.

Investment

$$i = I(y, r) \qquad I_1 > 0,\ I_2 < 0 \qquad (3)$$

So investment depends positively on output and negatively on interest rates. But why? Think about profit maximising firms: when it comes to their investment decision, they will compare the costs and benefits of an additional unit of investment.

Because investment is used to increase the capital stock, the benefit of investment accrues from the greater productive capacity the firm will possess. By how much will this capacity increase? This is given by the marginal product of capital, MPK, which measures the increase in output from an increase in the capital stock.

The cost is given by the interest rate, r, the reason being that a) the firm had to borrow the funds in order to invest and r is the cost of borrowing or, b) the firm could have purchased a bond paying r, instead of investing (opportunity cost).

Therefore, for a profit-maximising firm marginal cost = marginal revenue, or

MPK = r.

This is the equilibrium condition. When the left-hand side (lhs) is greater than the right-hand side (rhs) investment will rise, so we can derive the investment function as dependent on the ratio:

$$i = \bar{I}(\frac{MPK}{r}) \qquad \bar{I}' > 0$$

so when the ratio is greater than one (MPK>r) the firm will decide to increase investment. The link between this form and equation (3) is as follows. Given a production function, $y = f(k,n)$, for a given capital stock, an increase in labour will increase output and the marginal product of capital. So indirectly, whenever output increases (given a fixed capital stock), MPK rises, hence we can use (3) as a shortcut.

Government Spending

This will simply be assumed fixed.

Equilibrium

Now that we have a theory for the behaviour of each of the components of GDP, we can analyse what the model has to say about output. Combining the equations and solving yields:

$$y = C(y-t) + I(y,r) + g \qquad (4)$$

There are two endogenous variables (y,r) in one equation. This represents y,r combinations so that the economy satisfies (1) – (3).

Another way of writing the IS curve would be (assuming we knew the shape of C(.) and I(.)) to solve for (4) by putting y on the lhs and everything else on the rhs. Then this would yield:

$$y = \sigma(r, g, t) \qquad \sigma_1 < 0, \sigma_2 > 0, \sigma_3 < 0 \qquad (5)$$

Note that the IS equation (5) does not give a solution for y or r. It simply states that given a value for one of the variables, there is a corresponding unique value for the other variable such that (1) to (3) hold. In addition, $\sigma_1 < 0$ implies that there is a negative relationship between interest rates and output (that is, the IS curve slopes down). $\sigma_2 > 0$ implies that an increase in *g* will shift the IS to the right (i.e., a higher *g* results in higher *y*) and $\sigma_3 < 0$ that increases in taxes, *t,* shift the IS left.

It is sometimes said that the IS represents good market equilibrium, but this is not quite correct, since we haven't said anything about the production side.

THE LM CURVE

Here the assumption is that households allocate their financial wealth between bonds and money (currency), and again, households (we'll ignore firms holding currency) face a choice. Holding money has the benefit of liquidity, it eases transactions. However, bonds pay interest (because of competitive markets we'll also assume that interest on corporate and government bonds pay the same rate).

Hence, increases in interest rates imply rises in the opportunity costs of holding money. Additionally, since money is used for transactions (proxied by the level of income, *y*), increases in GDP imply more numerous transactions and therefore a greater demand for currency. These two variables, r and y, are therefore the main determinants of the demand for money.

But the above pertains to the demand for *real* money (*M/P)*, that is, the purchasing power of the currency held. That is:

$$\frac{M^d}{P} = L(y, r) \qquad L_1 > 0 > L_2 \qquad (6)$$

The monetary authority sets the money supply, so assuming money market equilibrium at all times:

$M^d = M^s$

inserting into (6):

$$\frac{M}{P} = L(y, r) \qquad (7)$$

Hence, for given value for real money balances, (7) implies a relationship between output and interest so that the money market is in equilibrium. Note that in order to draw the LM curve on a diagram one would want to have r on the lhs and everything else on the rhs.

AGGREGATE DEMAND

At any point both (5) and (7) must hold. Combining them we would end with an equation that involves output, real money balances, government spending and taxes. Assuming that we did this and solved for output:

$$y = \psi(\frac{M}{P}, g, t) \tag{8}$$

Note that all we have done was to combine the IS and LM equations into a single one. Recall your first year Macro and all this implies is that we have derived the equation for Aggregate Demand.

In terms of the maths, note that in each of the IS and LM curves there were two unknowns: output and interest rates. Once you know one in each equation you find the unique value for the other. However, each equation by itself is useless, that is, you can't just assume a value for interest rates in the IS to get output. The crucial point is that the IS and LM curves have positive and negative slopes in (y,r) space, respectively. This means (assuming they are fairly straight lines) that there is only one intersection and one equilibrium. This is important as it implies that an equilibrium exists and also, that it is unique.

In addition, although the IS and LM don't tell us much in isolation, when combined we get Aggregate Demand (which implies that the economy is on the IS and LM simultaneously).

Given the assumptions in the previous equations

$\psi_1 > 0,\ \psi_2 > 0,\ \psi_3 < 0$

Since M, g and t are exogenous, (8) gives a relationship between output and prices and the sign of the aggregate demand relationship is given by $-\psi_1$ (differentiate with respect to prices). In other words, aggregate demand slopes down.

It should be notes that the IS-LM model is often used without combining them to derive aggregate demand. It is important to understand what this means. By using the IS-LM model in isolation, one is (implicitly) assuming that prices are fixed, so that we can derive the implications of, say, higher government expenditure, on output and interest rates. But assuming that prices are fixed (and output is flexible) is another way of saying that the aggregate supply curve is horizontal. Therefore keep in mind that this is one of the main assumptions underlying analysis with the IS-LM model. This is arguably a good enough approximation in the (very) short run, but not in the medium or long run.

Now that the demand side of the model has been completed, it is time to look at how aggregate supply is derived. We will initially consider two main models: the Classical and the Keynesian aggregate supply functions.

The Classical Aggregate Supply

In the Classical model, all prices (including the price of labour) are fully flexible. This has the implication that all markets clear. Consequently, the real wage always equals the marginal product of labour and labour demand always equals labour supply. In other words, the nominal wage has no effect on the labour market.

The Classical aggregate supply function is made up of the following equations:

$$y = f(n) \qquad f'(n) > 0,\ f''(n) < 0$$

$$f'(n) = \frac{W}{P}$$

$$n = h(w) \qquad h'(w) > 0 \ \textit{and}\ w = \frac{W}{P}$$

The first equation is the production function, so that output depends on employment (we're ignoring the capital stock and any other factors that affect output). We'll always assume that its first derivative is positive and its second negative for all values of *n.* This simply means that higher employment will always yield higher output but at a diminishing rate.

The first equation represents firms' optimal choice: choose the level of output where marginal benefit of employing additional labour (its marginal product, denoted by the derivative of output with respect to employment) equals its marginal cost.

Lastly, the third equation is a simple assumption about labour supply: it depends (positively) on the real wage.

Looking at these equations, you'll see that there are only three variables (output, labour and the real wage) and three equations, so that nothing else is needed to determine the values of output, labour and the real wage in the Classical model, These are determined by the supply-side of the economy. The only purpose of the demand side is to determine nominal variables (prices/inflation). You may have noticed that the production function only contains labour as its argument, i.e., it ignores capital. This is only done as a simplification: we're assuming that capital is constant in the short run, so that it is simply a fixed number and here we've made it equal to one without loss of generality. A more realistic model would include it, but the main conclusions of the model would remain unchanged.

The Classical Model.

The full Classical model is simply the combination of the equations that make up aggregate demand, as described above, and the Classical aggregate supply function.[1] That is,

$$y = C(y - t) + I(y, r) + g$$

$$\frac{M}{P} = L(y, r)$$

[1] The demand side as presented here (IS-LM) is a Keynesian development. However, given that the supply side is independent of the behaviour of demand all the conclusions hold.

$$y = f(n)$$

$$f'(n) = \frac{W}{P}$$

$$n = h(w)$$

There are five equations and five endogenous variables (output, interest rates, labour, the nominal wage and prices).

Given the assumptions made above regarding each of the equations, one obtains a unique equilibrium (that is, there is only one value for each of the endogenous variables that satisfies all of the five equations), and alternative experiments can be conducted.

Let there be an increase in the money supply, *M*. Because we know that the real wage, output and labour are determined via the last three equations, these remain unchanged.

The first equation, the IS curve, depends on the interest rate and output and these haven't changed either, so the only variable that can react to the increase in the money supply in the LM equation is the price level, which will change by an identical amount to the *M*, so that the ratio remains the same as before. Consequently, in the Classical model monetary policy only affects prices (and nominal variables), but real variables are not affected. This is called the Classical dichotomy.

There is also an additional concept that you should be aware: monetary neutrality. When money is said to be neutral, it means that the quantity of money does not affect the allocation of resources, that is, real variables such as employment and output.

Additionally, superneutrality is similar, except that it refers to the growth rate (rather than the level) of money.

The Keynesian Aggregate Supply Function

The Keynesian model is a simple modification of the Classical model. Yet, it can yield drastically different conclusions. At its core is the assumption that some nominal variables (typically wages or prices of goods) are rigid. In the Classical model, the real wage equals the marginal product of labour, and the utility of the wage for a given amount of employment supplied equals the marginal disutility of that labour supplied.

In the Keynesian model, the second element is rejected. As a consequence, although firms may be on their demand for labour, households won't be on their labour supply curve, so that the labour market is not in equilibrium (labour demand does not equal labour supply).

But if labour demand does not equal labour supply, what determines the actual amount of labour employed? Is it demand or supply? This is not clear, nor why adjustment is not achieved. We will ignore this issue for now, but we will assume that the nominal wage is temporarily fixed (due to contracts) and analyse the consequences.

The model is virtually the same as the Classical model described above, bar some minor modifications. Crucially, a distinction between labour supplied and labour demanded is now necessary:

$$y = f(n^d)$$

$$f'(n^d) = \frac{W}{P}$$

$$n^s = h(w) \ (ignored)$$

$$W = \overline{W}$$

$$w = \frac{W}{P}$$

To determine what the actual amount of labour employed will be, we'll assume it is determined by firms, that is:

$$n = n^d$$

The idea here is that firms and labour agree on the nominal wage (for example, via unions), but then firms will choose the amount of labour to be employed. Although not plausible in the long-run, this is justified for short-run analysis.

Assuming that the nominal wage is fixed and that firms determine the actual amount of labour employed leads to some drastic consequences when compared to the Classical model.

This is how the model works: we have four equations and five unknowns (y, n, W, w and P).[2] The second equation implies that there is a direct relationship between labour (via its marginal product) and prices. As before, firms will set employment to the level where the marginal product of labour is equal to the real wage. However, notice the following (this

[2] Previously we could just consider w without needing to know W or P (just their ratio). This is not the case in the present model as W is fixed.

did not occur in the Classical model): if prices rise, given that the nominal wage is fixed real wages fall (labour is cheaper) and therefore firms demand more labour. As we've assumed that the amount of labour employed is labour demanded, employment rises. From the first equation (the production function) this implies that output rises. Similarly, if the nominal wage (assumed exogenous) rises, labour becomes more expensive. Firms cut down on employment and therefore GDP falls.

The overall conclusion that emerges is that the supply side of the model is no longer independent of demand. In other words, the Classical dichotomy no longer holds. This will imply that anything that affects demand (eg, fiscal and monetary policy) will now have the potential to affect supply.

The Keynesian Model

Combining the aggregate side of the economy analysed earlier with the Keynesian supply function, the complete Keynesian model is:

$$y = C(y - t) + I(y, r) + g$$

$$\frac{M}{P} = L(y, r)$$

$$y = f(n^d)$$

$$f'(n^d) = \frac{W}{P}$$

$$W = \overline{W}$$

In contrast to the Classical model, the level of output is no longer determined independently of nominal variables, so that the Classical dichotomy is not valid in the present model. Returning to the previous example, an increase in the money supply will shift the LM curve rightwards, lowering interest rates, increasing output and prices. Therefore, money is now able to affect output, in contrast to the Classical model.

Summary on Classical and Keynesian models.

We have explored the IS-LM model to derive aggregate demand and combined it with the supply-side of the economy to derive the Classical and Keynesian models of the economy. But there was also a difference in focus between the two main theories. For the Classicals, the analysis was divided between value theory (the behaviour of firms and households at the disaggregated level) and aggregate monetary theory (e.g., the quantity theory). Keynes replaced this distinction with a new one: that between micro- and macroeconomics. Furthermore, he wanted a theory where monetary neutrality would not hold (at least in the short-run).

In terms of trying to understand the workings of each model the following provides the simplest representation: the Classical model is supply driven. This means that if anything shifts the supply side it will affect the economy and then the demand side will adjust. In contrast, the Keynesian model is demand driven (the supply-side adjusts).

Other Theories.

The disagreements between Classical and Keynesian theorists finally led to what is called the Neoclassical Synthesis (NS). One way of interpreting the Neoclassical Synthesis is to have the standard Keynesian model explained above, plus the Phillips Curve (PC) to analyse the behaviour of output when it is below potential. Once GDP reaches its potential level, the Classical model applies. We will be discussing the PC later in the course, but a simple formulation can be put forward as:

$$y_t = y^* + \phi(p_t - p_t^e) \qquad \phi > 0$$

where y^* represents potential output and p_t^e is the expected price level prevailing at time *t-1*, so that when prices are higher than expected, output will be above its natural rate. In the long run, actual prices equal expected prices and output is equal to its potential. One final element remains to complete this model, and that is how expectations are formed. In the Neoclassical Synthesis, these were assumed to be formed adaptively. That is, expectations of current variables depend on past values and (possibly) adjusted for past errors in expectations:

$$p_t^e = p_{t-1}^e + \alpha(p_{t-1} - p_{t-1}^e) \qquad \alpha > 0$$

This equation can be interpreted as follows: expectations of current prices depend on the last period's expected value plus an adjustment that depends on the previous period's mistake in expectations. Therefore, the parameter α measures the adjustment to previous

mistakes. An alternative formulation often found in many textbooks and the literature is that rewriting the above equation one obtains:

$$p_t^e = \alpha p_{t-1} + (1-\alpha)\, p_{t-1}^e$$

The main punchline in the NS model is that if due to a shock (eg an unexpected increase in the money supply) causes prices to increase, output rises above its potential level, and because price expectations only adjust gradually, so that the economy can remain away from its long run values for a considerable amount of time.

Additional Reading:

Parkin and Bade provide a good introduction to the IS-LM model, but for a more detailed analysis on the Keynesian and Classical models Hillier (1991) [The Macroeconomic Debate: Models of the Closed and Open Economy, Blackwell] is excellent.