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InGaAs Laser Parameters Calculations

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Objective

The purpose of this paper is to design and calculate the different parameters of an *InGaAs* laser using an specific example.

Laser Parameters

Given the following parameters:

a) Lasing efficiency (η) = 10%
b) Output power (P_o) = 10mW
c) $\theta_J = 7^\circ C / Watt$
d) Laser stripe dimensions = 250 μm long, 4 μm wide, 0.3 μm deep
e) Chip size is 250 μm x 25 μm

Calculate:

1) $\lambda_{\min}$ *and* $\lambda_{\max}$
2) Output beam dimensions at $\lambda_{\min}$ *and* $\lambda_{\max}$
3) The $\Delta\lambda$ due to temperature
4) Mode (longitudinal) structure from $25^\circ C$ to its operating temperature

Solution

Figure 1 shows a diagram of the semiconductor laser that uses an *InGaAs* gain layer.

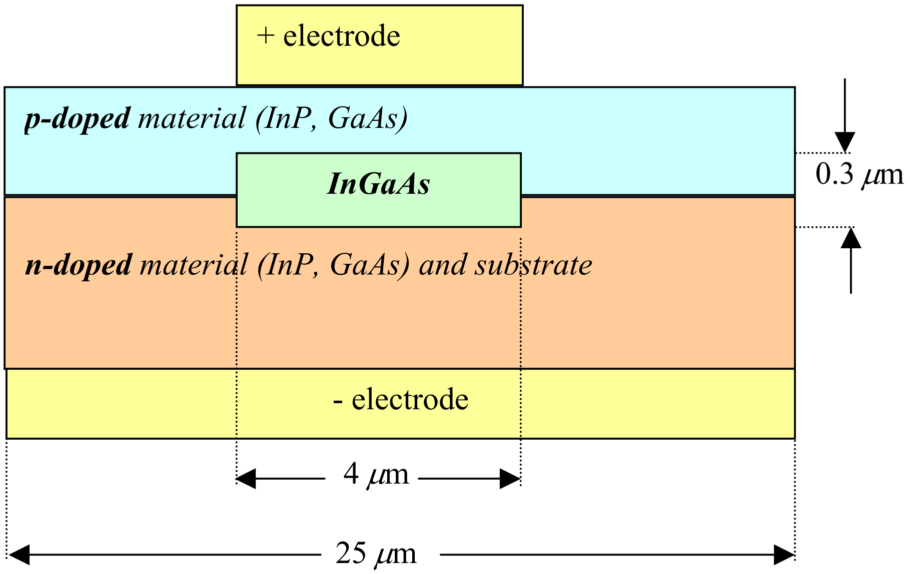


Figure 1. Front view of the *InGaAs* Laser

The length of the bar is 250 μm. It is assumed that the front of the bar is polished and that this surface results in a mirror with a reflection coefficient of 0.3, while the rear of the laser bar has a mirror with a high reflection coefficient (it will be assumed to be 0.95).

Minimum and Maximum Wavelength Emission

$In_xGa_{1-x}As$ has a wide range laser emission spectrum that basically depends on the Indium concentration, *x*. The laser light wavelength also depends on the stress induced by the p-doped and n-doped layers embedding the *InGaAs* stripe. “Standard” $In_xGa_{1-x}As$ has x=0.53 [5] which is the concentration required unstressed *InGaAs* [1]. For x=0.53 the *InGaAs* stripe lases at 1.68 μm [5].

To calculate the wavelength of the emitted light, regardless of the embedding layers, the following empirical equations are used:

$$E_g = \begin{cases} 1.508 - 1.47x + 0.375x^2 & T = 77^\circ K \\ 1.43 - 1.53x + 0.45x^2 & T = 300^\circ K \end{cases} \qquad \text{[1] (117 and 118)}$$

Which give us the bandgap energy for a given Indium concentration *x*. For other temperatures extrapolation is used. In this project, the equation for 300K will be used.

Recalling that the energy of the emitted photons is hv, we have:

$$E_g = hv \qquad \text{where } h \text{ is the Planck's constant} \qquad (1.0)$$

$$v = \frac{E_g}{h} \qquad (2.0)$$

$$\lambda = \frac{c}{v} = \frac{hc}{E_g} \qquad (3.0)$$

Assuming a minimum Indium concentration x=0.05 and a maximum Indium concentration = 0.95, we can compute the maximum and minimum laser wavelengths for *InGaAs*. These are:

$$\lambda_{\min} = \frac{hc}{E_{g,x=0.05}} = 0.915\mu m \qquad \text{and} \qquad \lambda_{\max} = \frac{hc}{E_{g,x=0.95}} = 3.24\mu m$$

All other wavelengths in the range have been plotted and are shown in Figure 2. The MATLAB script used to calculate and plot the wavelengths, *InGaAsCurve*, can be found on Appendix A.

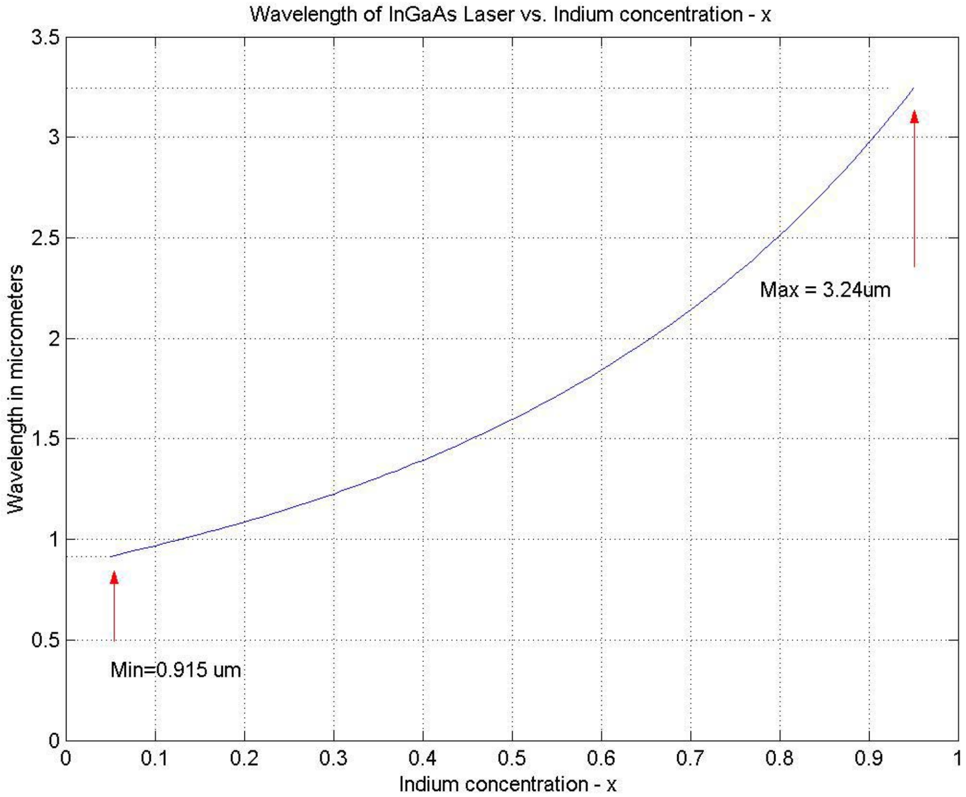


Figure 2. Wavelength emission of *InGaAs* vs. Indium concentration

These results are in agreement with experimental results shown on Figure 1.9 of reference [3] and Figure 2 of reference [4], which can be found on Appendix B.

Now, the standard bandgap of the InGaAs stripe is modified when buried between two other layers. Heterostructures with $In_xGa_{1-x}As$ quantum wells between *InP* barriers have the interesting property of being either strained or unstrained [1]. The structure is unstrained for $x \approx 0.53$ or under tensile or compressive strain for smaller or larger Indium concentration, respectively.

The bandgap of *InP* is:

$$E_g = 1.42667 - 0.000326436T \qquad [1]\ (119)$$

The *InP* barriers have the effect of shifting the heavy-holes and light-holes valence bands in *InGaAs*. To calculate exactly the total bandgap shift and therefore, the resulting wavelength, the following equations are used. Please see Appendix B for Tables 6-1 and 6-2 that contain several parameters used in these Equations.

The lattice mismatch parameter:

$$e_0 = (a_b - a_w)/a_b \qquad [1]\ (97)$$

Values for a_x in Table [1] 6-2

Hydrostatic potential:

$$\delta E_H = 2a_1 e_0 (C_{11} - C_{12})/C_{11} \qquad [1]\ (104)$$

Values for C_{ij} in Table [1] 6-2

Shear-deformation potential:

$$\delta E_S = a_2 e_0 (C_{11} + 2C_{12})/C_{11} \qquad [1]\ (105)$$

Valence band shift due to heavy-holes:

$$E_{hh} = \frac{\hbar^2 k^2}{2m_{hh}} - \delta E_H - \frac{1}{2}\delta E_S \qquad [1]\ (107)$$

Valence band shift due to light-holes:

$$E_{lh} = \frac{\hbar^2 k^2}{2m_{lh}} - \delta E_H + \frac{1}{2}\delta E_S \qquad [1]\ (108)$$

The bandgaps of *GaAs* and *InAs* can be found in Table [1] 6-1. To obtain the resulting bandgap of *InGaAs*, the following weighted formula is used:

$$\overline{E}_g = xE_{g,InAs} + (1-x)E_{g,GaAs}$$

Since sometimes the valence band of the light-holes is higher than the valence band of the heavy-holes and vice versa, the minimum bandgap between the conduction band and the higher valence band will be used to compute the energy of the emitted photons and therefore their wavelength. To automate this process and plot an equivalent curve of wavelength versus Indium concentration, a MATLAB script, *InPInGaAsCurve*, was written. The code can be found on Appendix A.

Figure 3 shows the new wavelengths emitted by an *InGaAs* stripe similar to the one illustrated in Figure 1.

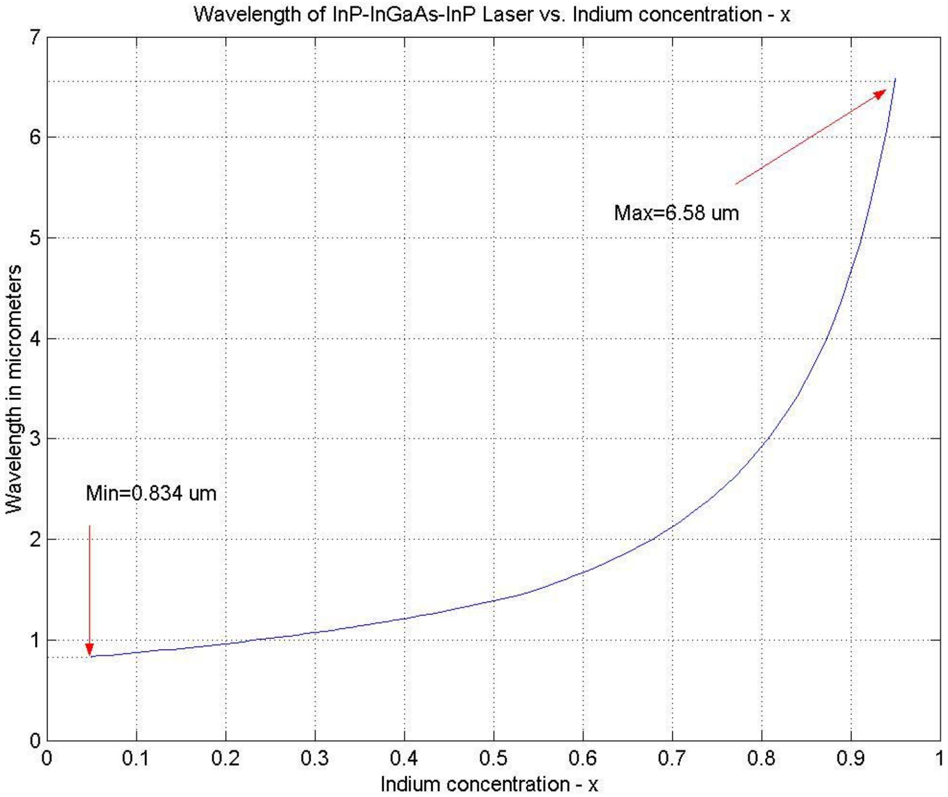


Figure 3. Wavelength emission of *InP-InGaAs-InP* vs. Indium concentration

It can be seen that the minimum wavelength is slightly smaller than before ($\lambda_{\min} = 0.834\mu m$), but the maximum wavelength is doubled ($\lambda_{\max} = 6.58\mu m$) when the *InGaAs* structure is under compressive stress. The resulting stress is due to difference in lattice structure and size between *InGaAs* and *InP* or any other material used for the barriers.

Laser beam size

The spatial distribution of emission from a semiconductor structure is elliptical in nature, with a preponderance of angular divergence along the growth axis [2]. Figure [2] 13-31 in Appendix B shows schematically how the emitted laser beam diverges along the vertical and horizontal axes.

To calculate the angles of beam divergence in both directions, parallel and perpendicular to the gain layer, the following equations will be used. Refer to Figure 4 showing the angles of divergence and the resulting beam spot (an ellipse) at certain distance *r*.

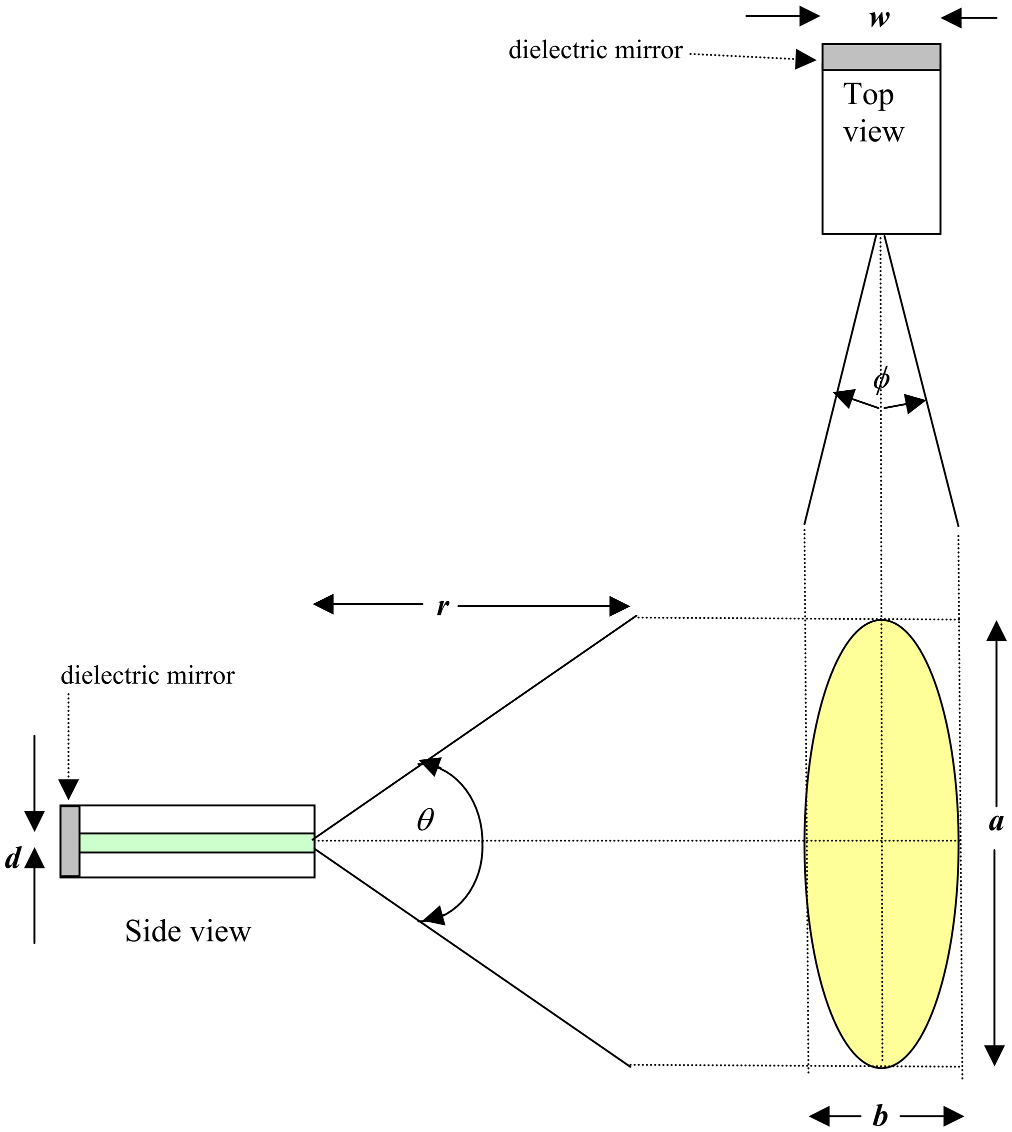


Figure 4. Laser beam spot front view

The corresponding angles are calculated approximately by:

$$\theta = \frac{\lambda}{d} radians \qquad (4.0)$$

$$\phi = \frac{\lambda}{w} radians \qquad (5.0)$$

and the resulting ellipse main and minor axis, *a* and *b*, at distance *r*, are calculated as follows:

$$a = 2r\sin\left(\frac{\theta}{2}\right) \qquad (6.0)$$

$$b = 2r\sin\left(\frac{\phi}{2}\right) \qquad (7.0)$$

The laser beam is considered to be “collimated” up to a distance equal 2 times the Rayleigh range which is:

$$Z_r = \frac{\pi w_0^2}{\lambda} \qquad (8.0)$$

where w_0 is the waist of the beam, which in this case is the smaller geometrical size, *d*, the thickness of the *InGaAs* stripe.

Table 1 presents the results for the minimum and maximum wavelengths previously calculated.

λ	θ	ϕ	$2Z_r$	**Spot size at 2*Z*$_r$**		**Spot size at 10cm**		**Spot size at 1m**	
				a	***b***	***a***	***b***	***a***	***b***
0.915µm	175°	13°	0.62µm	1.23µm	0.14µm	20cm	2.3cm	2m	0.23m
3.24µm	>180°	46°	0.17µm	N/A	N/A	N/A	N/A	N/A	N/A

Table 1. Laser beam size and angles of divergence for $d = 0.3\mu m$

It can be seen that due to the fact that the *InGaAs* stripe thickness is 3 times smaller than the minimum wavelength, the emitted light is highly dispersed. The angles of divergence are too big under these circumstances. To overcome this problem, a thicker *InGaAs* is required. To see an example, let’s assume that the thickness is equal to the width, which is larger than the maximum wavelength, that is, *d*=4µm. Table 2 shows the results.

λ	θ	ϕ	$2Z_r$	**Spot size at 2*Z*$_r$**		**Spot size at 10cm**		**Spot size at 1m**	
				a	***b***	***a***	***b***	***a***	***b***
0.915µm	13°	13°	110µm	25µm	25µm	2.3cm	2.3cm	0.23m	0.23m
3.24µm	46°	46°	31µm	7µm	7µm	7.88cm	7.88cm	0.78m	0.78m

Table 2. Laser beam size and angles of divergence for $d = 4\mu m$

In the case the beam spot has a circle shape. It can be seen that larger wavelengths diverge faster than smaller wavelengths.

Linewidth $(\Delta\lambda)$ **Calculation**

The laser emission linewidth $\Delta\nu$, is obtained with:

$$\Delta\nu = \frac{h\nu}{2\pi\tau_c^2 P_{out}}\left(1+\beta_e^2\right) \qquad [2]\ (13.G.17)$$

where β_e is a 'linewidth broadening factor' with typical value of 5 at room temperature (300K). P_{out} is the output power and τ_c is the photon lifetime, calculated as,

$$\tau_c = \frac{1}{\left(\alpha_p - \frac{1}{2L}\ln\left(R_1 R_2\right)\right)c'} \qquad [2]\ (4.23a)$$

where α_p are parasitic losses (10cm^{-1} will be assumed), L is the length of the *InGaAs* stripe (250µm), R_1 and R_2 are the reflection coefficients of the 2 mirrors (assumed 0.95 and 0.3), and c' is the speed of light in the semiconductor.

$c' = \frac{c}{n}$ where n is the index of refraction ($n \approx 3.4$ for *InGaAs*)

replacing all these values in Equation [2] (4.23a) we obtain,

$$\tau_c = \frac{n}{\left(\alpha_p - \frac{1}{2L}\ln\left(0.95*0.3\right)\right)c} = \frac{3.4}{\left(10cm^{-1} - \frac{1}{2(0.025cm)}\ln\left(0.285\right)\right)3*10^{10}\frac{cm}{s}}$$

$$\tau_c = 3.23ps \qquad (9.0)$$

We now need to calculate the linewidth for different wavelengths therefore, expressing Equation [2] (13.G.17) as a function of λ:

$$\Delta\nu = \frac{hc}{2\pi\tau_c^2\lambda P_{out}}\left(1+\beta_e^2\right) \qquad (10.0)$$

For $\lambda_{\min} = 0.915\mu m$ and $\lambda_{\max} = 3.24\mu m$ we obtain the following linewidths:

$\Delta\nu_{\lambda=0.915\mu m} = 8.62MHz$ $\Delta\nu_{\lambda=3.24\mu m} = 2.44MHz$

The equivalent wavelength shift is

$$\Delta\lambda = \Delta\nu\left|-\frac{\lambda_0}{\nu_0}\right| = \Delta\nu\left|-\frac{\lambda_0^{\,2}}{c}\right| \qquad \text{[6] (Page 231)}$$

therefore,

$$\Delta\lambda_{\lambda_0=0.915\mu m} = 0.0024nm \qquad \Delta\lambda_{\lambda_0=3.24\mu m} = 0.0085nm$$

Temperature Effect on Linewidth

So far we haven't been concerned with the effects of temperature on linewidth. From Equation [2] (4.23a) we can see that the photon lifetime depends on the length of the laser stripe *L*. Temperature produces dilation of the semiconductor structure and therefore changes its length and the photon lifetime. From the specifications we have,

$$\theta_J = 7^0 / W$$

since the efficiency of the laser is 10%,

$$P_{out} = \eta P_{in} \rightarrow P_{in} = \frac{P_{out}}{\eta} = \frac{10mW}{0.1} = 100mW \qquad (10.1)$$

the amount of power that is dissipated as heat by the *InGaAs* structure is the difference between the output power and the input power,

$$P_{heat} = P_{out} - P_{in} = 90mW \qquad (10.2)$$

the change in temperature is calculated as follows

$$\Delta T = \theta_J P_{heat} = 7^\circ Cx90mW = 0.63^\circ C = 273.63^\circ K \qquad (10.3)$$

We can calculate the new length, *L*, using the thermal expansion coefficient of InGaAs and then use it in Equation 9.0. Another approach is to calculate the change in linewidth using the equations shown in Figure 5 (corresponds to Figure 6.47 of reference [6]).

From this figure we can see that

$$\Delta h\nu \cong 2.75kT \qquad (11.0)$$

To see the effect of temperature on linewidth, let's assume that the InGaAs stripe is emitting at $\lambda = 1.68\mu m$, the standard wavelength.

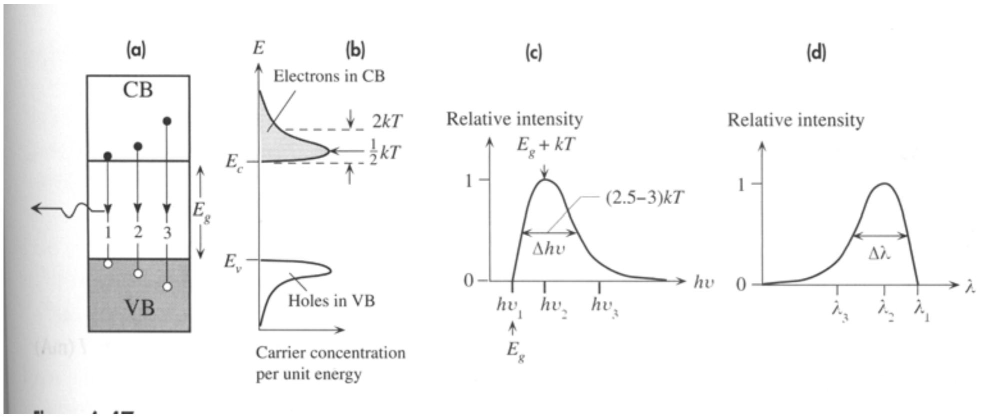


Figure 5. Effect of temperature on linewidth

The corresponding bandgap energy for this wavelength is:

$$E_g = \frac{hc}{\lambda} = \frac{hc}{1.68\mu m} = 0.7385eV$$

$$\Delta h\nu \cong 2.75kT = 2.75(1.38*10^{23})273.63*1.6*10^{-19} = 0.0648\text{eV}$$

The new bandgap energy is

$$E = E_g + \Delta h\nu = 0.7385eV + 0.0648eV = 0.8033eV$$

the wavelength of photons emitted with this energy is

$$\lambda = \frac{hc}{E} = \frac{6.626*10^{-34}x3x10^{8}}{0.8033eV} = 1.54\mu m$$

the change in wavelength is then

$$\Delta\lambda = 1.68\mu m - 1.54\mu m = 0.14\mu m = 140nm$$

Longitudinal Modes Calculation

The number of allowed modes in the *InGaAs* micro cavity are calculated with the following equation:

$$N_m = Int\left[2\frac{L}{\lambda_0}NA\right] + 1 \qquad [2](9.6)$$

where *Int* is the integer part function, *L* is the length of the stripe, λ_0 is the main wavelength and *NA* is the numerical aperture defined as:

$$NA = \sqrt{n_1^2 - n_2^2} \qquad [2]\ (9.7)$$

from these equations we obtain the requirement of the for a *single mode waveguide*:

$$\frac{L}{\lambda_0} < \frac{1}{2NA} \qquad [2]\ (9.8)$$

As an example, an *InGaAs* waveguide ($n_1 = 3.9$) sandwiched between two layers of AlGaAs ($n_2 = 3.0$) results in a numerical aperture of

$$NA = \sqrt{3.9^2 - 3.0^2} = 2.49$$

and for a wavelength of 1.68µm and length of 250µm we have

$$N_m = Int\left[2\frac{250\mu m}{1.68\mu m}2.49\right] + 1 = 742$$

Now, the temperature due to losses increases the length of the micro cavity, *L*. The thermal expansion coefficients of *InAs* and *GaAs* are respectively,

$$TC_{InAs} = 4.52 * 10^{-6} m/^\circ C \qquad TC_{GaAs} = 5.73 * 10^{-6} m/^\circ C$$

Since the concentration of indium for 1.68µm wavelength is 0.53, using a weighted average we obtain the thermal expansion coefficient of $In_{0.53}Ga_{0.47}As$ as follows:

$$TC_{In_{0.53}Ga_{0.47}As} = (0.53) * 4.52 * 10^{-6} m/^\circ C + (0.47) * 5.73 * 10^{-6} m/^\circ C = 5.08 * 10^{-6} m/^\circ C$$

since the temperature obtained in equation (10.3) was 0.63˚C we obtain the new length,

$$\Delta L = 0.63^\circ C * 5.08 \mu m/^\circ C = 3.2 \mu m$$

which is approximately 1% of the original length. Using this new length we have

$$N_m = Int\left[2\frac{253.2\mu m}{1.68\mu m}2.49\right] + 1 = 751$$ that is, 9 modes more than before.

CONCLUSIONS

Several parameters for an *InGaAs* laser were calculated. The wavelength of the emitted light can be tuned in a wide range by changing the concentration of indium or by changing the material of the barriers. The thickness of the InGaAs stripe has a big impact on the angles of divergence of the emitted laser beam. It was found that temperature has a direct effect on the linewidth of the laser as well as on the allowed modes.

REFERENCES

[1] Semiconductor-Laser Physics, textbook by Chow, Koch and Sargent III. Springer-Verlag, 1994. ISBN=3-540-57614-2

[2] Optoelectronics, textbook by Emmanuel Rosencher and Borge Vinter. Cambridge University Press, 2002. ISBN=0 521 77129 3

[3] Introduction to Laser Physics, textbook by Koichi Shimoda. Springer-Verlag, 1984. ISBN=3-540-16713-7

[4] Long-wavelength Semiconductor Lasers on InGaAs Ternary Substrates with Excellent Temperature Characteristics. Paper by Koji Otsubo, Yoshito Nishijima and Hiroshi Ishikawa. FUJITSU Science and Technology Journal, pp 212-222, December 1998

[5] What is InGaAs? An explanation of Indium Gallium Arsenide. Web Page located at: www.sensorsinc.com/company/ingaas.asp as of November 18, 2003.

[6] Principles of Electronic Materials and Devices, Second Edition, by S.O. Kasap. McGraw Hill 2002. ISBN=0-07-239342-4

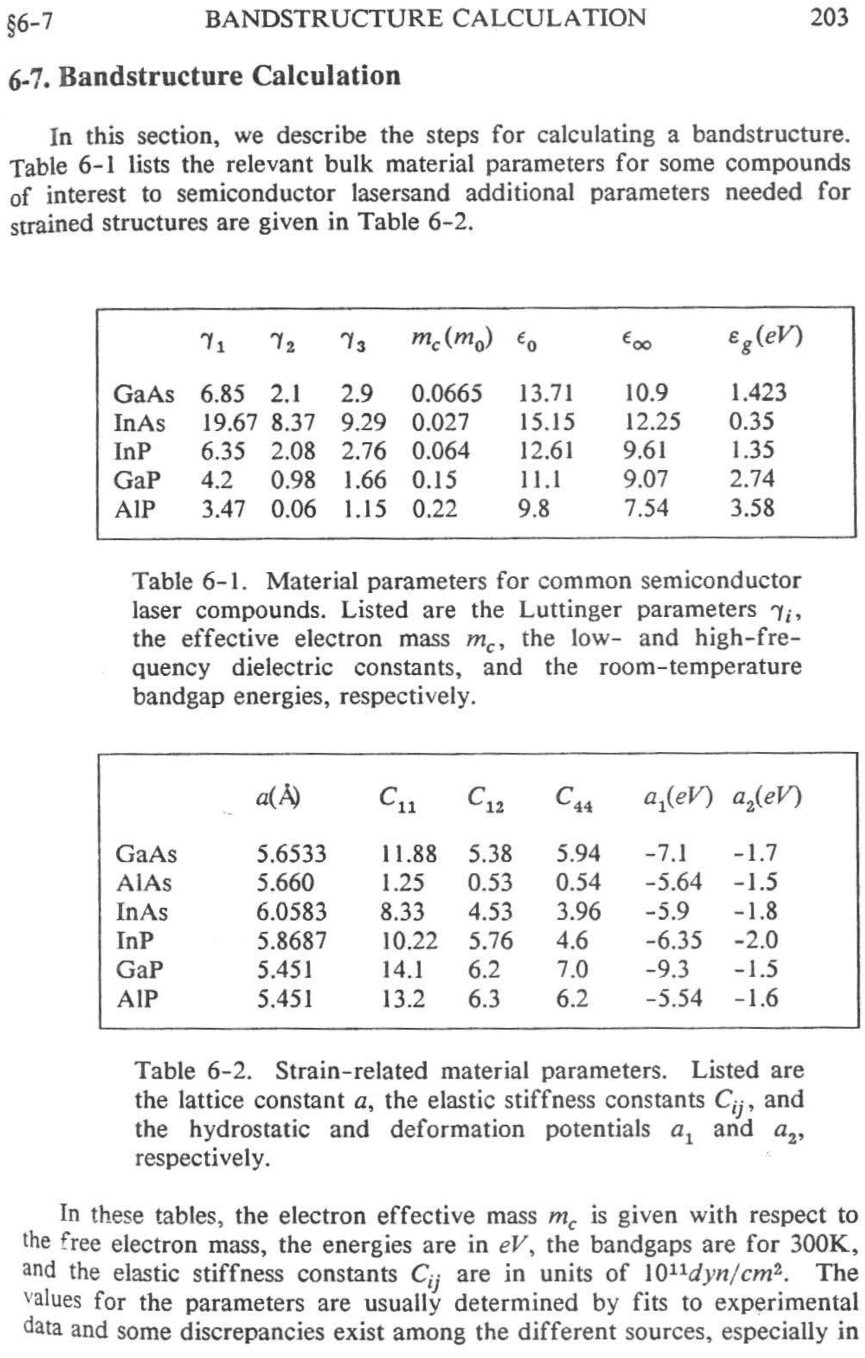
APPENDIX A. MATLAB SCRIPTS

```
%*********************************************************
% SCRIPT: InGaAsCurve
%*********************************************************
% This Script Plots the wavelength vs. indium concentration of
% an InGaAs stripe.
%*********************************************************
x=0.05:0.01:0.95;
N=length(x);
y=zeros(N);
for i=1:1:N
   y(i)=InGaAs(x(i))/(10^-6);
end
plot(x,y);
title('Wavelength of InGaAs Laser vs. Indium concentration - x');
xlabel('Indium concentration - x ');
ylabel('Wavelength in micrometers');
grid on;
```

```
%*****************************************************
% SCRIPT: InPInGaAsCurve
% ****************************************************
% This script plots the wavelength vs. Indium concentration
% of an InP-InGaAs-InP laser.
% ****************************************************
x=0.05:0.01:0.95;
N=length(x);
y=zeros(N,1);
for i=1:1:N
   [Ehh,Elh,Eg]=BandEv(x(i));
   if (Ehh > Elh) % determine highest valence band:
      y(i,1)=Lambda(Eg-Ehh)/10^-6;
   else
      y(i,1)=Lambda(Eg-Elh)/10^-6;
   end
end
plot(x,y);
title('Wavelength of InP-InGaAs-InP Laser vs. Indium concentration - x');
xlabel('Indium concentration - x ');
ylabel('Wavelength in micrometers');
grid on;
```

APPENDIX B. SELECT REFERENCES

REFERENCE 1



6-7. Bandstructure Calculation

In this section, we describe the steps for calculating a bandstructure. Table 6-1 lists the relevant bulk material parameters for some compounds of interest to semiconductor lasersand additional parameters needed for strained structures are given in Table 6-2.

	γ_1	γ_2	γ_3	$m_c(m_0)$	ϵ_0	ϵ_∞	$\varepsilon_g(eV)$
GaAs	6.85	2.1	2.9	0.0665	13.71	10.9	1.423
InAs	19.67	8.37	9.29	0.027	15.15	12.25	0.35
InP	6.35	2.08	2.76	0.064	12.61	9.61	1.35
GaP	4.2	0.98	1.66	0.15	11.1	9.07	2.74
AlP	3.47	0.06	1.15	0.22	9.8	7.54	3.58

Table 6-1. Material parameters for common semiconductor laser compounds. Listed are the Luttinger parameters γ_i, the effective electron mass m_c, the low- and high-frequency dielectric constants, and the room-temperature bandgap energies, respectively.

	a(Å)	C_{11}	C_{12}	C_{44}	$a_1(eV)$	$a_2(eV)$
GaAs	5.6533	11.88	5.38	5.94	-7.1	-1.7
AlAs	5.660	1.25	0.53	0.54	-5.64	-1.5
InAs	6.0583	8.33	4.53	3.96	-5.9	-1.8
InP	5.8687	10.22	5.76	4.6	-6.35	-2.0
GaP	5.451	14.1	6.2	7.0	-9.3	-1.5
AlP	5.451	13.2	6.3	6.2	-5.54	-1.6

Table 6-2. Strain-related material parameters. Listed are the lattice constant a, the elastic stiffness constants C_{ij}, and the hydrostatic and deformation potentials a_1 and a_2, respectively.

In these tables, the electron effective mass m_c is given with respect to the free electron mass, the energies are in eV, the bandgaps are for 300K, and the elastic stiffness constants C_{ij} are in units of $10^{11} dyn/cm^2$. The values for the parameters are usually determined by fits to experimental data and some discrepancies exist among the different sources, especially in

REFERENCE 3

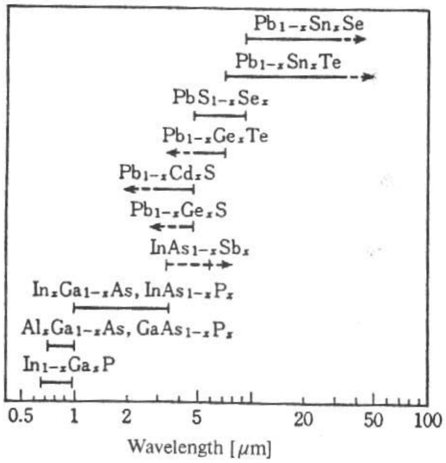


Fig. 1.9. Wavelength range of output of various tertiary semiconductor lasers

REFERENCE 4

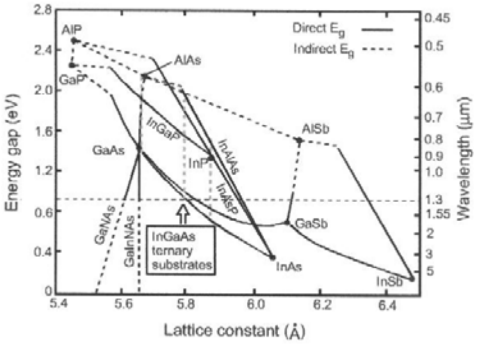


Figure 2
Bandgap energy versus lattice constant of III-V compound semiconductors.

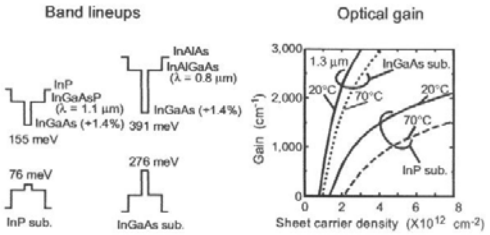


Figure 3
Band lineups of 1.3 μm quantum wells on InP substrates, and InGaAs substrates and their calculated optical gain.