

Contribution to inertial mass by reaction of the vacuum to accelerated motion

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Abstract

We present an approach to understanding the origin of inertia involving the electromagnetic component of the quantum vacuum and propose this as a step toward an alternative to Mach's principle. Preliminary analysis of the momentum flux of the classical electromagnetic zero-point radiation impinging on accelerated objects as viewed by an inertial observer suggests that the resistance to acceleration attributed to inertia may be at least in part a force of opposition originating in the vacuum. This analysis avoids the *ad hoc* modeling of particle-field interaction dynamics used previously by Haisch, Rueda and Puthoff (Phys. Rev. A **49**, 678, 1994) to derive a similar result. This present approach is not dependent upon what happens at the particle point, but on how an external observer assesses the kinematical characteristics of the zero-point radiation impinging on the accelerated object. A relativistic form of the equation of motion results from the present analysis. Its manifestly covariant form yields a simple result that may be interpreted as a contribution to inertial mass. We note that our approach is related by the principle of equivalence to Sakharov's conjecture (Sov. Phys. Dokl. **12**, 1040, 1968) of a connection between Einstein action and the vacuum. The argument presented may thus be construed as a descendant of Sakharov's conjecture by which we attempt to attribute a mass-giving property to the electromagnetic component — and possibly other components — of the vacuum. In this view the physical momentum of an object is related to the radiative momentum flux of the vacuum instantaneously contained in the characteristic proper volume of the object. The interaction process between the accelerated object and the vacuum (akin to absorption or scattering of electromagnetic radiation) appears to generate a physical resistance (reaction force) to acceleration suggestive of what has been historically known as inertia.

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I. INTRODUCTION

As discussed recently by Vigier [1], “the origin and nature of inertial forces... can be considered as an unsolved mystery in modern physics. It still sits, like Banco’s ghost, at any banquet of natural philosophers.” The instantaneous opposition to acceleration of all material objects is conventionally assumed to be a universal property of matter known as inertia. Historically there have been two views on the origin of inertial mass. It has been assumed to be either an inherent internal property of matter capable of no further explanation, or, in the view of Mach, a property that somehow originates externally in a collective linkage among all matter in the universe. This last, often referred to as Mach’s principle, may be exemplified in a thought experiment. Rotation is a form of acceleration. The inertia of matter manifests itself in the existence of centrifugal (and Coriolis) forces in the reference frame of a rotating object. Imagine a universe containing only a single object. In the view of Mach it would be an absurdity to claim that, in an otherwise empty universe, this object is capable of rotation. This would imply that centrifugal (and Coriolis) forces could not manifest and that therefore the single object in an empty universe should be devoid of inertia. If a single external object is now introduced, the phenomenon of rotation, by virtue of external reference, is again possible and the inertia of the rotating object should reappear. This allows the interpretation that the external object is the cause of the inertia of the rotating object. However it would be unphysical to assume that any external object no matter how minute should be capable of creating all at once the “full inertia” in the rotating object that it would otherwise possess in the “standard” universe. It can thus be argued in the Machian view that inertia must be an asymptotic function of surrounding matter that would gradually come into being as the universe is filled around the object in question.

A rigorous and quantitative formulation of Mach’s principle has never been successfully developed [2]. A tentative attempt by Sciama [3] to quantify Mach’s principle by associating inertial mass generation with a vector extension of the gravitational potential (analogous to a gravitation current) resulted in a prediction that was later shown to be inconsistent with experimental evidence. In the Sciama formulation, the asymmetrical distribution of surrounding matter in the Milky Way should result in a directional dependence of inertial mass with respect to galactic coordinates as measurable in a laboratory, this amounting to a variation on the order of $\Delta m/m = 10^{-7}$ whereas the experiments of Hughes and Drever subsequently indicated that $\Delta m/m \leq 10^{-20}$ [4].

That general relativity is not Machian is exemplified by the fact that it is possible to formulate solutions of the field equations for an empty universe and for a rotating universe. Recent examination on the relationship of the Lense-Thirring effect with general relativity further demonstrates the absence of a clear relationship and possible inconsistency between Mach’s principle and general relativity [5]. Additional conflicts between general relativity and Mach’s principle are presented by Vigier [1].

The Machian view would imply that it is entirely arbitrary whether one regards acceleration as motion (or rotation) of the object in question or as counter-motion (or counter-rotation) of the rest of the universe. However because the inertia reaction force occurs at the same instant that acceleration is applied to an object it becomes causally awkward to explain how Mach’s principle could be accommodated without the need for instantaneous propagation of some kind of back-reaction field involving infinite velocity, thus violating relativity and causality [6]. Preservation of causality is, of course, a strong argument for finding a basis of inertia that involves locally-originating forces and interactions. The approach of Vigier [1] is to find such a non-Machian basis in local interactions of a real Dirac subquantum aether model stemming from Einstein-de Broglie-Bohm causal stochastic quantum mechanics. The view presented herein substitutes for Mach’s principle in identifying the electromagnetic fields of the quantum vacuum as the external causative agent of inertia by providing a locally-originating reaction force. Limitations of treatment allow us to show this only for the electromagnetic vacuum, leaving the contributions of other vacuum fields for further extensions of the theory. An example of a contribution by other vacuum fields is precisely the one recently presented by Vigier [1].

The original development of this idea [7] proposed that the inertial property of matter could originate in Lorentz-force interactions between electromagnetically interacting particles at the level of their most

fundamental components (e.g., electrons and quarks) and the quantum vacuum (QV) [8]. This general idea is a descendent of a conjecture of Sakharov for the case of Einstein action [9] that can be extended by the principle of equivalence to the case of inertia. The approach of stochastic electrodynamics (SED) was used in [7] to study the classical dynamics of a highly idealized model of a fundamental particle constituent of matter (that contained a “parton”, i.e., a surrogate for a very fundamental particle component) responding to the driving forces of the so-called electromagnetic zero-point field (ZPF), the classical analog of the QV.

Reservations can be raised about the proposal of [7], for example: (a) the complexity, in that the analysis of the classical charged particle-ZPF dynamics involved an extensive calculational development which complicated the assessment of the physical validity of the approach; and (b) the introduction of *ad hoc* dynamical models for the interaction of the field and matter particularly at very high frequencies. Among these dynamical assumptions there were two of clear concern: (i) the idealized representation of a particle as a “parton” (Planck oscillator) and (ii) the use of the Abraham-Lorentz-Dirac (ALD) equation as the starting point (the ALD equation originates in Newton’s Laws). Details on problems with the development in [7] will be discussed in [10].

Consequently the primary purpose of the present paper is to outline a simple approach which avoids some of these model-related issues by examining how an opposing flux of radiative energy and momentum should arise under natural and suitable assumptions in an accelerated frame from the viewpoint of an inertial observer without regard to details of particle-field dynamics, i.e., independently of any dynamical models for particles. As the details of particle-field interactions are not of concern in the present case, the use of the classical electromagnetic ZPF formalism of SED looks quite natural. Using standard relativistic transformations for the electromagnetic fields, it is argued that upon acceleration a time rate of change of momentum density or momentum flux will arise out of the ZPF in the proper volume of any accelerating object, and that this turns out to be directed against and linearly proportional to the acceleration (Sec. II, III, IV and V). This arises after evaluation of the ZPF momentum density (Sec. IV, V) as it appears at a given point in an accelerated frame S , to an independent inertial laboratory observer due to transformations of the fields from the observer’s inertial laboratory reference frame I_* , to another inertial frame I_τ , instantaneously comoving with the object (Sec. II) and from the viewpoint of the observer in the laboratory inertial frame I_* . Absorption or scattering of this radiation by the accelerated charged particle is found to result in a force opposing the acceleration, yielding an $\mathbf{f} = m\mathbf{a}$ relation for subrelativistic motions. We follow standard notation in using $\mathbf{f}$ to refer to a three-force, and $\mathbf{F} = \gamma\mathbf{f}$ to refer to the corresponding space part of the relativistic four-force; cf. Eq. (9). The relativistic form of the force expression is obtained in Appendix D and presented in Sec. VI. For the execution of this development we again assume hyperbolic motion [7, 11, 12] (i.e., constant proper acceleration). Extension to an arbitrary accelerated motion is readily envisioned (Sec. VII). Section VIII concludes the paper. Important details or elaborations, extensions, and refinements are left for the appendices (A, B, C and D).

What we tentatively propose here is that when an object of rest mass m receives an impulse and is thereby accelerated by an external agent, the following two features deserve special attention.

(1) The scattering of the incoming ZPF flux within the object is what generates a reaction force heretofore attributed solely to the existence of an unexplained property called inertia. This clearly must be directed opposite to the direction of acceleration. As shown below this reaction force will turn out to be proportional to the acceleration $\mathbf{a}$ but in opposite direction to it, $\mathbf{f}^{zp} \sim -\mathbf{a}$. If there were an ideal body acting as a “perfect absorber”, i.e., capable of interacting with all the incoming flux of momentum from the ZPF up to the highest frequencies, an enormous maximum total reaction force would appear. In the case of a more realistic but still idealized body of characteristic proper volume V_0 that intercepts only a certain proportion η of the radiation ($0 < \eta < 1$), owing to this opacity there appears a reaction force on the body of the form $\mathbf{f}^{zp} \sim -V_0\eta\mathbf{a}$. The effect should clearly be larger for bodies of larger volume V_0 and/or such that η , the matter-radiation coupling coefficient, is larger. In this interpretation inertial mass becomes a function of such opacity. As the SED classical ZPF background is such a large reservoir of energy, this phenomenological coupling coefficient η can be extremely small for a substantial effect to still appear. We do not concern ourselves in this paper with the nature or strength of η , i.e., we omit consideration of the detailed dynamics of interaction of the ZPF

with matter in general or with material particles in particular. We report only on the necessary existence of a force of opposition by the ZPF as characterized by a change in the electromagnetic momentum density to the accelerated motion of the object without any concern for the details of the particle-radiation interaction embodied in the efficiency factor η .

(2) After the acceleration process is completed, from the point of view of an inertial observer attached to the stationary laboratory frame there appears associated with the body in motion a net flux of momentum density in the surrounding ZPF. In other words, on calculating the ZPF momentum contained in the object as referenced to the observer's own inertial frame, the observer would conclude that a certain amount of momentum is instantaneously contained within the proper volume, V_0 , of the moving object. This momentum is directly related to what would normally be called the physical momentum of the object. Calculated with respect to its own frame the object itself would find no net ZPF momentum contained within itself, consistent with the view that one's own momentum is necessarily always zero.

II. ZERO-POINT FIELD AND HYPERBOLIC MOTION

In the following we reference a small (in a sense to be specified below) and accelerated “object” consisting of elementary “particles” contained within a small volume. The term “object” can refer to either such a spatially extended but small entity or, in the context of reference frames, to its central point. We assume a non-inertial frame of reference, S , accelerated in such a way that the acceleration $\mathbf{a}$ as seen from an object fixed to a specific point, namely $(c^2/a, 0, 0)$, in the accelerated system, S , remains constant, i.e., the point $(c^2/a, 0, 0)$ is uniformly accelerated. Such condition leads as in [7, 11] to the well-known case of hyperbolic motion [12]. We again represent the classical electromagnetic ZPF in the traditional form and assume the same three reference systems I_* , I_τ , and S as in [7] and originally introduced in [11]. I_* is the inertial laboratory frame. S is the accelerated frame in which the object is placed at rest at the point $(c^2/a, 0, 0)$. τ is the object proper time as measured by a clock located at this same object point $(c^2/a, 0, 0)$ of S . I_τ is an inertial system whose $(c^2/a, 0, 0)$ point at proper time τ exactly (but only instantaneously) coincides with the object point of S . The acceleration of this $(c^2/a, 0, 0)$ point of S is $\mathbf{a}$ as measured from I_τ . Hyperbolic motion is defined such that $\mathbf{a}$ is the same for all proper times τ as measured in the corresponding I_τ frames at a point $(c^2/a, 0, 0)$ that in each one of these I_τ frames instantaneously comoves and coincides with the corresponding object point, namely $(c^2/a, 0, 0)$ of S . At proper time $\tau = 0$ this object point of the S -system also instantaneously coincides with the $(c^2/a, 0, 0)$ point of I_* and thus $I_* = I_\tau(\tau = 0)$. We refer to the observer’s laboratory time in I_* as t_* , chosen such that $t_* = 0$ at $\tau = 0$. For simplicity we let the object acceleration $\mathbf{a}$ at proper time τ take place along the x -direction so that $\mathbf{a} = a\hat{x}$ is the same constant vector, as seen at every proper time τ in every corresponding I_τ system. The acceleration of the $(c^2/a, 0, 0)$ point of S as seen from I_* is $\mathbf{a}_* = \gamma_\tau^{-3}\mathbf{a}$ [12]. Occasionally we refer to S as the Rindler non-inertial frame. We take it as a “rigid” frame [12]. It can be shown that as a consequence the acceleration $\mathbf{a}$ is not the same for the different points of S , but we are only interested in points inside a *small* neighborhood of the center of the accelerated object [12]. Specifically we are interested in a neighborhood of the object’s central point that contains the object and within which the acceleration is everywhere essentially the same.

Because of the hyperbolic motion [7,11,12], the velocity $u_x(\tau) = \beta_\tau c$ of the object point fixed in S with respect to I_* , is

$$\beta_\tau = \frac{u_x(\tau)}{c} = \tanh\left(\frac{a\tau}{c}\right) \quad (1)$$

and then

$$\gamma_\tau = (1 - \beta_\tau^2)^{-1/2} = \cosh\left(\frac{a\tau}{c}\right). \quad (2)$$

The ZPF in the laboratory system I_* is given by [7,11]

$$\mathbf{E}^{zp}(\mathbf{R}_*, t_*) = \sum_{\lambda=1}^2 \int d^3k \, \hat{\epsilon}(\mathbf{k}, \lambda) H_{zp}(\omega) \cos[\mathbf{k} \cdot \mathbf{R}_* - \omega t_* - \theta(\mathbf{k}, \lambda)], \quad (3a)$$

$$\mathbf{B}^{zp}(\mathbf{R}_*, t_*) = \sum_{\lambda=1}^2 \int d^3k \, (\hat{k} \times \hat{\epsilon}) H_{zp}(\omega) \cos[\mathbf{k} \cdot \mathbf{R}_* - \omega t_* - \theta(\mathbf{k}, \lambda)]. \quad (3b)$$

(See however statements on a normalization factor following Eq. (A5) in Appendix A. See also Ref. [13]). $\mathbf{R}_*$ and t_* refer respectively to the space and time coordinates of the point of observation of the field in I_* . At $t_* = 0$, the point $\mathbf{R}_* = (c^2/a)\hat{x}$ of I_* and the object in S coincide (see Eq. (6) below). The phase term $\theta(\mathbf{k}, \lambda)$ is a family of random variables, uniformly distributed between 0 and 2π , whose mutually independent elements are indexed by the wavevector $\mathbf{k}$ and the polarization index λ (or more technically, $\theta(\mathbf{k}, \lambda)$ is a stochastic process with index set $\{(\mathbf{k}, \lambda)\}$). Furthermore one defines,

$$H_{zp}^2(\omega) = \frac{\hbar\omega}{2\pi^2}. \quad (4)$$

As the coordinates $\mathbf{R}_*$ and time t_* refer to the particle point of the accelerated frame S as viewed from I_* we, for convenience, Lorentz-transform the fields from I_* to the corresponding I_τ frame tangential to S and then (omitting for simplicity to display explicitly the λ and $\mathbf{k}$ dependence in the polarization vectors, $\hat{\epsilon} = \hat{\epsilon}(\mathbf{k}, \lambda)$), obtain

$$\begin{aligned} \mathbf{E}^{zp}(0, \tau) = \sum_{\lambda=1}^2 \int d^3k \{ \hat{x}\hat{\epsilon}_x + \hat{y}\gamma_\tau[\hat{\epsilon}_y - \beta_\tau(\hat{k} \times \hat{\epsilon})_z] + \hat{z}\gamma_\tau[\hat{\epsilon}_z + \beta_\tau(\hat{k} \times \hat{\epsilon})_y] \} \\ \times H_{zp}(\omega) \cos[\mathbf{k} \cdot \mathbf{R}_* - \omega t_* - \theta(\mathbf{k}, \lambda)] \end{aligned} \quad (5a)$$

$$\begin{aligned} \mathbf{B}^{zp}(0, \tau) = \sum_{\lambda=1}^2 \int d^3k \{ \hat{x}(\hat{k} \times \hat{\epsilon})_x + \hat{y}\gamma_\tau[(\hat{k} \times \hat{\epsilon})_y + \beta_\tau\hat{\epsilon}_z] + \hat{z}\gamma_\tau[(\hat{k} \times \hat{\epsilon})_z - \beta_\tau\hat{\epsilon}_y] \} \\ \times H_{zp}(\omega) \cos[\mathbf{k} \cdot \mathbf{R}_* - \omega t_* - \theta(\mathbf{k}, \lambda)], \end{aligned} \quad (5b)$$

where the zero in the argument of the I_τ fields, $\mathbf{E}^{zp}$ and $\mathbf{B}^{zp}$ actually means the I_τ spatial point $(c^2/a, 0, 0)$. Here we observe three things. First, we take the fields that correspond to the ZPF as viewed from every inertial frame I_τ (whose $(c^2/a, 0, 0)$ point coincides with the particle point $(c^2/a, 0, 0)$ of S and instantaneously comoves with the object at the corresponding instant of proper time τ) to also represent the ZPF viewed instantaneously and from the single point $(c^2/a, 0, 0)$ in S . Note the dependence on the proper time τ of the object. Second, the corresponding fields in I_τ are obtained from a simple Lorentz rotation from I_* into I_τ . Hence for every proper time τ in the Rindler non-inertial frame, the fields $\mathbf{E}^{zp}$ and $\mathbf{B}^{zp}$ appear as expanded in terms of the four-vector $(k, \mathbf{k})$ whose components are the wavevector magnitude $k = \omega/c$ and wavevector $\mathbf{k}$ of I_* [11]. This will prove to be an advantageous simplifying feature that will help in establishing I_* as the ultimate reference frame in terms of which everything at the object point in S at any proper time τ is written.

The third and final point is crucial: Though the fields at the object point in S and in the corresponding tangential frame I_τ instantaneously coincide, this does not mean that detectors in S and in I_τ will be subject to the same effect, i.e., experience the same radiation-field time evolution. *Detectors need time to perform their measurements*: This necessarily involves integration over some interval of time and the evolution of the fields in S and in I_τ are obviously different. Hence a detector at rest in I_τ and the same detector at rest in S do not experience the same thing even during a short time interval. (This point touches on the origin of the Unruh-Davies effect, which is beyond the scope of the present paper.)

Consider any dynamical system such as, for example, a collection of interacting particles: An infinite number of different dynamical states can yield the same instantaneous snapshot of the system configuration even though the dynamical states of motion may be radically different. Two snapshots separated in time are necessary to distinguish differences among systems coordinates involving velocities; three will begin to distinguish accelerations, etc. The state of the system cannot be captured in an exact instant of time, but rather is a function of the time evolution. Summarizing, while the two fields, namely that of S and that of I_τ , are the same at a given space-time point, the evolution of the field in S and the evolution of the field in I_τ are by no means the same. Furthermore any field or radiation measurements in I_τ and in S both take some time and are not confined to a single space-time point.

We clarify the notation used in the sense that all polarization components are understood to be scalars, i.e., directional cosines, but written in the form $\hat{\epsilon}_i(\mathbf{k}, \lambda) \equiv \hat{\epsilon} \cdot \hat{x}_i$, where $\hat{x}_i = \hat{x}, \hat{y}, \hat{z}$; $i = x, y, z$. The karat means that they come from axial projections of the polarization unit vector $\hat{\epsilon}$. We use the same convention for components of the $\hat{k}$ unit vector where, e.g., $\hat{k}_x$ denotes $\hat{k} \cdot \hat{x}$. We can select space and time coordinates and orientation in I_* such that [11,12]

$$\mathbf{R}_*(\tau) \cdot \hat{x} = \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) \quad (6)$$

$$t_* = \frac{c}{a} \sinh \left(\frac{a\tau}{c} \right) \quad (7)$$

From Eqs. (1), (2) and (6–9) one obtains [7, 11]

$$\begin{aligned} \mathbf{E}^{zp}(0, \tau) = & \sum_{\lambda=1}^2 \int d^3k \times \\ & \left\{ \hat{x} \hat{e}_x + \hat{y} \cosh \left(\frac{a\tau}{c} \right) \left[\hat{e}_y - \tanh \left(\frac{a\tau}{c} \right) (\hat{k} \times \hat{e})_z \right] + \hat{z} \cosh \left(\frac{a\tau}{c} \right) \left[\hat{e}_z + \tanh \left(\frac{a\tau}{c} \right) (\hat{k} \times \hat{e})_y \right] \right\} \\ & \times H_{zp}(\omega) \cos \left[k_x \frac{c^2}{a} \cosh \left(\frac{a\tau}{c} \right) - \frac{\omega c}{a} \sinh \left(\frac{a\tau}{c} \right) - \theta(\mathbf{k}, \lambda) \right] \end{aligned} \quad (8a)$$

$$\begin{aligned} \mathbf{B}^{zp}(0, \tau) = & \sum_{\lambda=1}^2 \int d^3k \times \\ & \left\{ \hat{x} (\hat{k} \times \hat{e})_x + \hat{y} \cosh \left(\frac{a\tau}{c} \right) \left[(\hat{k} \times \hat{e})_y + \tanh \left(\frac{a\tau}{c} \right) \hat{e}_z \right] + \hat{z} \cosh \left(\frac{a\tau}{c} \right) \left[(\hat{k} \times \hat{e})_z - \tanh \left(\frac{a\tau}{c} \right) \hat{e}_y \right] \right\} \\ & \times H_{zp}(\omega) \cos \left[k_x \frac{c^2}{a} \cosh \left(\frac{a\tau}{c} \right) - \frac{\omega c}{a} \sinh \left(\frac{a\tau}{c} \right) - \theta(\mathbf{k}, \lambda) \right]. \end{aligned} \quad (8b)$$

This is the ZPF as instantaneously viewed from the object fixed to the point $(c^2/a, 0, 0)$ of S that is performing the hyperbolic motion.

III. DYNAMICAL SETTING: FORCE OF RESISTANCE TO ACCELERATED MOTION AND ZPF MOMENTUM FLUX

In 1968 Sakharov [9] published a brief conjecture regarding Einstein action. In this approach the concept of mass does not arise; objects simply move along geodesics. However one could go on to interpret this as gravity arising in certain perturbations by massive bodies of the surrounding vacuum fields, and the principle of equivalence would then imply that this should extend to inertia. A successful extension to a more general field theory than the one contemplated here would likely suggest that an inertia effect is caused by the vacuum of that theory, without the need of postulating any additional field solely for the purpose of giving mass to material entities (e.g. a mass-giving Higgs-type field).

The objective of the present paper is to study, by a completely different approach to that in [7], the hypothesis that inertial mass may be considered a vacuum effect, i.e., that within the limited context of our treatment, the parameter m of Newton's second law ($\mathbf{f} = m\mathbf{a}$) can be explained as an effect due to the ZPF, i.e., that the inertial rest mass can be explained, at least in part, as a coefficient involving the object, its electromagnetic coupling and other ZPF parameters. This is attempted now not by means of a dynamical analysis on a very specific model (as in [7]) but instead by careful examination of the structure of the fields viewed in relation to an object being compelled to perform accelerated motion by an external agent. The goal of such analysis is to find an expression and an explanation for the m parameter.

Newton's second law can be more generally written, but still in its traditional nonrelativistic form, as

$$\mathbf{f} = \frac{d\mathbf{p}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{p}}{\Delta t}, \quad (9a)$$

which is the limiting form of the space part of the relativistic four-force form of Newton's law:

$$\mathbf{F} = \frac{d\mathbf{p}}{d\tau} = \gamma \frac{d\mathbf{p}}{dt}, \quad (9b)$$

which for the case when $\beta \rightarrow 0$ and $\gamma \rightarrow 1$ (corresponding for us to the object in the $\tau \rightarrow 0$ limit when I_τ coincides with I_* and then $\gamma_\tau \rightarrow 1$) becomes

$$\mathbf{f} = \left. \frac{d\mathbf{p}}{d\tau} \right|_{\tau=0}. \quad (9c)$$

Having defined force in his second law as the rate of change of momentum imparted to an object by an agent, Newton then states in his third law that such a force will result in the creation of an equal and opposite reaction force back upon the accelerating agent. The concept of inertia becomes then a necessity: Inertia is thus necessarily attributed to the accelerating object in order to generate the equal and opposite reaction force upon the agent required by the third law. It is our proposition that resistance from the vacuum is what physically provides that reaction force. One can interpret this as either the origin of inertia of matter or as a substitute for the concept of innate inertia of matter. In other words, inertia becomes in a sense a placeholder for this heretofore undiscovered vacuum-based reaction force which is a necessary requirement of Newton's third law. Force is then seen to be a primary concept; inertia is not.

Newton's third law is essentially a statement about symmetry in nature for contact forces. In the static case (e.g., pressing one hand against the other), from symmetry alone an applied force $\mathbf{f}$, must necessarily result in a reaction force $\mathbf{f}_r$ such that

$$\mathbf{f} = -\mathbf{f}_r. \quad (10)$$

Inertia as the dynamical extension of this law can be made explicit by writing the $\mathbf{f} = m\mathbf{a}$ relation as

$$\mathbf{f} = -(-m\mathbf{a}), \quad (11)$$

which makes it clear that inertia as a resistance to acceleration is equivalent to a reaction force of the form

$$\mathbf{f}_r = -m\mathbf{a}. \quad (12)$$

To recapitulate our argument, Newton's third law states that if an agent applies a force to a point on an object, at that point there arises an equal and opposite force back upon the agent. Were this not the case, the agent would not experience the process of exerting a force and we would have no basis for mechanics. The law of equal and opposite macroscopic contact forces is thus fundamental both conceptually and perceptually, but it is legitimate to seek further underlying connections. In the case of a stationary object (fixed to the earth, say), the equal and opposite macroscopic forces can be said to arise in microscopic interatomic forces in the neighborhood of the point of contact which act to resist compression. This can be traced more deeply still to electromagnetic interactions involving orbital electrons of adjacent atoms or molecules, etc.

A similar experience of equal and opposite forces arises in the process of accelerating (pushing on) an object. It is an experimental fact that to accelerate an object a force must be applied by an agent and that the agent will thus experience an equal and opposite force so long as the acceleration continues. We argue that this equal and opposite force also has a deeper physical cause, which at least in part turns out to also be electromagnetic and is specifically due to the scattering or interaction with ZPF radiation. We demonstrate that from the point of view of a nearby inertial observer there exists a net energy and momentum flux (Poynting vector) of ZPF radiation transiting the accelerating object in a direction necessarily opposite to the acceleration vector. The scattering opacity of the object to the transiting flux creates the back-reaction force customarily called the inertial reaction. Inertia is thus, in part, a special kind of electromagnetic drag effect, namely one that is acceleration-dependent since only in accelerating frames is the ZPF perceived as asymmetric. In stationary or uniform-motion frames the ZPF is perfectly isotropic.

As a first step we must examine in precise detail how we estimate the change of momentum $\Delta\mathbf{p}$ implied in Eq. (9a) before taking the limit. Assume for concreteness that the massive object of rest mass m performs hyperbolic motion under the action of the external agent with corresponding constant proper acceleration $\mathbf{a}$ along the x -axis so that $\mathbf{a} = a\hat{x}$ as in Sec. II. At proper time $\Delta\tau$ the object is instantaneously at rest in the inertial coordinate frame $I_{\Delta\tau}$ at the point $(c^2/a, 0, 0)$ of that frame. Moreover at the object proper time $\tau = 0$ (that corresponds to the time $t_* = 0$ of I_*), the object was instantaneously detected at rest at the point $(c^2/a, 0, 0)$ of the laboratory inertial frame I_* by the observer located at that point. After a short lapse of laboratory time $\Delta t_* > 0$ that corresponds to the object proper time $\Delta\tau$, the object is seen, from the viewpoint of I_* , to have received from the accelerating agent the amount of impulse or momentum increment $\Delta\mathbf{p}_*$. The expression (9a) but as seen in I_* is thus

$$\mathbf{f}_* = \frac{d\mathbf{p}_*}{dt_*} = \lim_{\Delta t_* \rightarrow 0} \frac{\Delta\mathbf{p}_*}{\Delta t_*}. \quad (13)$$

At the corresponding object proper time $\Delta\tau$, the object is instantaneously at rest in the comoving inertial frame $I_{\Delta\tau}$. Consequently the momentum of the object at proper time $\Delta\tau$ and as viewed in $I_{\Delta\tau}$ is of course zero.

As proposed in [7], the force of opposition to the accelerating action imposed by the external agent does not come from the object itself but from the all-encompassing vacuum which is restrictively represented herein by only the electromagnetic ZPF. Our goal is to show that if this is so, the force of opposition, $\mathbf{f}_r$, in the subrelativistic case is strictly proportional to the negative of the acceleration, namely to $-\mathbf{a}$, as in Eq. (12), and can therefore reasonably be interpreted as a contribution to the inertia of the object.

Taking a vacuum-opposition-to-acceleration for granted, but not yet assuming that it is proportional to $-\mathbf{a}$, it then follows that if the force in Eq. (9a) accelerates the object and if our hypothesis is correct, there must, from Newton's third law, be an opposite matching reaction force due to the ZPF, $\mathbf{f}^{zp}$, such that (see Eq. 12):

$$\mathbf{f}^{zp} = \mathbf{f}_r = -\mathbf{f}. \quad (14)$$

The key is to find whether $\mathbf{f}^{zp}$ will prove, from relativistic electrodynamics, to be proportional to $-\mathbf{a}$. Actually, this should be true as viewed from any inertial frame whatsoever [14]. To recapitulate the direction of our argument, we assume in Eq. (13) that, in the sense of Rindler [12], Newton's second law just provides a definition for the force entity. Newtonian mechanics starts at this point, treating Eq. (13) as a postulate of physics. The equal and opposite force of the object on the pushing agent required by Newton's third law is traditionally assumed to be provided by the innate inertia of the accelerating object. Anticipating that the resistance comes instead from the vacuum, we write this equal and opposite reaction condition explicitly with the superscript ZP in Eq. (14). When we compare Eqs. (9), (12) and (14), it follows that if the accelerating agent by means of the force $\mathbf{f}$ gives to the object during a time interval $\Delta\tau$ an impulse or change of momentum $\Delta\mathbf{p}$, there must be a corresponding impulse (change of momentum) $\Delta\mathbf{p}^{zp}$ provided by the ZPF in exactly the same time interval and as viewed from the same inertial reference frame but in the opposite direction to $\Delta\mathbf{p}$ so that

$$\Delta\mathbf{p}^{zp} = -\Delta\mathbf{p}. \quad (15)$$

Hence $\Delta\mathbf{p}^{zp}$ is the matching reactive counter-impulse given by the ZPF that opposes the impulse $\Delta\mathbf{p}$ given by the accelerating agent. We refer both $\Delta\mathbf{p}^{zp}$ and $\Delta\mathbf{p}$ to the same inertial frame and in this case to the laboratory frame I_* and write

$$\Delta\mathbf{p}_*^{zp} = -\Delta\mathbf{p}_*. \quad (16).$$

As this momentum change for the object $\Delta\mathbf{p}_*$ is calculated with respect to the inertial frame (that conventionally we call the laboratory frame) I_* and not with respect to any other frame, (e.g., the inertial frame $I_{\Delta\tau}$) it is necessary to calculate the putative ZPF-induced opposing impulse $\Delta\mathbf{p}_*^{zp}$ with respect to the same inertial frame I_* (and not with respect to $I_{\Delta\tau}$ or any other frame). We write

$$\Delta\mathbf{p}_*^{zp} = \mathbf{p}_*^{zp}(\Delta t_*) - \mathbf{p}_*^{zp}(0) = \mathbf{p}_*^{zp}(\Delta t_*). \quad (17)$$

The momentum $\mathbf{p}_*^{zp}(\Delta t_*)$ is essentially the integral of $d\mathbf{p}_*^{zp}$ from I_* -frame time $t_* = 0$ to I_* -frame time $t_* = \Delta t_*$. The last equality follows from symmetry of the ZPF distribution as viewed in I_* that leads to

$$\mathbf{p}_*^{zp}(0) = 0. \quad (18)$$

In what follows we seek to find a mathematical expression for the ZPF-induced inertia reaction force $\mathbf{f}_*^{zp}$. For this purpose it is useful to state that from Newton's third law and the force defined above we can write that the following must be true if our hypothesis is correct:

$$\lim_{\Delta t_* \rightarrow 0} \frac{\Delta\mathbf{p}_*^{zp}}{\Delta t_*} = \mathbf{f}_*^{zp} = -\mathbf{f}_* = -\lim_{\Delta t_* \rightarrow 0} \frac{\Delta\mathbf{p}_*}{\Delta t_*}. \quad (19)$$

If the inertia origin propounded here is correct then Eq. (19), at least in the subrelativistic case, should yield a nonvanishing force $\mathbf{f}_*^{zp}$ that is parallel to the direction of the acceleration $\mathbf{a} = a\hat{x}$, opposite to it, and proportional to the acceleration magnitude $a = |\mathbf{a}|$.

IV. INERTIA REACTION FORCE AND THE ZPF MOMENTUM DENSITY

We use Eq. (19) both to evaluate and define the effect that we will identify with the ZPF inertia force $\mathbf{f}_*^{zp}$. We concern ourselves in this section with the ZPF momentum flux entering the object. Next, in Sec. V, we further develop this analysis (with the help of Appendix A) to the point of deriving the acceleration-dependent $\mathbf{f}_*^{zp}$. Finally in Appendix B we analyze the momentum content.

In order to fully grasp the situation we consider the following simple fluid analogy involving as a heuristic device a constant velocity and a spatially varying density in place of the usual hyperbolic motion through a uniform vacuum medium. Let a small geometric figure of a fixed proper volume V_0 move uniformly with constant subrelativistic velocity $\mathbf{v}$ along the x -direction. The volume V_0 we imagine as always immersed in a fluid that is isotropic, homogeneous and at rest, except such that its density $\rho(x)$ increases in the x -direction but is uniform in the y - and z -directions. Hence, as this small fixed volume V_0 moves in the x -direction, the mass enclosed in its volume, $V_0\rho(x)$, increases. In an inertial frame at rest with respect to the geometric figure the mass of the volume, $V_0\rho(x)$, is seen to grow. Concomitantly it is realized that the volume V_0 is sweeping through the fluid and that this $V_0\rho(x)$ mass grows because there is a net influx of mass coming into V_0 in a direction opposite to the direction of the velocity. In an analogous fashion, for the more complex situation envisaged in this paper, simultaneously with the steady growth of the ZPF momentum contained within the volume of the object discussed above, the object is sweeping through the ZPF of the I_* inertial observer and for him there is a net influx of momentum density coming from the background into the object and in a direction opposite to that of the velocity of the object.

As it is the ZPF radiation background of I_* in the act of being swept through by the object which we are calculating now, we fix our attention on a fixed point of I_* , say the point of the observer at $(c^2/a, 0, 0)$ of I_* , that momentarily coincides with the object at the object proper time $\tau = 0$, and consider that point as referred to the inertial frame I_τ that instantaneously will coincide with the object at a future generalized object proper time $\tau > 0$. Hence we compute the I_τ -frame Poynting vector, but evaluated at the $(c^2/a, 0, 0)$ space point of the I_* inertial frame, namely in I_τ at the I_τ space-time point:

$$ct_\tau = \frac{c^2}{a} \sinh\left(\frac{a\tau}{c}\right), \quad (20)$$

$$x_\tau = -\frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right), \quad y_\tau = 0, \quad z_\tau = 0. \quad (21)$$

This Poynting vector we shall denote by $\mathbf{N}_*^{zp}$. Everything however is ultimately referred to the I_* inertial frame as that is the frame of the observer that looks at the object and whose ZPF background the moving object is sweeping through. In order to accomplish this we first compute

$$\begin{aligned} \langle \mathbf{E}_\tau^{zp}(0, \tau) \times \mathbf{B}_\tau^{zp}(0, \tau) \rangle_x &= \langle E_{y\tau} B_{z\tau} - E_{z\tau} B_{y\tau} \rangle \\ &= \gamma_\tau^2 \langle (E_{y*} - \beta_\tau B_{z*})(B_{z*} - \beta_\tau E_{y*}) - (E_{z*} + \beta_\tau B_{y*})(B_{y*} + \beta_\tau E_{z*}) \rangle \\ &= -\gamma_\tau^2 \beta_\tau \langle E_{y*}^2 + B_{z*}^2 + E_{z*}^2 + B_{y*}^2 \rangle + \gamma_\tau^2 (1 + \beta_\tau^2) \langle E_{y*} B_{z*} - E_{z*} B_{y*} \rangle \\ &= -\gamma_\tau^2 \beta_\tau \langle E_{y*}^2 + B_{z*}^2 + E_{z*}^2 + B_{y*}^2 \rangle \end{aligned} \quad (22)$$

that we use in the evaluation of the Poynting vector [15]

$$\mathbf{N}_*^{zp} = \frac{c}{4\pi} \langle \mathbf{E}_\tau^{zp} \times \mathbf{B}_\tau^{zp} \rangle_* = \hat{x} \frac{c}{4\pi} \langle \mathbf{E}_\tau^{zp}(0, \tau) \times \mathbf{B}_\tau^{zp}(0, \tau) \rangle_x. \quad (23)$$

The integrals are now taken with respect to the I_* ZPF background (using then the k -sphere of I_* introduced in Appendix C) as that is the background that the I_* -observer considers the object to be sweeping through. This is why we denote this Poynting vector as $\mathbf{N}_*^{zp}$, with an asterisk subindex instead of a τ subindex, to indicate that it refers to the ZPF of I_* . Observe that in the last equality of Eq. (22) the term proportional to the x -projection of the ordinary ZPF Poynting vector of I_* vanishes. The net amount of momentum of

the background the object has swept through after a time t_* , as judged again from the I_* -frame viewpoint, is

$$\mathbf{p}_*^{zp} = \mathbf{g}_*^{zp} V_* = \frac{\mathbf{N}_*^{zp}}{c^2} V_* = -\hat{x} \frac{1}{c^2} \frac{c}{4\pi} \gamma_\tau^2 \beta_\tau \frac{2}{3} \langle \mathbf{E}_*^2 + \mathbf{B}_*^2 \rangle V_*, \quad (24)$$

which is the complement and clear counterpart of Eq. (B8) of Appendix B, i.e., the negative of the expression for $\mathbf{p}_*$ evaluated in Eq. (B9). Furthermore by means of Eq. (19) we will calculate the force $\mathbf{f}_*^{zp}$ directly from the expression for $\mathbf{p}_*^{zp}$. These steps are presented in the next Section. Prior to that however we present a discussion of the conceptual origin of the momentum flux expression of Eq. (24) complemented with a more detailed derivation of the cross product of Eq. (23) that is performed in Appendix A.

V. MOMENTUM FLUX AND NEWTONIAN INERTIA

Any observer at rest in an inertial frame sees the ZPF isotropically distributed around himself. The Poynting vector $\mathbf{N}^{zp}$ and the momentum density $\mathbf{g}^{zp} = \mathbf{N}^{zp}/c^2$ of such ZPF vanish for that observer. This is of course the case for the observer at rest in I_* . Consider now another inertial observer located at a geometric point that, with respect to I_* , moves uniformly with *constant* velocity, $\mathbf{v} = \hat{x}v_x = \hat{x}\beta c$. Imagine the instant of time when the geometric point is passing and in the immediate neighborhood of the stationary I_* observer. Both observers necessarily see the ZPF symmetrically and isotropically distributed around themselves in their own frames. However, the ZPF for each observer is not, because of the Doppler shifts, isotropically distributed with respect to the other frame. In the terminology of Appendix C the I_* -observer is located at the center of his own k -sphere, but the moving point is necessarily located off-center of the I_* -observer's k -sphere. Hence, for the I_* -observer the ZPF Poynting vector, $\mathbf{N}_*^{zp}$, and the corresponding momentum density, $\mathbf{g}_*^{zp}$, impinging on the moving point should appear to be non-vanishing. Furthermore, because the motion of the geometric point is uniform, not hyperbolic, both the $\mathbf{N}_*^{zp}$ and $\mathbf{g}_*^{zp}$ at the moving geometric point appear to the I_* -observer to be time-independent constants of the motion.

Extend now the consideration above to all the points inside a small ϵ -neighborhood of the previous geometric point that comove with *constant* velocity $\mathbf{v} = \hat{x}\beta c$. Let V_0 be the proper volume of that neighborhood. Because of length contraction such neighborhood has, in I_* , the volume $V_* = V_0/\gamma$. Clearly to the observer in I_* the neighborhood's $\mathbf{g}_*^{zp}$ and $\mathbf{N}_*^{zp}$ do not appear as vanishing because of the uniform motion with constant velocity, $\mathbf{v} = \hat{x}\beta c$, inducing Doppler shifts of all the neighborhood's points with respect to I_* . If the said neighborhood exactly coincides with the location and geometry of a moving object of proper volume V_0 and rest mass m_0 that has the neighborhood's central geometric point at its center, then according to ordinary mechanics, the object appears to the observer in I_* as carrying a mechanical momentum $\mathbf{p}_* = \gamma m_0 \mathbf{v}$.

We turn now to the object's corresponding ZPF momentum. Because the object occupies its proper volume V_0 and coincides with the uniformly moving ϵ -neighborhood, it has for the observer at rest in I_* an amount of ZPF momentum, $V_* \mathbf{g}_* = (V_0/\gamma) \mathbf{g}_*$, as described above. We re-emphasize that when measured and from the point of view of the *inertial observer comoving with the object*, both the object momentum and the Poynting vector of the ZPF do exactly vanish, the last because in k -space the object is at the center of that observer's k -sphere (Appendix C). In the present case of a constant velocity and zero acceleration for the object, as opposed to the general case we have been considering of accelerated hyperbolic motion, the momenta $\mathbf{p}_*$ and $\mathbf{p}_*^{zp}$ above are both of course constants. Hence their time derivatives in Eq. (19) both vanish. We return to our original hyperbolic motion problem.

Let us go back to the paragraph immediately preceding Eq. (22). We again compute Eqs. (23) and (24) but perform (23) in more detail. From Appendix A, we can compute the Poynting vector of Eq. (A4) that the radiation should have at the $(c^2/a, 0, 0)$ point of I_τ but referred to I_* with the coordinates of Eq. (22), viz,

$$\begin{aligned} \mathbf{N}_*^{zp}(\tau) &= \frac{c}{4\pi} \langle \mathbf{E}^{zp} \times \mathbf{B}^{zp} \rangle \\ &= \hat{x} \frac{c}{4\pi} \langle E_y B_z - E_z B_y \rangle \\ &= -\hat{x} \frac{c}{4\pi} \frac{8\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \end{aligned} \quad (25)$$

where $\mathbf{E}^{zp}$ and $\mathbf{B}^{zp}$ stand for $\mathbf{E}_\tau^{zp}(0, \tau)$ and $\mathbf{B}_\tau^{zp}(0, \tau)$ respectively as in the case of Eq. (23) and where as in Eqs. (22), (23) and (24) the integration is understood to proceed over the k -sphere of I_* . The object now is not in uniform but instead in accelerated motion. If suddenly at proper time τ the motion were to switch from hyperbolic back to uniform because the accelerating action disappeared, we would just need to replace in Eq. (25) the constant rapidity s at that instant for $a\tau$, and β_τ in Eq. (1) would then become $\tanh(s/c)$. (But then $\mathbf{N}^{zp}$ would cease to be, for all times onward, a function of τ and force expressions as Eq. (28) below would vanish.) Observe that we make explicit the τ dependence of this as well as of the

subsequent quantities below. $\mathbf{N}_*^{zp}(\tau)$ represents energy flux, i.e., energy per unit area and per unit time in the x -direction. It also implies a parallel, x -directed momentum density, i.e., field momentum per unit volume incoming towards the object position, $(c^2/a, 0, 0)$ of S , at object proper time τ and as estimated from the viewpoint of I_* . Explicitly such momentum density is

$$\mathbf{g}_*^{zp}(\tau) = \frac{\mathbf{N}_*^{zp}(\tau)}{c^2} = -\hat{x} \frac{8\pi}{3} \frac{1}{4\pi c} \sinh\left(\frac{2a\tau}{c}\right) \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega, \quad (26)$$

where we now introduce the henceforth frequency-dependent coupling coefficient, $0 \leq \eta(\omega) \leq 1$, that quantifies the fraction of absorption or scattering at each frequency. Let V_0 be the proper volume of the object, namely the volume that the object has in the reference frame I_τ where it is instantaneously at rest at proper time τ . From the viewpoint of I_* , however, such volume is then $V_* = V_0/\gamma_\tau$ because of Lorentz contraction. The amount of momentum due to the radiation inside the volume of the object according to I_* , i.e., the radiation momentum in the volume of the object viewed at the laboratory is

$$\mathbf{p}_*^{zp}(\tau) = V_* \mathbf{g}_*^{zp} = \frac{V_0}{\gamma_\tau} \mathbf{g}_*^{zp}(\tau) = -\hat{x} \frac{4V_0}{3} c \beta_\tau \gamma_\tau \left[\frac{1}{c^2} \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \right], \quad (27).$$

which is again Eq. (24).

At proper time $\tau = 0$, the $(c^2/a, 0, 0)$ point of the laboratory inertial system I_* instantaneously coincides and comoves with the object point of the Rindler frame S in which the object is fixed. The observer located at $x_* = c^2/a$, $y_* = 0$, $z_* = 0$ instantaneously, at $t_* = 0$, coincides and comoves with the object but because the latter is accelerated with constant acceleration $\mathbf{a}$, the object *according to* I_* should receive a time rate of change of incoming ZPF momentum of the form:

$$\frac{d\mathbf{p}_*^{zp}}{dt_*} = \frac{1}{\gamma_\tau} \frac{d\mathbf{p}_*^{zp}}{d\tau} \Big|_{\tau=0}. \quad (28)$$

We *postulate* that such rate of change may be identified with a force from the ZPF on the object. Such interpretation, intuitively at least, looks extremely natural. In this respect Rindler [12] in introducing Newton's second law makes the following important epistemological point: "This is only 'half' a law; for it is a mere definition of force," and this is precisely the sense in which we introduce it here as a definition of the force of reaction by the ZPF. If the object has a proper volume V_0 , the force exerted on the object by the radiation from the ZPF as seen in I_* at $t_* = 0$ is then

$$\frac{d\mathbf{p}_*^{zp}}{dt_*} = \mathbf{f}_*^{zp} = - \left[\frac{4}{3} \frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \right] \mathbf{a}. \quad (29)$$

Furthermore

$$m_i = \left[\frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \right] \quad (30)$$

is an invariant scalar with the dimension of mass. The expression for m_i differs considerably from the corresponding one in [7] because here, on purpose, no interaction features were included in the analysis. Such ZPF-particle interactions will be taken up in future work. Observe that in Eq. (30) we have neglected a factor of 4/3. Such factor must be neglected because a fully covariant analysis (Appendix D) will show that it disappears. The corresponding form of m_i as written (and without the 4/3 factor) is then susceptible of a very natural interpretation: Inertial mass of an object is that fraction of the energy of the ZPF radiation enclosed within the object that interacts with it (parametrized by the $\eta(\omega)$ factor in the integrand). Further discussion of this point we leave for the Appendices B and D. Clearly if the acceleration suddenly ceases at proper time τ , Eqs. (28) and (29) identically vanish signaling the fact that acceleration is the reason that the vacuum produces the opposition that we identify with the force of reaction known as inertia. From the proper time instant τ when the acceleration $\mathbf{a}$ is turned off, the object continues in uniform motion. The object proceeds onwards with the rapidity s it acquired up to that point, namely $a\tau$. Thus β_τ in Eq. (1)

and all quantities from Eqs. (25) to (27) become constants, as the rapidity s ceases to depend on the proper time τ . Because of the Lorentz invariance of the ZPF energy density spectrum [16], the object is left at rest in the inertial frame I_τ and at the center of the k -sphere of the I_τ observer but off-center of the k -sphere of the I_* one (Appendix C). From the I_* perspective the object appears to possess a momentum (which reflects the ZPF momentum inside V_0 , a point that will become clear in Appendix B). Observe furthermore that in Eq. (30) and previous equations some cut-off procedure is implicit in that $\eta(\omega)$ subsides at high frequencies. (Otherwise we recall the cut-off referred to in Appendix A according to the prescription of [17].)

VI. RELATIVISTIC FORCE EXPRESSION

The coefficient m_i that we identify with the ZPF contribution to inertial mass, corresponds then just to the ZPF-induced part of the rest mass of the object. In order to simplify the discussion that follows, however, we will usually take this ZPF contribution as all of inertial mass. If the vacuum exerts an opposition force on the accelerated object of magnitude $-m_i \mathbf{a}$ as in Eq. (29) and if Newton's third law (Eq. (14)) holds, then the accelerating agent must exert an active force $\mathbf{f}$ of amount $\mathbf{f} = m_i \mathbf{a}$ to produce the acceleration. This is Newton's equation of motion. The radiative opposition made by the vacuum precisely coincides time-wise with the onset of acceleration at every point throughout the interior of the accelerated object, continues exactly so long as the acceleration persists and is in direct proportion to the amount of mass associated with that small region. Herein lies the intuitive power of the approach. The only other alternative is the traditional one of Newton that assumes such inertial opposition comes from the object itself "because it has a fundamental property called mass." This however leaves the origin of inertia unexplained. Inertia we argue is a phenomenon created by the vacuum as in Eq. (30). It is important to add that our analysis yields not just the nonrelativistic Newtonian case but it also embodies a fully relativistic description within special relativity [12] at least for the case of longitudinal forces, i.e., forces parallel to the direction of motion (See however Sec. VII).

From the definition of the momentum $\mathbf{p}_*^{zp}$ in Eq. (27), from Eqs. (28), (29), and the force equation (14) it immediately follows that the momentum of the object is

$$\mathbf{p}_* = m_i \gamma_\tau \vec{\beta}_\tau c, \quad (31)$$

in exact agreement with the momentum expression for a moving object in special relativity [12]. The expression for the space vector component of the four-force [12] is then

$$\mathbf{F}_* = \gamma_\tau \frac{d\mathbf{p}_*}{dt_*} = \frac{d\mathbf{p}_*}{d\tau}, \quad (32)$$

and as the force is pure in the sense of Rindler [12], the correct form for the four-force immediately follows (recalling (12) for the space part):

$$\mathcal{F} = \frac{d\mathcal{P}}{d\tau} = \frac{d}{d\tau}(\gamma_\tau m_i c, \mathbf{p}) = \gamma_\tau \left(\frac{1}{c} \frac{dE}{dt}, \mathbf{f} \right) = \gamma_\tau \left(\mathbf{f} \cdot \vec{\beta}_\tau, \mathbf{f} \right) = \left(\mathbf{F} \cdot \vec{\beta}_\tau, \mathbf{F} \right), \quad (33)$$

in the ordinary way anticipated above. Consistency with Special Relativity is established. (For a detailed exposition pertaining to Eqs. 31–33 see Appendix D.) In Eq. (33) we have dropped the * subscript notation for generality.

VII. GENERAL MOTION

Our analysis so far has been restricted to the case of simple hyperbolic motion, i.e., rectilinear motion with uniformly constant proper acceleration $\mathbf{a}$ that remains the same throughout the trajectory of the object. Here we give reasons suggesting an extension of the argument to the case of general motion, i.e., a motion along a nonrestricted trajectory where the proper acceleration, henceforth denoted as $\mathbf{a}(\tau)$, does not remain constant, neither in magnitude nor in direction, but on the contrary changes from one instant to the next of object instantaneous proper time and becomes thereby a function of τ . We observe that in our derivation of Eq. (33) the subrelativistic result $\mathbf{f} = m_i \mathbf{a}$ depended only on the instantaneous value of the acceleration $\mathbf{a}$, i.e., on the hyperbolic moving object instantaneous proper acceleration, and not in any way whatsoever on the history of the object motion.

For the case of general motion mentioned above, let us consider a second identical object also performing hyperbolic motion but with a trajectory that has exactly the same constant proper acceleration, $\mathbf{a}_0$, as the general motion object at one instant of the first's proper time $\tau = \tau_0$. We thus write $\mathbf{a}(\tau_0) = \mathbf{a}_0$. But for the second object, i.e., that undergoing hyperbolic motion, we have shown that Eq. (33) holds and hence that the accelerating agent must be applying a force $m_i \mathbf{a}_0 = m_i \mathbf{a}(\tau_0)$. Moreover, as argued above, this expression displays complete independence from the past history of the object's motion, i.e., expression (33) and related previous expressions are memoryless. It is then quite reasonable to assume that as the force (in the frame that instantaneously comoves with the object performing hyperbolic motion) is such that it is exactly proportional to the acceleration and the result is indeed independent of the previous history of the object motion, the same result should hold for the identical object in the case of general motion. This means that for the case of general motion it should instantaneously hold that

$$\mathbf{f}(\tau) = m_i \mathbf{a}(\tau). \quad (34)$$

The instantaneous proper force is equal to m_i times the instantaneous proper acceleration and both are collinear vectors in the same direction as was the special case of the hyperbolically moving object. What will be required for the confirmation of this argument however are detailed calculations of several concrete examples of nonhyperbolic but accelerated motion; e.g., the case of ordinary circular motion with constant angular velocity that yields a centripetal acceleration $\mathbf{a}(\tau)$ of constant magnitude.

VIII. OUTLOOK AND CLOSING REMARKS

The dynamical approach of [7] required mathematical steps and approximations that led to a certain complexity of presentation. The new development here is simpler in that it does not deal with the dynamics, but exclusively with the form of the ZPF in relation to an accelerated object. The final result is derived using standard relativistic field transformations and does not involve approximations. A fully covariant analysis is presented in Appendix D.

The viewpoint of SED has been taken and matter has been assumed to be exclusively made of electromagnetically interacting particles or entities. Neutrons (presumed to consist of three charged quarks plus several neutral gluons) are polarizable electromagnetically-interacting particles displaying at least several experimentally known multipole moments. Neutrons are thus in principle treatable by our model whereas neutrinos are presumably strictly neutral and not polarizable. It is however not clear if neutrinos do actually have a rest mass and consequently display subrelativistic behavior or if instead they have no rest mass and consequently are strictly relativistic. In either case neutrinos lie beyond the scope of our present considerations as they do not interact electromagnetically. An explanation for the inertia of non-electromagnetically interacting particles and fields, such as neutrinos, may follow similar lines in a more sophisticated development. If as proposed in this paper (and in [7]) inertia appears because of the opposition of the vacuum (the electromagnetic ZPF in the case of SED) to the motion through it of accelerated particles that interact with its fields ($\mathbf{E}^{zp}$ and $\mathbf{B}^{zp}$ fields in the case of the ZPF of SED), one can naturally conjecture that the same happens, in a more general way with the zero-point fluctuations of other fields (like those of the weak and of the strong interactions) that oppose the accelerated motion through them of accelerated entities capable of interacting with them. The general idea is that rather than postulating an *ad hoc* mass-giving field on top of all the other fields, to examine instead if inertia can be explained by means of the already well-established (vacuum) fields, as e.g. the approach of Vigier [1].

There is one further conjecture that the present work suggests. The four-momentum that the accelerating agent transmits to the object during the acceleration process should go directly to the surrounding vacuum field [14]. The question is then whether the corresponding transferred linear momentum and associated translational kinetic energy should be radiated away to infinity in the ordinary manner of electromagnetic *radiation* fields or if instead they stay around in the immediate vicinity of the object in the usual way of *bound* fields [15]. The arguments of Sections III to VII point rather in the direction of this latter (second) possibility. We conjecture that no radiated four-momentum (to infinity) is produced. Presumably then only bound or velocity fields, in the manner of spatially rapidly decaying evanescent fields outside the accelerating object, are to be expected. As the moving object, made of electromagnetically interacting matter, enters a given space region, the electromagnetic modes structure in that region is modified accordingly. But then the ZPF is thereby also changed in that region. For this to be possible it may also help to realize that at the extremely high frequencies involved [7] a strong nonlinear coupling between matter and ZPF exists. Non-linearities in field equations at very high energies are a commonplace theoretical possibility [18], e.g., in our case of the simple electromagnetic field, a generalized form of the equations of classical electrodynamics may turn out to display non-linearities at very high energies. Ordinary Maxwell-Lorentz type equations would just then represent the linear version of more general nonlinear equations applicable at all frequencies.

The above view may suggest that molecules, atoms and even simple electromagnetically interacting particles, create around themselves and in their immediate vicinity some bound and nondissipative solitonic-like waves that accompany those particles wherever they go. This speculative view is not new. Proponents of source theories have for a long time [19] taken for granted evanescent fields that surround material particles. These “source-fields” are presumably able to theoretically substitute for the action of the more mundane electromagnetic vacuum of QED in derivations of “vacuum” effects like the Casimir force, the Van der Waals forces, etc. [19,20].

Last but not least we comment on where the concept presented here fits within the context of ordinary theory and in particular of standard classical theory. By looking at what we assumed in the course of this development we can easily see what the concept presented here does affect and also *what it does not*. We

very explicitly used the ordinary notion of what force is. So we cannot claim any direct explanation of that concept, not even a clarification of what force means. With respect to this classical force concept what we believe we have done is the following. Newton's third law requires that the motive force defined in the second law be counterbalanced by a reaction force. This has traditionally been satisfied implicitly by assuming the existence of inertia of matter. We propose to have found an explicit origin for this reaction force, viz. the acceleration-dependent scattering of ZPF radiation that the accelerated object is forced to move into. Our analysis presupposed electrodynamics and special relativity and other aspects of ordinary classical theory: Electrodynamics and some aspects of special relativity have been used in our developments since we used SED (that besides Maxwell's equations also presupposes the Lorentz force). As far as radiation reaction is concerned we merely suspect that it is somewhat connected with the developments here (and possibly also those in [7] and/or [10]) but so far this is only a suspicion. As to what we perceive as the core result of this work, we claim that our development represents a first step in the direction of clarifying the origin of what constitutes the essence of inertial mass. But it surely falls short in explicating all aspects of the origin of inertia. Indeed the analysis deliberately excluded any local consideration of details on particle-radiation interactions and only the electromagnetic aspect of the physical vacuum was involved. Only when our approach here can be implemented by more detailed models and when it can be extended to other components of the vacuum (other vacuum fields, e.g., weak, strong interactions) will inertial mass be able to be made assimilable to a property material objects of all sorts have because material objects affect the structure of the underlying modes of the vacuum fields in which they are permanently immersed.

Finally we make two disclaimers. First, our development was exclusively made within the context of classical theory (SED), so it is too early to say much about connections with quantum theory [21]. There for example remains the more technical question of how the SED renormalization procedure proposed herein (Appendix C) matches with its QED counterpart as both lead to essentially the same result. Second, we are not prepared to face the issue of how and in what sense our development might possibly affect or relate to general relativity (beyond what was briefly mentioned in Section I concerning Sakharov's hypothesis).

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APPENDIX A: ZPF-AVERAGED PRODUCTS FOR THE ZPF POYNTING VECTOR

Here we proceed with the explicit evaluation of the averaged products of the electric and magnetic field components $\langle E_i B_j \rangle$; $i, j = x, y, z$, that enter in the expression for the ZPF Poynting vector corresponding to the radiation being swept through by the accelerated object as calculated from the viewpoint of the observer at rest at $(c^2/a, 0, 0)$ of I_* . The resulting averaged products are part of the more extensive presentation of the approach implemented in Section V. (Note that for simplicity we generally drop the ZP-superscript notation for $\mathbf{E}$ and $\mathbf{B}$.)

We will see that the diagonal ones $\langle E_i B_i \rangle$; $i = x, y, z$, all vanish necessarily leading to the consequence that

$$\langle \mathbf{E} \cdot \mathbf{B} \rangle = 0 \quad (A1)$$

as was to be expected. Of the remaining $\langle E_i B_j \rangle$ averaged products we shall show that those products involving one component in the direction of the acceleration $\mathbf{a}$, i.e., one x -component of a field, E_x or B_x , vanish irrespectively of the other component:

$$\langle E_x B_j \rangle = 0; \quad j = x, y, z \quad (A2)$$

and

$$\langle E_i B_x \rangle = 0; \quad i = x, y, z. \quad (A3)$$

The only products that are spared are $\langle E_y B_z \rangle$ and $\langle E_z B_y \rangle$. They, moreover, will turn out to be the opposite of each other (see below) so that the momentum density (Eq. [15]) becomes

$$\mathbf{g}_*^{zp} = \frac{\mathbf{N}_*^{zp}}{c^2} = \frac{1}{c^2} \frac{c}{4\pi} \langle \mathbf{E}^{zp} \times \mathbf{B}^{zp} \rangle = \hat{x} \frac{1}{c^2} \frac{c}{4\pi} \langle E_y B_z - E_z B_y \rangle = \hat{x} \frac{1}{c^2} \frac{c}{2\pi} \langle E_y B_z \rangle. \quad (A4)$$

This corresponds then to the I_* fluid swept by the motion of the object across I_* according to the observer at rest in I_* . We only compute the Poynting vector $\mathbf{N}_*^{zp}$ that should be understood in the same sense and exactly as defined in Sec. IV. Explicit calculations evaluating Eqs. (A1–A4) follow below. In all those calculations, we do the averaging over the phases with

$$\begin{aligned} & \left\langle \cos \left[k_x \frac{c^2}{a} \cosh \left(\frac{a\tau}{c} \right) - \frac{\omega c}{a} \sinh \left(\frac{a\tau}{c} \right) - \theta(\mathbf{k}, \lambda) \right] \right. \\ & \quad \cdot \cos \left[k'_x \frac{c^2}{a} \cosh \left(\frac{a\tau}{c} \right) - \frac{\omega' c}{a} \sinh \left(\frac{a\tau}{c} \right) - \theta(\mathbf{k}', \lambda') \right] \Bigg\rangle \\ & = \frac{1}{2} \delta_{\lambda\lambda'} \delta(\mathbf{k} - \mathbf{k}'). \end{aligned} \quad (A5)$$

Notice that this equation is not properly normalized. A normalization factor $(2\pi)^3/V$, where V is the so called electromagnetic cavity volume, has been omitted for brevity on the right hand side. A corresponding compensating normalization factor $(V/(2\pi)^3)^{1/2}$ has also been omitted from the expressions for the fields, starting from Eqs. (3) and (4) above. These normalization factors, standard in physical optics, have been introduced also to SED [13], but as they mutually cancel after phase averaging we do not make them explicit in this paper.

After using Eqs. (10) and (11) and phase averaging with (A5), we find the first component for the case $i = x, j = x$:

$$\langle E_x B_x \rangle = \frac{1}{2} \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_x \quad (A6)$$

and as

$$\sum_{\lambda=1}^2 \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_x = 0 \quad (A7)$$

in partial confirmation of Eq. (A3),

$$\langle E_x B_x \rangle = 0. \quad (A8)$$

Note that we perform all k -integrations in the k -coordinates of I_* , centered in the k -origin of I_* as stated in Sections IV and V. In an analogous way to Eq. (A6), we easily find after averaging with Eq. (A5) that

$$\langle E_x B_y \rangle = \frac{1}{2} \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \hat{\epsilon}_x \cosh\left(\frac{a\tau}{c}\right) \left[(\hat{k} \times \hat{\epsilon})_y + \tanh\left(\frac{a\tau}{c}\right) \hat{\epsilon}_z \right], \quad (A9)$$

and as

$$\sum_{\lambda=1}^2 \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_y = \frac{k_z}{k} = \hat{k}_z, \quad (A10)$$

$$\sum_{\lambda=1}^2 \hat{\epsilon}_x \hat{\epsilon}_z = -\hat{k}_x \hat{k}_z, \quad (A11)$$

and because of the angle integrations in $\int d^3 k \dots = \int k^2 dk \int d\Omega \dots = \int k^2 dk \int \sin\theta d\theta \int d\phi \dots$,

$$\int \hat{k}_z d\Omega = 0, \quad (A12)$$

$$\int \hat{k}_x \hat{k}_z d\Omega = 0, \quad (A13)$$

we can confirm that as stated in Eqs. (A2) and (A3)

$$\langle E_x B_y \rangle = 0. \quad (A14)$$

As the problem is symmetric around the acceleration direction, i.e., the x -axis, it follows that if $\langle E_x B_y \rangle = 0$, then also

$$\langle E_x B_z \rangle = 0. \quad (A15)$$

For the yx -component, after phase averaging, we find

$$\langle E_y B_x \rangle = \frac{1}{2} \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) (\hat{k} \times \hat{\epsilon})_x \cosh\left(\frac{a\tau}{c}\right) \left[\hat{\epsilon}_y - \tanh\left(\frac{a\tau}{c}\right) (\hat{k} \times \hat{\epsilon})_z \right]. \quad (A16)$$

Furthermore

$$\sum_{\lambda=1}^2 \hat{\epsilon}_y (\hat{k} \times \hat{\epsilon})_x = -\hat{k}_z \quad (A17)$$

and

$$\sum_{\lambda=1}^2 (\hat{k} \times \hat{\epsilon})_x (\hat{k} \times \hat{\epsilon})_z = -\hat{k}_x \hat{k}_z \quad (A18)$$

that yield, as in Eq. (A8), vanishing solid angle integrations. We obtain

$$\langle E_y B_x \rangle = 0. \quad (A19)$$

In analogous fashion:

$$\langle E_y B_y \rangle = \frac{1}{2} \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \cosh^2 \left(\frac{a\tau}{c} \right) \left[\hat{\epsilon}_y - \tanh \left(\frac{a\tau}{c} \right) (\hat{k} \times \hat{\epsilon})_z \right] \left[(\hat{k} \times \hat{\epsilon})_y + \tanh \left(\frac{a\tau}{c} \right) \hat{\epsilon}_z \right], \quad (A20)$$

for which we use the identities

$$\sum_{\lambda=1}^2 \hat{\epsilon}_y (\hat{k} \times \hat{\epsilon})_y = 0, \quad (A21)$$

$$\sum_{\lambda=1}^2 \hat{\epsilon}_z (\hat{k} \times \hat{\epsilon})_z = 0, \quad (A22)$$

$$\sum_{\lambda=1}^2 \hat{\epsilon}_y \hat{\epsilon}_z = -\hat{k}_y \hat{k}_z, \quad (A23)$$

and

$$\sum_{\lambda=1}^2 (\hat{k} \times \hat{\epsilon})_y (\hat{k} \times \hat{\epsilon})_z = -\hat{k}_y \hat{k}_z \quad (A24)$$

to show that

$$\langle E_y B_y \rangle = \frac{1}{2} \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \cosh^2 \left(\frac{a\tau}{c} \right) \tanh \left(\frac{a\tau}{c} \right) (-\hat{k}_y \hat{k}_z + \hat{k}_y \hat{k}_z) = 0. \quad (A25)$$

In a similar way and after phase averaging, it follows that

$$\langle E_y B_z \rangle = \frac{1}{2} \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \cosh^2 \left(\frac{a\tau}{c} \right) \left[\hat{\epsilon}_y - \tanh \left(\frac{a\tau}{c} \right) (\hat{k} \times \hat{\epsilon})_z \right] \left[(\hat{k} \times \hat{\epsilon})_z - \tanh \left(\frac{a\tau}{c} \right) \hat{\epsilon}_y \right] \quad (A26)$$

where we use

$$\sum_{\lambda=1}^2 \hat{\epsilon}_y (\hat{k} \times \hat{\epsilon})_z = \hat{k}_x, \quad (A27)$$

$$\sum_{\lambda=1}^2 (\hat{k} \times \hat{\epsilon})_z (\hat{k} \times \hat{\epsilon})_z = 1 - \hat{k}_z^2 \quad (A28)$$

and

$$\sum_{\lambda=1}^2 \hat{\epsilon}_y^2 = 1 - \hat{k}_y^2, \quad (A29)$$

so that

$$\begin{aligned} \langle E_y B_z \rangle = & \frac{1}{2} \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \cosh^2 \left(\frac{a\tau}{c} \right) \cdot \\ & \left\{ \left[1 + \tanh^2 \left(\frac{a\tau}{c} \right) \right] \hat{k}_x - \tanh \left(\frac{a\tau}{c} \right) (1 - \hat{k}_y^2) - \tanh \left(\frac{a\tau}{c} \right) (1 - \hat{k}_z^2) \right\}. \end{aligned} \quad (A30)$$

The first term inside the curly brackets, proportional to $\hat{k}_x$, vanishes after angle integrations. For the others we use the fact that $2 - \hat{k}_y^2 - \hat{k}_z^2 = 1 + \hat{k}_x^2$ and after minimal algebra then

$$\langle E_y B_z \rangle = -\frac{1}{4} \sinh \left(\frac{2a\tau}{c} \right) \int d\omega \frac{\hbar\omega^3}{2\pi^2 c^3} \int d\Omega (1 + \hat{k}_x^2) \quad (A31)$$

where the divergence of the integration may be damped by a well-known convergence form factor (or more roughly a frequency cut-off) that we do not need to make explicit at this stage. Such a feature physically represents a frequency beyond which no material subparticle however small is going to be able to react to the radiation [17]. This is not a cut-off in the ZPF itself but is introduced because wavelengths smaller than the size of the minimal relevant entities, say partons [7], cannot produce any translational interactions but only presumably internal deformation of the “parton” [17]. We thus obtain

$$\langle E_y B_z \rangle = -\frac{4\pi}{3} \sinh \left(\frac{2a\tau}{c} \right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \quad (A32)$$

where, as explained above, an implicit cut-off or convergence form factor [17] in the frequency integration may henceforth be implemented [7]. For $\langle E_z B_x \rangle$ we obtain from symmetry around the x -axis and Eq. (A19) that

$$\langle E_z B_x \rangle = 0. \quad (A33)$$

The case of $\langle E_z B_y \rangle$ is done exactly as that for $\langle E_y B_z \rangle$ and gives the opposite value

$$\begin{aligned} \langle E_z B_y \rangle = & \frac{1}{2} \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \cosh^2 \left(\frac{a\tau}{c} \right) \tanh \left(\frac{a\tau}{c} \right) (2 - \hat{k}_y^2 - \hat{k}_z^2) \\ = & \frac{4\pi}{3} \sinh \left(\frac{2a\tau}{c} \right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \end{aligned} \quad (A34)$$

Due to the cylindrical symmetry around the x -axis, the case of $\langle E_z B_z \rangle$, as the one for $\langle E_y B_y \rangle$, must vanish:

$$\langle E_z B_z \rangle = 0. \quad (A35)$$

APPENDIX B: THE ZPF MOMENTUM CONTENT OF AN ACCELERATED OBJECT

The development of this ZPF momentum content approach serves to complement the ZPF momentum flux approach of Sections IV and V but is *totally independent* from it. Newton's third law as expressed in Eq. (19) implies that $\mathbf{f}_*^{zp}$ is the opposite of $\mathbf{f}_*$. In Section V we calculate $\mathbf{f}_*^{zp}$. Here instead we calculate $\mathbf{f}_*$. To calculate $\mathbf{f}_*$ we use the fact, also given in Eq. (19), that $\mathbf{f}_*$ is just the time derivative of $\mathbf{p}_*$. This $\mathbf{p}_*$ is not the ZPF momentum flowing through the object that we identified with $\mathbf{p}_*^{zp}$ above, but rather the momentum contained within the body that should be due to the ZPF, as will become clear as the argument progresses. This means that we need to reach an expression for $\mathbf{p}_*$, the object momentum, in terms of the ZPF and the particle electromagnetic coupling parameters.

First we develop a connection between $\mathbf{p}_*$ and $\mathbf{p}_*^{zp}$ before defining $\mathbf{p}_*$ more precisely, so we examine the $\Delta\mathbf{p}_*^{zp} = -\Delta\mathbf{p}_*$ condition of Eq. (16) referring everything, including the ZPF background, to the I_* inertial frame. We need to estimate the net ZPF momentum, $\Delta\mathbf{p}_*^{zp}$, and momentum density that entered into the object in a time interval Δt_* because of the object's sweeping accelerated motion through the electromagnetic background (Secs. IV and V). As we take the integrated impulses $\mathbf{p}_*^{zp}$ and $\mathbf{p}_*$ both to be zero at time $t_* = 0$ and they start to change at the same time, owing to Eq. (19) we should end up with

$$\mathbf{p}_*^{zp} = -\mathbf{p}_* \quad (B1)$$

where we have integrated $\mathbf{f}_*^{zp}$ and $\mathbf{f}_*$ over the I_* -frame time t_* . Moreover, because both $\mathbf{p}_*^{zp}$ and $\mathbf{p}_*$ are referred to the laboratory inertial frame I_* , then, $\mathbf{p}_*^{zp} = V_*\mathbf{g}_*^{zp}$ and $\mathbf{p}_* = V_*\mathbf{g}_*$ that with Eq. (B1) lead to

$$\mathbf{g}_*^{zp} = -\mathbf{g}_* \quad (B2)$$

We have calculated $\mathbf{p}_*^{zp}$ in Secs. IV and V. After confronting it with the result for $\mathbf{p}_*$ (which we calculate below), we shall verify Eq. (B1), as is necessary for a self-consistent interpretation of momentum. Therefore, as it will be explicitly confirmed below, the previous result for $\mathbf{g}_*^{zp}$ will be the negative of the expression (B7) for $\mathbf{g}_*$ below, that is calculated here in a direct and independent way. Eq. (B2) also makes sense intuitively with the aid of the fluid analogy of Sec. IV. In the short time interval Δt_* the momentum density inside the object increases in an amount $\Delta\mathbf{g}_*$. The corresponding change of $\mathbf{g}_*^{zp}$ in the same time interval is $\Delta\mathbf{g}_*^{zp}$. Hence if $\Delta\mathbf{g}_*$ is the internal change in momentum per unit volume of the ZPF in the interior of the object and referred to I_* , then $-\Delta\mathbf{g}_*^{zp}$ is the corresponding net amount of momentum per unit volume that in the same time interval Δt_* is swept inside the object volume due to the hyperbolic motion of the object through the ZPF of I_* . We now calculate the momentum density $\mathbf{g}_*$ in a direct manner.

If the central idea that the ZPF is the entity responsible for a contribution to the inertia reaction force is valid, then, *from the point of view of the I_* frame*, it follows that when the accelerating agent applies the external force $\mathbf{f}_*$ and accelerates the object, necessarily the accelerating agent is doing work against the vacuum fields and hence the energy provided by the accelerating agent is going to be stored somewhere in that vacuum. We envision a dichotomous situation, i.e., two possible alternative ways in which the vacuum may store this energy and its corresponding momentum provided by the accelerating agent. One way is that the energy is “radiated away”, i.e., the vacuum traveling modes radiate it out far away from the accelerated body. The other is that such energy and momentum are not indeed radiated out, but on the contrary stay bound in the manner of velocity fields [15] in and around the body presumably in association to the ZPF electromagnetic bound modes created by microscopic charge and current structures within the body. We propose that the first hypothesis, i.e., that the four-momentum is radiated away to infinity is less natural than the second, i.e., than the one that states that the four-momentum remains bound to the body: If a billiard ball is hit by the billiard club (its accelerating agent) receiving this way energy and momentum, i.e., four-momentum, and if this four-momentum were to escape to infinity, there would not be a readily natural way to explain how the ball in its turn subsequently transmitted all of its acquired four-momentum at the instant of a subsequent collision with another identical billiard ball that was standing still on the billiard table. The original ball's four-momentum had been radiated away to infinity! Explanations can probably be concocted but not one that is in an obvious way natural. We assume then only the second

hypothesis in this dichotomy which assumes that the acquired four-momentum is not stored far away from the accelerated object but, on the contrary, that it is stored within and in the body's immediate vicinity. Imagine an idealized body made of a simple electromagnetic cavity. The modes of the cavity contain ZPF energy and if the cavity moves they can transport electromagnetic momentum.

We therefore, in spite of its still very preliminary nature, have to follow this second hypothesis, namely that when a massive body moves with respect to some inertial frame the moving body drags with itself as velocity fields within its ZPF-bound electromagnetic modes the corresponding translational kinetic energy and momentum the body has with respect to that frame. Hence we assume that the momentum $\mathbf{p}_*$ of the moving object referred to the inertial frame I_* is due to the ZPF that interpenetrates the object. In order to find $\mathbf{p}_*$ we first calculate the ZPF momentum density $\mathbf{g}_*$ associated with the exact position of our accelerated object attached to the frame S , at the point $(c^2/a, 0, 0)$ of that frame. As the motion is hyperbolic and in the x -direction we only need to calculate

$$\mathbf{g}_* = \hat{x} g_{*x} = \frac{\mathbf{N}_*}{c^2} = \hat{x} \frac{1}{c^2} \frac{c}{4\pi} \langle \mathbf{E}_*^{zp}(0, \tau) \times \mathbf{B}_*^{zp}(0, \tau) \rangle_x \quad (B3)$$

where by this we mean the momentum density $\mathbf{g}_*$ and the associated Poynting vector $\mathbf{N}_* = \hat{x} N_{*x}$ [15] due to the ZPF as measured in the I_* frame at the object's space-time point at object proper time τ , namely :

$$t_* = \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right), \quad x_* = \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right), \quad y_* = 0, \quad z_* = 0, \quad (B4)$$

of that I_* frame.

However, as we are calculating the ZPF momentum associated with the object and the object is instantaneously at rest in the I_τ frame, the calculation should be done with this in mind. This means (Appendix C) that the integrals should be performed over the k -sphere of the I_τ frame. Additional support for this view comes from the clear interpretation it yields. In order to evaluate Eq. (B3) we need then to compute the relevant averaged cross-product of the electric and magnetic fields in I_* but evaluated at the object space-time point of Eq. (B4):

$$\begin{aligned} \langle \mathbf{E}_*^{zp}(0, \tau) \times \mathbf{B}_*^{zp}(0, \tau) \rangle_x &= \langle E_{y*} B_{z*} - E_{z*} B_{y*} \rangle \\ &= \gamma_\tau^2 \langle (E_{y\tau} + \beta_\tau B_{z\tau})(B_{z\tau} + \beta_\tau E_{y\tau}) - (E_{z\tau} - \beta_\tau B_{y\tau})(B_{y\tau} - \beta_\tau E_{z\tau}) \rangle \\ &= \gamma_\tau^2 \beta_\tau \langle E_{y\tau}^2 + B_{z\tau}^2 + E_{z\tau}^2 + B_{y\tau}^2 \rangle + \gamma_\tau^2 (1 + \beta_\tau^2) \langle E_{y\tau} B_{z\tau} - E_{z\tau} B_{y\tau} \rangle \\ &= \gamma_\tau^2 \beta_\tau \langle E_{y\tau}^2 + B_{z\tau}^2 + E_{z\tau}^2 + B_{y\tau}^2 \rangle \end{aligned} \quad (B5)$$

where we have made a Lorentz transformation of the fields [15] from those of I_* to those of I_τ . One of the resulting terms is found to be zero because it is proportional to the x -component of $\mathbf{N}_\tau$, the statistically averaged ZPF Poynting vector as viewed in the inertial frame I_τ at a point fixed in that frame. Furthermore

$$\langle E_{y\tau}^2 + B_{z\tau}^2 + E_{z\tau}^2 + B_{y\tau}^2 \rangle = \frac{2}{3} \langle \mathbf{E}_\tau^2 + \mathbf{B}_\tau^2 \rangle = \frac{2}{3} 8\pi U_\tau = 2 \cdot \frac{8\pi}{3} \int \frac{\hbar \omega'^3}{2\pi^2 c^3} d\omega' \quad (B6)$$

where U_τ means the ZPF energy density in I_τ given by the frequency integral that, as mentioned above, is carried out over the k -sphere of I_τ . In the last step we have simplified the notation such that hereafter primes denote quantities referred to I_τ , and unprimed quantities mean quantities referred to I_* , so we do not need to carry everywhere the τ and/or $*$ subindices. From Eqs. (B3), (B5) and (B6) then

$$\mathbf{g}_* = \hat{x} \frac{1}{c^2} \frac{c}{2\pi} \frac{8\pi}{3} \int \frac{\hbar \omega'^3}{2\pi^2 c^3} d\omega' \sinh\left(\frac{a\tau}{c}\right) \cosh\left(\frac{a\tau}{c}\right). \quad (B7)$$

From this we can get the ZPF momentum corresponding to the volume of the object all as referred to the inertial observer of I_* ,

$$\mathbf{p}_* = \mathbf{g}_* V_* = \hat{x} \frac{1}{c^2} \frac{c}{4\pi} \gamma_\tau^2 \beta_\tau \frac{2}{3} \langle \mathbf{E}_\tau^2 + \mathbf{B}_\tau^2 \rangle V_*, \quad (B8)$$

where V_* is the volume that the object presents to the observer in I_* , namely $V_* = V_0/\gamma$ because of Lorentz-contraction (V_0 is the proper volume of the object). As $\hat{x}\beta_\tau c$ is the object velocity and $\langle \mathbf{E}_\tau^2 + \mathbf{B}_\tau^2 \rangle V_0$ is proportional to the proper ZPF energy contained in the volume of the object, it is a simple matter to realize that Eq. (B8) does indeed have the form of a relativistic momentum ($\mathbf{p} = \gamma m_0 \mathbf{v}$ with m_0 the rest mass).

Having established that Eq. (B8) correctly represents the total ZPF momentum instantaneously contained in the proper volume V_0 of the object in question we next reintroduce a coupling parameter, η , which is now a function of frequency, $\eta(\omega)$, and which will parametrize the amount of interaction (absorption or scattering) at every frequency ω between the object and the radiative momentum flux associated with it. This will provide us the effective physical momentum of the object. With this in mind we write:

$$\mathbf{p}_* = \hat{x} \frac{4V_0}{3} c \beta_\tau \gamma_\tau \left[\frac{1}{c^2} \int \eta(\omega') \frac{\hbar \omega'^3}{2\pi^2 c^3} d\omega' \right]. \quad (B9)$$

Recognizing that this is the momentum of the body and that the accelerating force $\mathbf{f}_*$ applied by the external agent is just the rate of change of $\mathbf{p}_*$ with time we have $\mathbf{f}_* = d\mathbf{p}_*/dt_*$. From Eq. (19) (Newton's Third Law) we then obtain the expression for the force $\mathbf{f}_*^{zp} = -\mathbf{f}_*$ that the ZPF applies back to the object in opposition to the external agent's accelerating action

$$\mathbf{f}_*^{zp} = -\mathbf{f}_* = -\frac{d\mathbf{p}_*}{dt_*} = -\frac{1}{\gamma_\tau} \frac{d\mathbf{p}_*}{d\tau} \Big|_{\tau=0} = -\left[\frac{4}{3} \frac{V_0}{c^2} \int \eta(\omega') \frac{\hbar \omega'^3}{2\pi^2 c^3} d\omega' \right] \mathbf{a}. \quad (B10)$$

As already mentioned above, the 4/3 factor becomes unity when a fully covariant evaluation is performed (Appendix D). Observe that Eq. (B10) exactly reproduces the result for the inertia reaction force of Eq. (29) and its associated inertial mass of Eq. (30). Observe furthermore that m_i , when the 4/3 factor is obliterated (Appendix D), has a clear interpretation, namely it is exactly the amount of energy of the ZPF that lies inside the object's proper volume V_0 and that does actually interact with it (as depicted by the frequency-dependent coupling efficiency function $\eta(\omega)$ divided by c^2 to give it in units of mass). According to the view presented here it is the parameter V_0 and the spectral function $\eta(\omega)$ that determine the inertial mass of the object. The expression for m_i would precisely fit the energy of the ZPF (divided by c^2) that couples to the object and that lies within its volume as properly assumed in the approach of this Appendix. This interpretation is not so clearly found for the physical assumptions of the approach of Sections IV and V and Appendix A, but still in that approach the same m_i was found as required. Interestingly enough this 4/3 factor is the same factor found in the electrodynamics of classical charged particles [12, 15]. We found here further motivation for pursuing the fully covariant analysis of Appendix D.

APPENDIX C: DISCUSSION ON THE EVALUATION OF THE TIME RATE OF CHANGE OF ZPF MOMENTUM

This article has simplicity as one of its objectives. Insights however, are gained if we pay the price of going through a few intricacies. In our analysis there appears a momentum density from the ZPF radiation in accelerated frames that is dependent on the proper time of the accelerated object. Such proper time may be removed by means of an artifice. We believe that doing so would be physically incorrect and propose that such time dependence actually has an important physical meaning. This entails the adoption of a convention for the evaluation of otherwise indefinite integrals. Removal of the proper time would imply a completely different situation than the one we are dealing with here. The momentum density mentioned above depends on the object proper time.

A. Improper integrals and “time removal” technique

First of all we notice that the implicitly unbounded k and ω integrations of Sections IV, V and Appendices A and B when there is no $\eta(\omega)$ (i.e. set $\eta(\omega) \equiv 1$) and the limits for k or ω are indeed 0 and ∞ , constitute what are customarily called improper integrals. Improper integrals require some definition as they do not yield a unique well-defined value. In Sections IV and V and Appendices A and B we implicitly adopted a definition. As the inertia resistance force expression of Eq. (29) resulted from integrating over the ω or k of I_* in an explicit manner and as that result was physically sound, as follows from the clear delineation of Section III, we adopted the way of integrating implicit in that procedure as the actual definition of the improper integral. That this result is not necessarily unique is however seen from what follows. By performing a change of variable of integration, from that of $I_*=I_\tau(\tau=0)$ to that of $I_\tau(\tau>0)$, it can be shown that a different result is obtained. Moreover the procedure removes the proper time or τ -dependence of the expressions. Thus it is sometimes referred to as a “time removal procedure” [22].

The procedure leads to a seemingly paradoxical situation as the integration apparently yields two different values. The paradox is easily resolved. Its solution yields insight into the origin of $d\mathbf{p}_{*}^{zp}/dt_*$, the time rate of change of the radiation momentum in Eq. (28), and on its proposed connection to Newtonian inertia.

For the particular SED form of the “time removal” procedure [23,24], we start from the expression derived in Appendix A,

$$\begin{aligned} \langle E_y B_z \rangle &\equiv \langle E_y(0, \tau) B_z(0, \tau) \rangle = \left\langle \sum_{\lambda=1}^2 \sum_{\lambda'=1}^2 \int d^3k \int d^3k' \right. \\ &\times \cosh(A) \left[\hat{\epsilon}_y - \tanh(A)(\hat{k} \times \hat{\epsilon})_z \right] \cosh(A) \left[(\hat{k}' \times \hat{\epsilon}')_z - \tanh(A)\hat{\epsilon}'_y \right] H_{zp}(\omega) H_{zp}(\omega') \cdot \\ &\left. \cos \left[k_x \frac{c^2}{a} \cosh(A) - \frac{\omega c}{a} \sinh(A) - \theta(\mathbf{k}, \lambda) \right] \cos \left[k'_x \frac{c^2}{a} \cosh(A) - \frac{\omega' c}{a} \sinh(A) - \theta(\mathbf{k}', \lambda') \right] \right\rangle, \end{aligned} \quad (C1)$$

where we define

$$A \equiv \frac{a\tau}{c} \quad (C2)$$

and the averaging we perform by means of Eq. (A5). This yields again as in Appendix A:

$$\langle E_y B_z \rangle = \frac{1}{2} \sum_{\lambda=1}^2 \int d^3k H_{zp}^2(\omega) \cosh^2(A) \left[\hat{\epsilon}_y - \tanh(A)(\hat{k} \times \hat{\epsilon})_z \right] \left[(\hat{k} \times \hat{\epsilon})_z - \tanh(A)\hat{\epsilon}_y \right]. \quad (C3)$$

Of course that Eq. (C3) is identical to Eq. (A26) and if we follow in the ordinary way the command dictated by the summation over lambda operator and the integration over $\mathbf{k}$ -operator of Eq. (C3) the result cannot be

different from the one of Eq. (A31). At this stage however, one may decide to change the dummy variable of integration and instead of integrating over the four-vector components $(k, \mathbf{k})$ of I_* , integrate over the four-vector components $(k', \mathbf{k}')$ of I_τ by means of the corresponding transformation where of course $k' = \omega' c$:

$$k = c\omega = k' \cosh(A) + k'_x \sinh(A) = k' \gamma_\tau + k'_x \beta_\tau \gamma_\tau, \quad (C4a)$$

$$k_x = k'_x \gamma_\tau + k' \beta_\tau \gamma_\tau = k'_x \cosh(A) + k' \sinh(A), \quad (C4b)$$

$$k_y = k'_y, \quad k_z = k'_z. \quad (C4c)$$

Moreover we know that

$$\frac{d^3 k}{k} = \frac{d^3 k'}{k'}. \quad (C5)$$

Both integrations, the one over $d^3 k$ in the coordinates of I_* , and the one over $d^3 k'$ in the coordinates of I_τ , are at least formally performed over all k -space. Moreover as all space for k corresponds to all space for k' , and furthermore the Jacobian of the transformation that follows easily from Eqs. (C4a) and (C5)

$$\frac{d^3 k}{d^3 k'} = J(\mathbf{k}, \mathbf{k}') = \gamma_\tau \left(1 + \frac{k'_x}{\omega'} c \beta_\tau \right) \quad (C6)$$

is, except for $\omega' = 0$, nonsingular, then the integrations are both extended over all space whether when done over k or when performed over k' . Changing the integration variable by Eq. (C5) and using again Eqs. (A27–A29) we obtain

$$\begin{aligned} \langle E_y B_z \rangle_R &= \frac{1}{2} \int \frac{d^3 k'}{k'} \frac{\hbar c}{2\pi^2} k^2 \cosh^2(A) \cdot \left\{ \left[1 + \tanh^2(A) \right] \frac{k_x}{k} - \tanh(A) \left(1 - \frac{k_y^2}{k^2} \right) - \tanh(A) \left(1 - \frac{k_z^2}{k^2} \right) \right\} \\ &= \frac{\hbar c}{(2\pi)^2} \int \frac{d^3 k'}{k'} k' k'_x. \end{aligned} \quad (C7)$$

The subindex R stresses the difference between the case here and the one in the body of the paper. Observe that the proper time τ and hence A of Eq. (C2) have disappeared from the integral and this even before performing the integration: the proper time τ has been “removed”. The last equality demanded minor omitted algebra. Then as

$$\int k'_x d\Omega' = 0 \quad (C8)$$

the integral in Eq. (C7) vanishes identically and thus

$$\langle E_y B_z \rangle_R = 0. \quad (C9)$$

Symmetry then yields that also

$$\langle E_z B_y \rangle_R = 0. \quad (C10)$$

It has been a simple matter to establish by this procedure that the non-zero products of Section IV and Appendix A vanish. It is equally simple to establish that all the other products that already vanished there also vanish here and then that

$$\langle E_i B_j \rangle_R = 0, \quad i, j = x, y, z. \quad (C11)$$

Consequently according to this viewpoint the Poynting vector and hence the momentum density $\mathbf{g}_R$ identically vanish

$$c^2 \mathbf{g}_R = \mathbf{N}_R = 0. \quad (C12)$$

This is not so surprising if we propose the view that what we have done by means of the transformation from $(k, \mathbf{k})$ to $(k', \mathbf{k}')$, i.e., from the variables of I_* to the variables of I_τ , is to reevaluate the averaged Poynting vector incident on the moving object not now as the observer at rest in I_* estimates it, in which case such $\mathbf{N}$ does not vanish, but as viewed from the space point $(c^2/a, 0, 0)$ of the inertial frame I_τ where the object is found at rest at proper time τ and from where the ZPF of I_τ is necessarily seen as homogeneously and isotropically distributed.

Next we show how the formalism of Eq. (C4), that involves a transformation of the four-wavevector $(k, \mathbf{k})$ from I_* to I_τ and its corresponding inverse transformation that transforms from the $(k', \mathbf{k}')$ four-wavevector of I_τ back into that of I_* , take us, at least formally, from the Poynting vector $\mathbf{N}$ impinging on the object as estimated in I_* to the same $\mathbf{N}$ but as estimated in I_τ , and the second transformation back from the $\mathbf{N}$ impinging on the object as viewed in I_τ to the same but as viewed in I_* .

From Eq. (A4) the incident $\mathbf{N}$ on the object, as viewed in I_* , is $(c/2\pi)$ times the expression $\langle E_y B_z \rangle$ of Eqs. (A30) and (C3). Hence because of Eqs. (C7) and (A4) the $(k, \mathbf{k}) \rightarrow (k', \mathbf{k}')$ transformation of (C4) takes us from the $\mathbf{N}$ of Eq. (25) to

$$\mathbf{N} = \frac{\hbar c^2}{(2\pi)^3} \int \frac{d^3 k'}{k'} k'_x \quad (C13)$$

that as shown below is the $\mathbf{N}$ at proper time τ impinging on the object according to an inertial observer instantaneously coinciding and comoving with the object, hence at rest in I_τ . Eq. (C13) of course vanishes (after the angle integration Eq. (C8)) as it should because the ZPF in every inertial frame is homogeneous and isotropic. Recall that our ultimate proper time of evaluation is indeed $\tau = 0$ when I_τ becomes $I_* = I_\tau(\tau = 0)$. The corresponding $\mathbf{N}(\tau)$ at proper time τ exactly equal to zero, $\mathbf{N}(\tau = 0)$, also vanishes as follows from Eq. (B6).

Next we perform the inverse four-wavevector transformation $(k', \mathbf{k}') \rightarrow (k, \mathbf{k})$ from the four-wavevector of I_τ to that of I_* on the expression that corresponds to the $\mathbf{N}$ of Eq. (C13), that is the $\mathbf{N}$ viewed in I_τ . Nevertheless, we first derive the explicit form of the $\mathbf{N}$ of I_τ and show that it indeed corresponds to Eq. (C13) and only then, at a second stage, perform the aforementioned transformation.

Consider the ZPF $\mathbf{E}'$ and $\mathbf{B}'$ fields of I_τ at time t_τ of I_τ and for concreteness at the point $x_\tau = c^2/a$, $y_\tau = 0$, $z_\tau = 0$, that is the point of I_τ that at object proper time τ coincides and comoves with the object. Take the averaged Poynting product of these $\mathbf{E}'$ and $\mathbf{B}'$ fields,

$$\mathbf{N} \Big|_{I_\tau} = \frac{c}{4\pi} \langle \mathbf{E}' \times \mathbf{B}' \rangle = \hat{x} \frac{c}{4\pi} \langle (\mathbf{E}' \times \mathbf{B}')_x \rangle = \hat{x} \langle E'_y B'_z - E'_z B'_y \rangle, \quad (C14)$$

where the second equality follows from symmetry. We use the primed notation for all fields and variables of I_τ . From Eqs. (3), (4) and (5) then

$$\mathbf{N} \Big|_{I_\tau} = \hat{x} \frac{c}{4\pi} \sum_{\lambda'_1 \lambda'_2} \int d^3 k'_1 \int d^3 k'_2 H(\omega'_1) H(\omega'_2) \left[\hat{e}'_{1y} (\hat{k}' \times \hat{e}')_{2z} - (\hat{k}' \times \hat{e}')_{1y} \hat{e}'_{2z} \right]. \quad (C15)$$

$$\langle \cos(k'_{1x} x' + k'_{1y} y' + k'_{1z} z' - \omega'_1 t_\tau + \theta(\mathbf{k}'_1, \lambda'_1)) \cos(k'_{2x} x' + k'_{2y} y' + k'_{2z} z' - \omega'_2 t_\tau + \theta(\mathbf{k}'_2, \lambda'_2)) \rangle.$$

Applying the phase averaging of (A5) we have

$$\begin{aligned}
& \langle \cos(k'_{1x}x' + k'_{1y}y' + k'_{1z}z' - \omega'_1 t_\tau + \theta(\mathbf{k}'_1, \lambda'_1)) \cos(k'_{2x}x' + k'_{2y}y' + k'_{2z}z' - \omega'_2 t_\tau + \theta(\mathbf{k}'_2, \lambda'_2)) \rangle \\
& = \frac{1}{2} \delta_{\lambda'_1 \lambda'_2} \delta(\mathbf{k}'_1 - \mathbf{k}'_2),
\end{aligned} \tag{C16}$$

and we get

$$\mathbf{N} \Big|_{I_\tau} = \hat{x} \frac{c}{4\pi} \sum_{\lambda'=1}^2 \int d^3 k' H_{zp}^2(\omega') \frac{1}{2} [\hat{\epsilon}'_y (\hat{k}' \times \hat{\epsilon}')_z - (\hat{k}' \times \hat{\epsilon}')_y \hat{\epsilon}'_z], \tag{C17}$$

where the subindex on $\mathbf{N}$ just emphasizes that this is now the Poynting vector as seen in I_τ . Next we use the summation expressions (A27) and (A17) with one circular permutation of the subindices of the form $x \rightarrow y$, $y \rightarrow z$, $z \rightarrow x$ and then

$$\mathbf{N} \Big|_{I_\tau} = \hat{x} \frac{c}{4\pi} \sum_{\lambda'=1}^2 \int d^3 k' H_{zp}^2(\omega') \frac{1}{2} 2\hat{k}'_x, \tag{C18}$$

that from Eq. (5) and as $\omega' = ck'$ yields

$$\mathbf{N} \Big|_{I_\tau} = \hat{x} \frac{\hbar c}{(2\pi)^3} \int \frac{d^3 k'}{k'} k' k'_x = \mathbf{N} = c^2 \mathbf{g}, \tag{C19}$$

where the last expression serves to emphasize that we recover the $\mathbf{N}$ of Eq. (C13) as was to be expected. We need not emphasize that Eq. (C19) vanishes when integrated over the angles. Next we proceed to perform the inverse transformation to that of Eqs. (C4abc). This last takes us back from the $(k', \mathbf{k}')$ four-vector of I_τ into the $(k, \mathbf{k})$ four-vector of I_* , namely

$$\begin{aligned}
k' &= k \cosh(A) - k_x \sinh(A) \\
k'_x &= k_x \cosh(A) - k \sinh(A) \\
k'_y &= k_y \\
k'_z &= k_z
\end{aligned} \tag{C20}$$

with A as defined in Eq. (C2). Invoking again Eq. (C5) and after some algebra (Eq. (C19)) yields

$$\begin{aligned}
\mathbf{N} &= -\hat{x} \left(\frac{c}{4\pi} \right) \left(\frac{\hbar c}{2\pi^2} \right) \int d^3 k k (1 + \hat{k}_x^2) \cosh(A) \sinh(A) \\
&= -\hat{x} \left(\frac{c}{4\pi} \right) \left(\frac{8\pi}{3} \right) \sinh \left(\frac{2a\tau}{c} \right) \int d\omega \frac{\hbar \omega^3}{2\pi^2 c^3} \\
&= \mathbf{N}(\tau) \Big|_{I_*},
\end{aligned} \tag{C21}$$

that is the Poynting vector impinging on the object point in S according to the observer at I_* . So we recover the Poynting vector derived in (25).

Of course both the $\mathbf{N}$ of Eq. (C19) and the $\mathbf{N}$ of Eq. (C21) vanish at object proper time $\tau = 0$: that of Eq. (C19) because of symmetry after the angle integrations and that of Eq. (C21) since in that case the evaluation is made for vanishing proper time. In Eq. (C21) we have recovered the proper time dependence. This shows how the $(k, \mathbf{k})$ four-vector transformation takes us from the $\mathbf{N}$ of the object point in S as viewed in I_* that we derived in Eq. (25) to the corresponding $\mathbf{N}$ but as viewed in I_τ that was derived in Eq. (C19).

The inverse transformation takes us back in exactly the opposite way from the $\mathbf{N}$ as viewed in I_τ to the corresponding $\mathbf{N}$ as viewed in I_* .

This last example illustrates how and in what sense one can formally at least remove and place back the time variable into the expressions. A more fundamental explanation follows.

B. Spheres of integration in k -space: k -spheres

Expression (26) for the momentum density $\mathbf{g}(\tau)$ was obtained with an integration carried out over the k -space of the laboratory system I_* . The momentum density referred however to the estimate that an observer, at rest at $(c^2/a, 0, 0)$ of I_* , can make concerning the momentum density the observer views should be incident on the accelerated object. At proper time τ the object is instantaneously at rest at the point $(c^2/a, 0, 0)$ of the inertial frame I_τ that moves with respect to I_* with the velocity $\beta_\tau c \hat{x}$ given in Eq. (1).

From Eqs. (1), (2) and (C20), the last written as

$$\begin{aligned} k' &= \gamma_\tau (k - \beta_\tau k_x) \\ k'_x &= \gamma_\tau (k_x - \beta_\tau k) \\ k'_y &= k_y \\ k'_z &= k_z \end{aligned} \tag{C22}$$

one can express Eq. (26) in terms of wavevectors only

$$\mathbf{g}(\tau) = \frac{\hbar}{(2\pi)^2} \int \frac{d^3k}{k} k' \mathbf{k}', \tag{C23}$$

where the primed wavevector refers to that of I_τ and again the unprimed one is that of I_* . If subsequently the integration is carried over k , the wavevector of I_* , the result is again Eq. (26). However if we transform from the wavevector integration of I_* to that of I_τ by means of Eq. (C5), we obtain an integral exclusively over k of the form

$$\mathbf{g}(\tau) = \frac{\hbar}{(2\pi)^2} \int d^3k' \mathbf{k}', \tag{C24}$$

that when simply integrated of course vanishes because of symmetry. This is another way of viewing the same seemingly paradoxical situation of Eq. (A4) versus Eq. (C12). The inconsistency originates in the improper integral. When the integral is modified by a cut-off factor a more clear picture emerges. If we multiply the integrand in Eq. (C23), that is equivalent to that in Eq. (26) by the cut-off factor $\exp(-\omega'/\omega_c)$ and integrate over k it can readily be seen that the integral vanishes [25]. If after this we take the limit $\omega_c \rightarrow \infty$ the integral is left at zero value. However if instead we multiply by a cut-off factor of the form $\exp(-\omega/\omega_c)$ the result is

$$\begin{aligned} \mathbf{g}(\tau) &= \frac{\hbar}{(2\pi)^3} \int \frac{d^3k}{k} k' \mathbf{k}' \exp\left(-\frac{\omega}{\omega_c}\right) \\ &= -\hat{x} \frac{2\hbar}{3\pi^2 c^4} \gamma_\tau^2 \beta_\tau (3! \omega_c^4), \end{aligned} \tag{C25}$$

that corresponds essentially to the integral in Eq. (26) but with the integration over $\omega^3 d\omega$ carried out between the integration limits 0 and the cut-off frequency ω_c .

There are clearly two different cases to consider. (i) For an observer at $(c^2/a, 0, 0)$ of I_* , at object proper time τ and corresponding I_* time t_* , the object appears to be moving with velocity $\hat{x} \beta_\tau c$ and with mechanical momentum $m \gamma_\tau \beta_\tau c \hat{x}$ (where m is the rest mass) and the ZPF looks spherically symmetric with

respect to the $k = 0$ point, i.e., the origin of the k -space corresponding to I_* . (ii) For an observer that at the same object proper time τ is instead located at the point $(c^2/a, 0, 0)$ of I_τ and thus instantaneously coincides and comoves with the object, the object is of course at rest with no momentum and no velocity and the ZPF looks spherically symmetric around the origin of the corresponding k' -space that is the k -space of I_τ .

So, for the I_* observer the ZPF *cannot* be spherically symmetrically distributed around the object. However, for the I_τ observer at object proper time τ the ZPF does appear to be spherically symmetrically distributed around the object. We emphasize that these two assertions, that of I_* and that of I_τ , refer to the object at the same proper time instant τ .

From the above discussion we extract the concept of the ZPF k -sphere of an inertial observer which is the sphere in k -space up to the cut-off radius $k_c = \omega_c/c$ around the origin at $k = 0$ of the k -space corresponding to the observer's rest inertial frame. Inertial observers automatically have a unique k -sphere which is the one corresponding to the inertial frame with respect to which they are at rest. The ZPF radiation for an inertial observer is the ZPF radiation contained in the inertial observer's k -sphere where ω_c is the cut-off associated with the ZPF spectral distribution. If no cut-off is considered for the ZPF then the sphere has infinite radius as we let the radius of the sphere in the corresponding k -space go to infinity, namely, $k_c = \omega_c/c \rightarrow \infty$.

We illustrate the concept of the ZPF k -sphere for the case of the analysis performed in Section V, in particular for Eq. (34) and related equations. At proper time $\tau = 0$ and corresponding I_* time $t_* = 0$, when the object instantaneously coincides with the observer stationed in I_* at the point $(c^2/a, 0, 0)$, this observer claims that the ZPF k -sphere is symmetrically distributed around the object. Therefore, when the momentum density is calculated under this condition in general it should vanish as also follows from Eq. (26) or Eq. (C21) when $\tau = 0$. In the same way the object momentum is viewed to be zero. This is however not the case after a lapse of proper time $\Delta\tau$ when the acceleration of the hyperbolic motion has taken the object to be moving with velocity $\beta_{\Delta\tau}c\hat{x}$ with respect to such an observer stationed in I_* . For the observer in I_* still of course the ZPF distribution remains the same, i.e., spherically symmetric around himself and with a cut-off at $\omega = \omega_c$. But as the object is moving with respect to this observer, at rest at the point $(c^2/a, 0, 0)$ of I_* , he would claim that the object is not located at the center of his k -sphere. So the observer at the point $(c^2/a, 0, 0)$ of I_* at proper time $\Delta\tau > 0$, should see two things: (i) the object moving with momentum $m_i\gamma_{\Delta\tau}\beta_{\Delta\tau}c\hat{x}$ and (ii) a net non-zero momentum density of ZPF radiation $\mathbf{g}(\Delta\tau)$ given by Eq. (26), with τ replaced by $\Delta\tau$, that is impinging on the object as argued in Sections III, IV and V, and in Appendix A.

In summary, every inertial observer has an associated k -sphere, and he is at the central point of that sphere. Hence the ZPF of his inertial frame is isotropically and homogeneously distributed around him. And this is true for all inertial observers. An entirely different question is how an inertial observer with his associated k -sphere assesses the situation for a moving point from the perspective of his inertial frame. The observer's ZPF is in general neither homogeneously nor isotropically distributed around the moving point because the moving point is located off-center in the observer's k -sphere.

APPENDIX D: COVARIANT APPROACH

The analysis of Sections IV and V and of Appendix B considered only the momentum density $\mathbf{g}$ (or equivalently the Poynting vector $\mathbf{N} = c^2 \mathbf{g}$) contribution to the electromagnetic momentum $\mathbf{p}$. There is however an additional contribution to the momentum that was neglected there. Here we calculate that additional contribution. The price we pay is the need to invoke a more abstract, not so directly intuitive, formalism. The advantage, besides theoretical generality, is, very importantly, a simpler expression and a more direct interpretation for the inertial mass m_i of Eq. (30) [26].

The standard expression for the four-momentum is

$$P^\mu = \left(\frac{U}{c}, \mathbf{p} \right). \quad (D1)$$

In mechanics, the space part is

$$\mathbf{p} = m\gamma\mathbf{v} = \gamma\mathbf{p}_N, \quad (D2)$$

where $\mathbf{p}_N$ is the standard newtonian momentum

$$\mathbf{p}_N = m\mathbf{v}, \quad (D3)$$

and m the rest mass. U corresponds to the kinetic energy

$$U = \gamma mc^2. \quad (D4)$$

In the case of an electromagnetic field of total energy U and momentum $\mathbf{p}$, the four-momentum (D1) can be expressed as an integral over the electromagnetic energy-momentum stress tensor [27]

$$P^\mu = \int \Theta^{\mu\nu} d\sigma_\nu, \quad (D5)$$

where $d\sigma^\nu$ is a component of a planar three-dimensional space-like hypersurface element $d^3\sigma$ such that

$$d\sigma^\nu = \eta^\nu d^3\sigma \quad (D6)$$

and

$$d^3\sigma = \eta^\nu d\sigma_\nu \quad (D7)$$

with η^ν the normal unit vector to the hyperplane. This unit four-vector η^ν is then timelike, and in the $(-+++)$ convention we have

$$\eta^\nu \eta_\nu = -1, \quad (D8)$$

and along the direction of the worldline trajectory of the object at the point event, so

$$\eta^\nu = (\gamma, \gamma\vec{\beta}) \quad (D9)$$

and $\vec{\beta} = \mathbf{v}/c$ is the vectorial velocity of the object in units of c . In our case the object is viewed from the laboratory inertial frame I_* , and at proper time τ it is instantaneously at rest in the inertial frame I_τ . We thus select η^ν as $(\gamma_\tau, \gamma_\tau \vec{\beta}_\tau)$, with $\vec{\beta}_\tau c$ the object velocity as measured in I_* , to be the unit normal vector to the hypersurface as viewed in I_* and then $d\sigma^\nu$ has both time and space components. In I_τ however, the object velocity at proper time τ is zero by definition and so $\eta^\nu = (1, 0, 0, 0)$ and thus

$$d\sigma^\nu = (d^3x', 0, 0, 0), \quad (D10)$$

where d^3x' is the volume element in the three-space of I_τ . It can easily be seen, when Eq. (D7) is applied to the case of I_τ in Eq. (D10), that

$$d^3\sigma = \eta^\nu d\sigma_\nu = d^3x' \quad (D11)$$

is an invariant hypersurface element equal to a 3-space volume element. Thus $d^3\sigma$ has the same value in I_* and in I_τ .

The space part of the energy-momentum stress tensor $\Theta^{\mu\nu}$ is the Maxwell stress-tensor $\mathbf{T}$ with symmetric matrix components T_{ij} ; $i, j = x, y, z$ of the form

$$T_{ij} = \frac{1}{4\pi} \left[E_i E_j + B_i B_j - \frac{1}{2} (E^2 + B^2) \delta_{ij} \right], \quad (D12)$$

and yielding for the symmetric $\Theta^{\mu\nu}$ the components

$$\begin{aligned} \Theta^{00} &= \frac{1}{8\pi} (E^2 + B^2) = U, \\ \Theta^{0i} &= \Theta^{i0} = \frac{1}{4\pi} (\mathbf{E} \times \mathbf{B})_i = cg_i, \\ \Theta^{ij} &= -T^{ij}. \end{aligned} \quad (D13)$$

The four-momentum (D5) becomes then

$$P^0 = \gamma \int \left(\frac{U}{c} - \mathbf{v} \cdot \mathbf{g} \right) d^3\sigma, \quad (D14a)$$

$$\mathbf{p} = \gamma \int \left(\mathbf{g} + \frac{\mathbf{T} \cdot \mathbf{v}}{c^2} \right) d^3\sigma. \quad (D14b)$$

Expression (D14b) gives us the clue of how to correct the noncovariant approaches of Sections IV and V and of Appendix B and make them fully covariant. We can do this either to the momentum-flux approach of Section IV and V or to the momentum-content approach of Appendix B. The second case, namely the momentum content approach of Appendix B is more in line with the particular interpretation that we would like to emphasize for the inertial mass of Eq. (30). So we go to the momentum density $\mathbf{g}_*$ and the momentum $\mathbf{p}_*$ of the electromagnetic radiation inside the volume of the object as viewed from I_* represented in Eqs. (B7) and (B8) of Appendix B. The momentum of Eq. (B8) was calculated noncovariantly and then directly from the momentum density of (B7) as equal to the I_* momentum density multiplied by the volume that the object has in I_* , namely V_* , mimicking the purely newtonian case, as for the $\mathbf{p}_N$ of Eq. (D2) and (D3) above. In order to obtain the proper space part of the four-momentum (Eqs. (D1) and (D2) above) we need to replace the momentum density expression $\mathbf{g}_*$ by the corrected expression: $\mathbf{g}_* + (\mathbf{T}_* \cdot \mathbf{v})/c^2$.

When we integrate over the volume of the object in Eq. (D14b) we use the invariant element d^3x' of Eq. (D11), i.e., the three-volume element of I_τ . As the object volume was assumed so small, the integrand was taken as constant in Eq. (B6), so just a multiplication by the object volume $V_* = \gamma V_0$ as seen in I_* was required. Here we can do the same. Eq. (D14b) yields then instead of Eq. (B6) the expression

$$\mathbf{p}_* = \gamma \left(\mathbf{g}_* + \frac{\mathbf{T}_* \cdot \mathbf{v}}{c^2} \right) V_0, \quad (D15)$$

where $\mathbf{T}$ signifies the ZPF Maxwell stress tensor (D12) after the stochastic averaging (as in Eq. (A5)) has been performed, i.e., more explicitly

$$T_{ij} = \frac{1}{4\pi} \left\langle E_i E_j + B_i B_j - \frac{1}{2} (E^2 + B^2) \delta_{ij} \right\rangle, \quad (D16)$$

with $i, j = x, y, z$. The product $\mathbf{T} \cdot \mathbf{v}$ with $\mathbf{v} = \hat{x}v$ gives the column vector

$$\mathbf{T}_* \cdot \mathbf{v} = \begin{bmatrix} T_{xx_*} v \\ T_{yx_*} v \\ T_{zx_*} v \end{bmatrix} = (\hat{x}T_{xx_*} + \hat{y}T_{yx_*} + \hat{z}T_{zx_*}) v. \quad (D17)$$

It is a simple matter to show that the y and z components vanish as must be expected on physical grounds. We first show that $T_{yx_*} = 0$.

$$T_{yx_*} = \frac{1}{4\pi} \langle E_{y*} E_{x*} + B_{y*} B_{x*} \rangle \quad (D18)$$

but

$$\begin{aligned} \langle E_{y*} E_{x*} \rangle &= \langle E_{x\tau} (\gamma_\tau [E_{y\tau} + \beta_\tau B_{z\tau}]) \rangle \\ &= \gamma_\tau \langle E_{x\tau} E_{y\tau} \rangle + \gamma_\tau \beta_\tau \langle E_{x\tau} B_{z\tau} \rangle \end{aligned} \quad (D19)$$

and $\langle E_{x\tau} E_{y\tau} \rangle$ involves the factor

$$\sum_{\lambda=1}^2 \hat{e}_x \hat{e}_y = -\hat{k}_x \hat{k}_y \quad (D20)$$

that vanishes upon angular integration. In analogous fashion $\langle E_{x\tau} B_{z\tau} \rangle$ involves the factor

$$\sum_{\lambda=1}^2 \hat{e}_x (\hat{k} \times \hat{e})_z = -\hat{k}_y \quad (D21)$$

that also vanishes when integrated over the angles. The derivation of T_{zx_*} goes in entirely symmetric fashion and of course also yields zero. Next we compute T_{xx_*} .

$$T_{xx_*} = \frac{1}{8\pi} \langle E_{x*}^2 + B_{x*}^2 - E_{y*}^2 - B_{y*}^2 - E_{z*}^2 - B_{z*}^2 \rangle, \quad (D22)$$

but

$$\begin{aligned} \langle E_{x*}^2 \rangle &= \langle E_{x\tau}^2 \rangle \\ \langle B_{x*}^2 \rangle &= \langle B_{x\tau}^2 \rangle \\ \langle E_{y*}^2 \rangle &= \langle \gamma_\tau (E_{y\tau} + \beta_\tau B_{z\tau}) \gamma_\tau (E_{y\tau} + \beta_\tau B_{z\tau}) \rangle \\ &= \gamma_\tau^2 \langle E_{y\tau}^2 \rangle + \gamma_\tau^2 \beta_\tau^2 \langle B_{z\tau}^2 \rangle + 2\gamma_\tau^2 \beta_\tau \langle E_{y\tau} B_{z\tau} \rangle, \end{aligned} \quad (D24)$$

and analogously it follows that

$$\langle B_{y*}^2 \rangle = \gamma_\tau^2 \langle B_{y\tau}^2 \rangle + \gamma_\tau^2 \beta_\tau^2 \langle E_{z\tau}^2 \rangle - 2\gamma_\tau^2 \beta_\tau \langle E_{z\tau} B_{y\tau} \rangle \quad (D25a)$$

$$\langle E_{z*}^2 \rangle = \gamma_\tau^2 \langle E_{z\tau}^2 \rangle + \gamma_\tau^2 \beta_\tau^2 \langle B_{y\tau}^2 \rangle - 2\gamma_\tau^2 \beta_\tau \langle E_{z\tau} B_{y\tau} \rangle \quad (D25b)$$

$$\langle B_{z*}^2 \rangle = \gamma_\tau^2 \langle B_{z\tau}^2 \rangle + \gamma_\tau^2 \beta_\tau^2 \langle E_{y\tau}^2 \rangle + 2\gamma_\tau^2 \beta_\tau \langle E_{y\tau} B_{z\tau} \rangle. \quad (D25c)$$

Furthermore

$$\langle E_{i\tau}^2 \rangle = \frac{1}{3} \langle E_\tau^2 \rangle = \frac{1}{3} \langle B_\tau^2 \rangle = \langle B_{i\tau}^2 \rangle, \quad (D26)$$

where $i = x, y, z$, and as

$$U = \frac{1}{8\pi} \langle E_\tau^2 + B_\tau^2 \rangle = \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega, \quad (D27)$$

we have that

$$\langle E_{i\tau}^2 \rangle = \langle B_{i\tau}^2 \rangle = \frac{4\pi}{3} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \quad (D28)$$

for $i = x, y, z$. Hence

$$\begin{aligned} \hat{x} \frac{T_{xx} v}{c^2} &= \hat{x} \frac{1}{c^2} c\beta_\tau \frac{1}{8\pi} \frac{4\pi}{3} (2 - 4\gamma_\tau^2 - 4\gamma_\tau^2 \beta_\tau^2) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \\ &\quad + \hat{x} \frac{1}{8\pi} \frac{1}{c^2} c\beta_\tau 2\gamma_\tau^2 \beta_\tau \langle E_{y\tau} B_{z\tau} - E_{z\tau} B_{y\tau} \rangle \\ &= -\hat{x} \frac{1}{c^2} \frac{1}{3} c\beta_\tau \gamma_\tau^2 \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega - \hat{x} \frac{1}{c^2} c\beta_\tau \gamma_\tau^2 \beta_\tau^2 \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \end{aligned} \quad (D29)$$

The triangular brackets term in the second equality vanishes because it is propotional to the ZPF Poynting vector of I_τ in the x -direction and the integrations, as clearly explained in Appendix B, should be carried over the k -sphere of I_τ . It is furthermore straightforward to show that each one of the summands $\langle E_{y\tau} B_{z\tau} \rangle$ and $\langle E_{z\tau} B_{y\tau} \rangle$ vanishes individually. So

$$\mathbf{g}_* + \frac{\mathbf{T}_* \cdot \mathbf{v}}{c^2} = \hat{x} \frac{1}{c^2} c\beta_\tau \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \quad (D30)$$

that results after a mutual cancellation of two factors of the form $\gamma_\tau^2(1 - \beta_\tau^2) = 1$. We then replace Eq. (D30) into (D15) and obtain

$$\begin{aligned} \mathbf{p}_* &= \hat{x} c\gamma_\tau \beta_\tau \frac{V_0}{c^2} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \\ &= \hat{x} \left(\frac{V_0}{c^2} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \right) c \sinh\left(\frac{a\tau}{c}\right). \end{aligned} \quad (D31)$$

The inertia reaction force of Eq. (B10) becomes now

$$\begin{aligned} \mathbf{f}_*^{zp} &= -\frac{d\mathbf{p}_*}{dt_*} \\ &= -\frac{1}{\gamma_\tau} \frac{d\mathbf{p}_*}{d\tau} \Big|_{\tau=0} \\ &= -\left(\frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \right) \mathbf{a} \\ &= -m_i \mathbf{a}, \end{aligned} \quad (D32)$$

where we have written $\mathbf{a} = \hat{x}a$, the acceleration that goes in the x -direction and have introduced again the radiation coupling factor $\eta(\omega)$. Observe that the $4/3$ factor obtained in Sec. V and in Appendix B becomes unity in the present case. The inertial mass m_i is the same as Eq. (30) [28].

The zero-component of the four-momentum we write as

$$cP^0 = \gamma_\tau \int (U - \mathbf{g} \cdot \mathbf{v}) d^3\sigma \rightarrow \gamma_\tau \left[\frac{\langle E_*^2 + B_*^2 \rangle}{8\pi} - c\beta_\tau \mathbf{g}_* \right] V_0. \quad (D33)$$

It is a simple matter to check that

$$\frac{\langle E_*^2 + B_*^2 \rangle}{8\pi} = \frac{1}{3} (1 + 2\gamma_\tau^2 + 2\gamma_\tau^2 \beta_\tau^2) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \quad (D34)$$

The $\mathbf{g}_*$ in Eq. (D33) is the same of (B5) that we write as

$$c\mathbf{g}_* = \frac{4}{3} \beta_\tau \gamma_\tau^2 \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \quad (D35)$$

From Eqs. (D33–D35) we easily obtain

$$cP^0 = \gamma_\tau V_0 \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega = m_i c^2 \gamma_\tau, \quad (D36)$$

which as expected is the energy of the interacting part of the ZPF radiation inside the volume of the object V_0 . From Eqs. (D36) and (D31) we recover the standard mechanical four-momentum expression for an object of rest mass m_i and four-velocity

$$v^\mu = (c\gamma_\tau, \mathbf{v}\gamma_\tau), \quad (D37)$$

viz,

$$P^\mu = m_i v^\mu = (m_i c\gamma_\tau, m_i \mathbf{v}\gamma_\tau). \quad (D38)$$

This is the same four-momentum that in Eq. (33) we wrote as $\mathcal{P}$.

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$$\mathbf{g}(\tau) = \hat{x} \frac{\hbar}{(2\pi)^2} \int \frac{d^3k}{k} k' \mathbf{k}' \exp\left(-\frac{ck'}{\omega_c}\right).$$

Integration over the azimuth and over the frequency $\omega = ck$ yields an expression proportional again to ω_c^4 and to an integral expression on the latitude angle. This last it can be shown identically vanishes. So, even if the limit $\omega_c \rightarrow \infty$ is taken at the end, the result is still zero.

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in the $\eta(\omega)$ response of the particle) then the obliteration of the $4/3$ factor becomes more a matter of theoretical preference.

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