Triangles

Pyramids have triangles as their sides and polygons as their bases. We know how to find the areas of triangles given the base and the height. But some triangles are given only the sides. How can we find the area of a triangle given only the sides?
For any triangle with base b and height h the area is

$$A = \frac{1}{2}bh.$$

How can we find the area of a triangle with sides a, b and c and no height given?

Solution: Use Heron's Formula. Given a, b, c, Find the area. First find s where s represents the semiperimeter and the formula follows from here.

$$s = \frac{1}{2}(a+b+c),$$

then

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Take for instance the triangle with sides 6, 8, 10 which is a right triangle and the area is 24 from the elementary formula. Using the elementary formula

$$A = \frac{1}{2}bh = \frac{1}{2}(6)(8) = \frac{1}{2}(48) = 24$$

Using Heron's Formula

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(6+8+10) = \frac{1}{2}(24) = 12$$

$$\begin{aligned} s &= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{12(12-6)(12-8)(12-10)} \\ &= \sqrt{12(6)(4)(2)} = \sqrt{2\cdot 2\cdot 3\cdot 2\cdot 3\cdot 2\cdot 2\cdot 2} \\ &= \sqrt{\underline{2\cdot 2}\cdot\underline{2\cdot 2}\cdot\underline{2\cdot 2}\cdot\underline{3\cdot 3}} = 2\cdot 2\cdot 2\cdot 3 = 24. \end{aligned}$$

Another example is the triangle with sides 5, 12, 13 another right triangle. Heron's formula works for any triangle.

$$A = \frac{1}{2}bh = \frac{1}{2}(5)(12) = \frac{1}{2}(60) = 30.$$

Using Heron's formula

$$\begin{aligned} s &= \frac{1}{2}(a+b+c) = \frac{1}{2}(5+12+13) = \frac{1}{2}(30) = 15 \\ A &= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15(15-5)(15-12)(15-13)} \\ &= \sqrt{15(10)(3)(2)} = \sqrt{3\cdot 5\cdot 2\cdot 5\cdot 3\cdot 2} \\ &= \sqrt{\underline{2\cdot 2}\cdot\underline{3\cdot 3}\cdot\underline{5\cdot 5}} = 2\cdot 3\cdot 5 = 30. \end{aligned}$$

A very nice result. This by no means is a proof of Heron's formula. You can search the world wide web to find one.

Try to find the area of the following triangles with the following sides. A calculator may be helpful.

(1) 5, 8, 11

(2) 6, 7, 8